SLAC–PUB–4842
February 1989
(T)

ANNIHILATION CONTRIBUTIONS IN A WILSON LOOP FORMALISM*

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ABSTRACT

The neglected annihilation contributions in the Wilson loop formalism of Eichten and Feinberg are defined and calculated. An annihilation time scale is introduced to discriminate the glueball contributions from true annihilation contributions to isoscalar-pseudoscalar meson states. In this formalism, it is found that there is no mixing between two quarkonium states with different quark masses through annihilation diagram. In a fixed state the annihilation contributions oscillate as functions of the annihilation time scale, and so provide a possible mechanism to shift the mass of η upward, instead of downward as in the QED case.

INTRODUCTION

It is well known that the masses of isoscalar-pseudoscalar mesons such as η and η' have large contributions from the annihilation of the constituent quark and anti-quark pair.[1] In a somewhat different point of view, this was known as the

Submitted to Physical Review D

* Work supported in part by the Department of Energy, contract DE–AC03–76SF00515, and by the Korea Science and Engineering Foundation and the Ministry of Education of Korea.

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U(1) problem[2] and has been resolved by the formalism of axial-vector anomaly.[3] Furthermore, Witten has argued that the annihilation diagram viewpoint and the anomaly viewpoint are not incompatible in terms of $1/N$ expansion. However, there is as yet no precise calculation of the annihilation contributions and mixing angles between isoscalar mesons. Only qualitative arguments[4] and crude approximation methods[5] are known to estimate the mixings, and one major problem to be overcome is the treatment of the quark and anti-quark bound state.

The description of bound states between a quark and an anti-quark pair cannot be done as in the QED case because the exact form of confining potential has not been obtained from first principles. In QCD, the static potential results from an infinite set of graphs through the interactions of full Yang-Mills couplings, and so we have to introduce an approximation method to treat these infinite graphs. One method is to calculate directly by computers on suitably chosen lattices. However, in this method, there exist restrictions on the number of lattice sites and only quenched approximation cases can be calculated.[6] Another approximation method to account for the confinement is to introduce a boundary which forms a bag containing quark, anti-quark, and gluons. In this bag model, the quarks move freely and relativistically, and are described by eigenmodes determined by the form and the size of the bag. The hadron spectra can be calculated and fitted fairly well to the observed values, but if we want to calculate diagrams, containing internal propagators such as annihilation diagrams we have to write down the propagators as sums over bag-model eigenmodes.[7] These propagators result not only in algebraic complexity but also in some ambiguity concerning the bound-state description connected with the boundary conditions imposed on the eigenmodes.

The other method to account for the confining potential is to assume an appropriate potential form; this method is known to be convenient for quantitative calculations of spectra and decay processes. The non-relativistic potential model calculations were first carried out for the heavy charmonium ($c\bar{c}$) system just after the J/ψ had been observed. Later, the same calculations were applied to the heavier ($b\bar{b}$) system and it was realized that the spin-dependences between the quark and the anti-quark should be accounted for a systematic description of quarkonium systems. There have been several attempts to derive the spin-dependent forces; however, in a theoretical viewpoint of quark confinement, the derivations of Eichten and Feinberg[8] have provided a clear basis for other calculations. They,used the Wilson loop formalism[9] and obtained the spin-dependent potentials up to order $1/m^2$ in a perturbative expansion with respect to the inverse quark masses and with respect to the spatial part of the gauge covariant derivative in the equation for the quark propagator. In this expansion, they neglected the annihilation contributions as small short-distance effects in higher order in the effective coupling constant α_s. The first-order results can be used to explain nearly all the meson masses except for several states including the lowest lying isoscalar-pseudoscalar mesons.[10] In order to explain the isoscalar-pseudoscalar meson masses it is apparent that we have to calculate the annihilation contributions; furthermore, the annihilation diagrams are known to be related to the mixings between states with the same quantum numbers. State mixings are also important in the study of non-$q\bar{q}$ states such as glueballs and exotic states.[11]

There have been several attempts to calculate the annihilation diagrams. As is well known, the two-photon annihilation diagrams in QED produce a negative energy shift[12] in contradiction to the case of η in QCD. To get a positive energy shift, Donoghue and Gomm replaced one physical gluon with one instantaneous Coulomb interaction in a bag model calculation,[7] and Jaronski and Long[13] argued that the mass shift becomes positive if the quark motion is highly relativistic. However, their calculations were carried out by assuming quasifree quarks and gluons, and so it is necessary to improve the treatment of bound states between a quark and an anti-quark.

In this paper, we will concentrate on the Wilson loop formalism of bound states and calculate the neglected annihilation contributions in the derivations of Eichten and Feinberg. General formalisms are presented in the next section, and calculations are carried out in the third section. The final section is devoted to discussions.

GENERAL FORMALISMS

The QCD Lagrangian for a system with quark of mass m is given by

$$L = \int \left[-\frac{1}{2} Tr \ (F_{\mu\nu} F^{\mu\nu}) + \bar{\Psi} \ (i\gamma^\mu \partial_\mu + g\gamma^\mu A_\mu - m) \ \Psi \right] d^3x \quad , \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$, and $A_\mu = A_\mu^a t^a$ with $\{t^a\}$ the representation matrices for the quarks in the fundamental representation of the gauge group SU(3). In order to derive the Wilson loop form of potential it is convenient to introduce the four-point function

$$I = \left\langle 0 | T^* \left(\bar{\Psi}(y_2) \bar{\Gamma}_b P \begin{bmatrix} y_2 \\ y_1 \end{bmatrix} \Psi(y_1) \right) \left(\bar{\Psi}(x_1) \Gamma_a P \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Psi(x_2) \right) |0 \right\rangle \quad , \tag{2}$$

in which T^* means time ordering, and Γ has the appropriate Dirac and flavor structure; the path-ordered exponential

$$P \begin{pmatrix} x \\ y \end{pmatrix} \equiv P \exp \left(ig \int_y^x dz_\mu A^\mu(z) \right) \tag{3}$$

is included to maintain gauge invariance. The four-point function can be reduced into two terms: one is the connected Wilson loop term, and the other is the annihilation contribution. Introducing the fermion propagation function S(x, y;A) by

$$(i\gamma^\mu \partial_\mu + g\gamma^\mu A_\mu - m) \ S(x, y; A) = \delta^4(x - y) \quad , \tag{4}$$

the I becomes

$$I = \left[Tr \left\{ S \left(x_2, y_2; -i\frac{\delta}{\delta J} \right) P \begin{bmatrix} y_2 \\ y_1 \end{bmatrix} \bar{\Gamma}_b S \left(y_1, x_1; -i\frac{\delta}{\delta J} \right) P \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Gamma_a \right\} \right.$$

$$\left. - Tr \left\{ S \left(y_1, y_2; -i\frac{\delta}{\delta J} \right) P \begin{bmatrix} y_2 \\ y_1 \end{bmatrix} \bar{\Gamma}_b \right\} Tr \left\{ S \left(x_2, x_1; -i\frac{\delta}{\delta J} \right) P \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Gamma_a \right\} \right] Z(J)|_{J=0}, \tag{5}$$

where $Z(J) = W(J)/W(0)$ and

$$W(J) = Det\left[S\left(-i\frac{\delta}{\delta J}\right)\right]\int[dA^{\mu}]\exp\left\{i\int d^4x\left[-\frac{1}{2}Tr(F_{\mu\nu}F^{\mu\nu}) + J^a_\mu A^\mu_a\right]\right\}. \tag{6}$$

The double trace term is the annihilation contribution, and since we are interested in the lowest order term we can neglect the fermion determinant factor $Det[S(-i\delta/\delta J)]$, which produces quark loops. The Wilson loop form can be obtained by inserting the non-relativistic propagation function S_0 into the first term of Eq. (5). Ignoring the spatial motion of the quark in Eq. (4), the S_0 satisfies the equation

$$\left(i\gamma^0\frac{\partial}{\partial t} + g\gamma^0 A_0 - m\right)S_0(x,y;A) = \delta^4(x-y) \quad , \tag{7}$$

and the explicit solution is given by

$$\begin{aligned} S_0(x,y;A^0) = &- i\theta(x^0-y^0)e^{-im(x^0-y^0)}\frac{1+\gamma^0}{2}P\begin{bmatrix}x^0\\y^0\end{bmatrix}\delta(\vec{x}-\vec{y}) \\ &- i\theta(y^0-x^0)e^{-im(y^0-x^0)}\frac{1-\gamma^0}{2}P\begin{bmatrix}x^0\\y^0\end{bmatrix}\delta(\vec{x}-\vec{y}) \quad . \end{aligned} \tag{8}$$

With this S_0, the first term of Eq. (5) becomes

$$\begin{aligned} |^{NR} = &-e^{-2imT}\delta(\vec{x_1}-\vec{y_1})\delta(\vec{x_2}-\vec{y_2})\ Tr\left(\frac{1+\gamma^0}{2}\ \bar{\Gamma}_b\ \frac{1-\gamma^0}{2}\Gamma_a\right) \\ &\times Tr\left(P\exp\left\{ig\oint_{C(R,T)} dz_\mu\left[-i\frac{\delta}{\delta J_\mu(z)}\right]\right\}\right)Z(J)|_{J=0} \quad , \end{aligned} \tag{9}$$

where $T = |x^0 - y^0|$, $R = |\vec{x_2} - \vec{x_1}|$, and C(R,T) is the path shown in Fig. 1, which forms the Wilson loop. The spin-dependent potentials can be obtained from the Eq. (5) by considering the relativistic corrections to the propagator S_0, which come from the spatial part of the gauge covariant derivative.

CALCULATIONS

The annihilation contributions come from the two diagrams in Fig. 2. We assume that the quark loops are in rectangular shapes, so that we can use the S_0 for the temporal side propagators and neglect time dependences on the propagators for the spatial sides. The solution of the naive spatial equation

$$(-\vec{\gamma}\cdot\vec{D}-m)\ S(\vec{x},\vec{y})=\delta^3(\vec{x}-\vec{y}) \tag{10}$$

is given by

$$S(\vec{x},\vec{y})=i\frac{m^{3/2}}{(2\pi)^{3/2}}\frac{\vec{r}\cdot\vec{\gamma}}{r^{3/2}}\ K_{-3/2}(mr)-\frac{m}{4\pi}\frac{e^{-mr}}{r}\quad , \tag{11}$$

with $\vec{r}=\vec{x}-\vec{y}$, $\vec{D}=-i\vec{\nabla}+g\vec{A}$, and K is the modified Bessel function. However, in order to estimate the annihilation contributions we need to compare them with the first term in Eq. (5) in which there appears the uncalculable Wilson loop integral as in the Eq. (9). To get a similar Wilson loop integral, we introduce another path-ordered exponential factor and obtain

$$S_s(x,y;A)=\left\{P\exp\left[-ig\int_{\vec{y}}^{\vec{x}}d\vec{z}\cdot\vec{A}(z)\right]\right\}\delta(x^0-y^0)S(\vec{x},\vec{y})\quad , \tag{12}$$

which satisfies

$$(-\vec{\gamma}\cdot\vec{D}-m)\ S_s(x,y;A)=\delta^4(x-y)\quad . \tag{13}$$

Now the propagator $S(x_2,x_1;A)$ can be written as

$$S(x_2,x_1;A)=-g^2\int S_0(x_2,x_3)\gamma^\mu A_\mu(x_3)S_s(\vec{x_3},\vec{x_4})\gamma^\nu A_\nu(x_4)S_0(x_4,x_1)d^4x_3d^4x_4, \tag{14}$$

and the annihilation term I_A becomes

$$
\begin{aligned}
&-Tr\left\{S\left(y_1,y_2;-i\frac{\delta}{\delta J}\right)P\begin{bmatrix}y_2\\ y_1\end{bmatrix}\bar{\Gamma}_b\right\}Tr\left\{S\left(x_2,x_1;-i\frac{\delta}{\delta J}\right)P\begin{bmatrix}x_1\\ x_2\end{bmatrix}\Gamma_a\right\}Z(J)|_{J=0}\\
&=-g^4\int d^4y_4d^4y_3d^4x_4d^4x_3Tr\left\{S_0(y_1,y_4)\gamma^\mu S_s(\vec{y_4},\vec{y_3})\gamma^\nu S_0(y_3,y_2)P\begin{bmatrix}y_2\\ y_1\end{bmatrix}\bar{\Gamma}_b\right\}\\
&\times Tr\left\{S_0(x_2,x_3)\gamma^\alpha S_s(x_3,x_4)\gamma^\beta S_0(x_4,x_1)P\begin{bmatrix}x_1\\ x_2\end{bmatrix}\Gamma_a\right\}\\
&\times\left\{\left(-i\frac{\delta}{\delta J^\mu(y_4)}\right)\left(-i\frac{\delta}{\delta J^\nu(y_3)}\right)\left(-i\frac{\delta}{\delta J^\alpha(x_3)}\right)\left(-i\frac{\delta}{\delta J^\beta(x_4)}\right)\right\}Z(J)|_{J=0}\quad .
\end{aligned}
\tag{15}
$$

By inserting the expressions of Eqs. (8) and (12) with different quark masses for the initial and final states, we get

$$
\begin{aligned}
I_A =&g^4\int d^4y_4d^4y_3d^4x_4d^4x_3[dA_\mu]e^{-im_2(y_1^0-y_4^0+y_2^0-y_3^0)-im_1(x_3^0-x_2^0+x_4^0-x_1^0)}\\
&\times\delta(\vec{y_1}-\vec{y_4})\delta(y_4^0-y_3^0)\delta(\vec{y_3}-\vec{y_2})\delta(\vec{x_2}-\vec{x_3})\delta(x_3^0-x_4^0)\delta(\vec{x_4}-\vec{x_1})\\
&\times Tr\left\{\frac{1+\gamma^0}{2}\gamma^\mu\left[\frac{im_2^{3/2}}{(2\pi)^{3/2}}\frac{\vec{r_y}\cdot\vec{\gamma}}{r_y^{3/2}}K_{-3/2}(m_2r_y)-\frac{m_2}{4\pi}\frac{e^{-m_2r_y}}{r_y}\right]\gamma^\nu\frac{1-\gamma^0}{2}\bar{\Gamma}_b\right.\\
&\qquad\qquad\left.\times P\exp\left(ig\oint_{C_y}dz_\mu A^\mu(z)\right)\right\}\\
&\times Tr\left\{\frac{1-\gamma^0}{2}\gamma^\alpha\left[\frac{im_1^{3/2}}{(2\pi)^{3/2}}\frac{\vec{r_x}\cdot\vec{\gamma}}{r_x^{3/2}}K_{-3/2}(m_1r_x)-\frac{m_1}{4\pi}\frac{e^{-m_1r_x}}{r_x}\right]\gamma^\beta\frac{1+\gamma^0}{2}\gamma_5\right\}\\
=&-\frac{2m_1^{3/2}}{(2\pi)^{3/2}}\frac{K_{-3/2}(m_1r_x)}{r_x^{3/2}}\epsilon_{\alpha\beta j}r_x^j.
\end{aligned}
\tag{19}
$$

Now I_A is given by

$$I_A = 4g^4 \frac{(m_1 m_2)^{3/2}}{(2\pi)^3} \int [dA_\mu] d^4y_4 d^4y_3 d^4x_4 d^4x_3 e^{-im_2(y_1^0 - y_4^0 + y_2^0 - y_3^0) - im_1(x_3^0 - x_2^0 + x_4^0 - x_1^0)}$$

$$\times \delta(\vec{y_1} - \vec{y_4}) \delta(y_4^0 - y_3^0) \delta(\vec{y_3} - \vec{y_2}) \delta(\vec{x_2} - \vec{x_3}) \delta(x_3^0 - x_4^0) \delta(\vec{x_4} - \vec{x_1})$$

$$\times \frac{K_{-3/2}(m_2 r_y)\ K_{-3/2}(m_1 r_x)}{(r_y r_x)^{\frac{3}{2}}} \epsilon_{\mu\nu i} r_y^i \epsilon_{\alpha\beta j} r_x^j$$

$$\times [D_{\mu\beta}(y_4, x_4) D_{\nu\alpha}(y_3, x_3) + D_{\mu\alpha}(y_4, x_3) D_{\nu\beta}(y_3, x_4)]$$

$$\times Tr \left\{ P \exp \left[ig \oint_{C_y} dz_\mu A^\mu(z) \right] \right\} Tr \left\{ P \exp \left[ig \oint_{C_x} dz_\mu A^\mu(z) \right] \right\} e^{iS_{YM}(A)} \quad , \tag{20}$$

where only spatial indices of $\mu,\ \nu,\ \alpha,\ \beta$ will survive. Since there is no temporal index, we can choose the temporal gauge to calculate the gluon propagators.

There are two cases we have to consider for the gluon propagator parts contracted with those of quarks; one with $r_x = r_y$, and the other with $r_x \neq r_y$. The first case is the annihilation contribution in a given state with the same initial and final quark mass and the second one is the contribution between different states which is responsible for state mixings. For the latter state mixing contributions we have in Eq. (20)

$$\epsilon_{\mu\nu i} r_y^i \epsilon_{\alpha\beta j} r_x^j \{ D_{\mu\beta}(y_4, x_4) D_{\nu\alpha}(y_3, x_3) + D_{\mu\alpha}(y_4, x_3) D_{\nu\beta}(y_3, x_4) \}$$

$$= \frac{\vec{r_x} \cdot \vec{r_y}}{8\pi^4} \left\{ \frac{1}{(y_4 - x_4)^2 - i\varepsilon} \cdot \frac{1}{(y_3 - x_3)^2 - i\varepsilon} - \frac{1}{(y_4 - x_3)^2 - i\varepsilon} \cdot \frac{1}{(y_3 - x_4)^2 - i\varepsilon} \right\}, \tag{21}$$

so that

$$
\begin{aligned}
I_{AM} = & \left(\frac{g^2}{4\pi}\right)^2 \frac{(m_1 m_2)^{3/2}}{\pi^5} e^{im_1(x_1^0+x_2^0)-im_2(y_1^0+y_2^0)} K_{-3/2}(m_1 r_x) K_{-3/2}(m_2 r_y) \frac{\vec{r_x}\cdot\vec{r_y}}{(r_x r_y)^{3/2}} \\
& \times \int dy_4^0 dx_4^0 e^{2i(m_2 y_4^0 - m_1 x_4^0)} \\
& \times \left[\frac{1}{(y_4^0-x_4^0)^2-(\vec{y_1}-\vec{x_1})^2-i\varepsilon} \cdot \frac{1}{(y_4^0-x_4^0)^2-(\vec{y_2}-\vec{x_2})^2-i\varepsilon} \right. \\
& \left. - \frac{1}{(y_4^0-x_4^0)^2-(\vec{y_1}-\vec{x_2})^2-i\varepsilon} \cdot \frac{1}{(y_4^0-x_4^0)^2-(\vec{y_2}-\vec{x_1})^2-i\varepsilon} \right] \\
& \times \int [dA_\mu] Tr \left\{ P \exp\left[ig \oint_{C_y} dz_\mu A^\mu(z) \right] \right\} Tr \left\{ P \exp\left[ig \oint_{C_x} dz_\mu A^\mu(z) \right] \right\} e^{iS_{YM}(A)}
\end{aligned}
\tag{22}
$$

by integrating out the delta functions. For simplicity, let's introduce the notation

$$
r_{ij} = |\vec{y_i} - \vec{x_j}| \quad , \tag{23}
$$

where i and j refer to 1 or 2. In the center-of-mass system we have

$$
r_{11} = r_{22} \quad , \qquad r_{12} = r_{21} \quad , \tag{24}
$$

and the time integrals become

$$
\begin{aligned}
I_t = \int dy_4^0 dx_4^0 e^{2i(m_2 y_4^0 - m_1 x_4^0)} & \left\{ \frac{1}{[(y_4^0-x_4^0)^2 - r_{11}^2 - i\varepsilon]^2} \right. \\
& \left. - \frac{1}{[(y_4^0-x_4^0)^2 - r_{12}^2 - i\varepsilon]^2} \right\} \quad .
\end{aligned}
\tag{25}
$$

Now we cannot freely integrate over the interval from 0 to T because of the uncalculable Wilson loop factors. In order to estimate the annihilation contribution, we have to find out a way to compare the Wilson loop integral in Eq. (9) with those in Eq. (22). In fact, the integral

$$W = \int [dA_\mu] Tr \left\{ P \exp \left[ig \oint_{C(R,T)} dz_\mu A^\mu(z) \right] \right\} e^{iS_{YM}(A)} \tag{26}$$

is related for large T limit to the static potential energy $\epsilon(R)$ as

$$\lim_{T\to\infty} \left\{ -\frac{1}{T} \ln W_E \right\} = \epsilon(R) \quad , \tag{27}$$

where E stands for Euclidean space. Since we cannot calculate the integral, we usually introduce an appropriate form of $\epsilon(R)$ to determine the radial excitations and the spin splittings of various spectra. Therefore, it is necessary to use an approximation method to compare the integral W with those in Eq. (22). When the spatial distances $r_x = |\vec{x_2} - \vec{x_1}|$ and $r_y = |\vec{y_2} - \vec{y_1}|$ have large differences, as shown in Fig. 3(a), we can assume that the energy difference between the two states is large, and the probability for mixing can be assumed to be suppressed.[14] If the annihilation time is not negligible with respect to the total time scale T, as in Fig. 3(b), we cannot estimate the difference between the W and those in Eq. (22), and we have to introduce another loop factor connected to the two gluons. The extra loop factor is related to the unknown glueball contributions which must be considered independently. Excluding the above two cases, we can assume that the annihilation contributions between different states of nearly the same spatial size come from the case in which the annihilation time interval is short enough to neglect the differences between the loop integral W over $C(R,T)$ and the products of the two integrals over C_x and C_y. The annihilation time interval is restricted by the above condition, and we assume that

$$-\Delta \leq y_4^0 - x_4^0 \leq \Delta \quad , \tag{28}$$

with Δ an appropriate parameter. Then the time integral I_t becomes

$$I_t \cong \int_0^T dx_4^0 e^{2i(m_2-m_1)x_4^0} \int_{-\Delta}^{\Delta} dt e^{2im_2 t} \left[\frac{1}{(t^2 - r_{11}^2 - i\varepsilon)^2} - \frac{1}{(t^2 - r_{12}^2 - i\varepsilon)^2}\right] \qquad (29)$$

neglecting some boundary effects, which is reasonable for $T \to \infty$. Assuming that Δ is independent of x_4^0, the calculated result is given by

$$I_t = \exp[i(m_2 - m_1)T] \frac{\sin[(m_2 - m_1)T]}{m_2 - m_1} f(r_{11}, r_{12}, \Delta) \quad , \qquad (30)$$

where

$$\begin{aligned} f(r_{11}, r_{12}, \Delta) = \frac{1}{2}\Bigg[\frac{1}{r_{11}^3}\Big(& \cos(2m_2 r_{11}) \; \{ci[2m_2(r_{11}+\Delta)] - ci[2m_2(r_{11}-\Delta)]\} \\ & + \sin(2m_2 r_{11}) \; \{si[2m_2(r_{11}+\Delta)] - si[2m_2(r_{11}-\Delta)]\}\Big) \\ - \frac{1}{r_{12}^3}\Big(& \cos(2m_2 r_{12}) \; \{ci[2m_2(r_{12}+\Delta)] - ci[2m_2(r_{12}-\Delta)]\} \\ & + \sin(2m_2 r_{12}) \; \{si[2m_2(r_{12}+\Delta)] - si[2m_2(r_{12}-\Delta)]\}\Big)\Bigg] \quad , \end{aligned} \qquad (31)$$

and

$$si(x) = -\int_x^\infty \frac{\sin t}{t} dt \quad , \qquad (32.a)$$

$$ci(x) = -\int_x^\infty \frac{\cos t}{t} dt \quad . \qquad (32.b)$$

Now the mixing annihilation contribution I_{AM} becomes

$$I_{AM} = \frac{(m_1 m_2)^{3/2}}{\pi^5}\alpha_s^2 \exp[-i(m_1+m_2)T]\ \frac{\sin[(m_2-m_1)T]}{m_2-m_1}$$
$$\times K_{-3/2}(m_2 r_y)K_{-3/2}(m_1 r_x)\frac{\vec{r_x}\cdot\vec{r_y}}{(r_x r_y)^{3/2}}f(r_{11},r_{12},\Delta)$$
$$\times \int [dA_\mu] Tr\left\{P\exp\left[ig\oint_{C_y} dz_\mu A^\mu(z)\right]\right\}Tr\left\{P\exp\left[ig\oint_{C_x} dz_\mu A^\mu(z)\right]\right\}e^{iS_{YM}(A)}, \tag{33}$$

with

$$r_x = r_{12} - r_{11} \quad , \qquad r_y = r_{12} + r_{11} \quad . \tag{34}$$

In order to estimate the I_{AM}, we need to write down the sum of I_{NR} and I_{AM} ;

$$\begin{aligned} I_{NR} + I_{AM} \cong\ & 2e^{-2im_1T}\delta(\vec{x_1}-\vec{y_1})\delta(\vec{x_2}-\vec{y_2})W \\ & + \frac{2}{3}\alpha_s^2\frac{(m_1m_2)^{3/2}}{\pi^5}\ \exp[-i(m_1+m_2)T]\ \frac{\sin[(m_2-m_1)T]}{m_2-m_1} \\ & \times K_{-3/2}(m_2r_y)K_{-3/2}(m_1r_x)\frac{\vec{r_x}\cdot\vec{r_y}}{(r_xr_y)^{3/2}}\ f(r_{11},r_{12},\Delta)W \quad , \end{aligned} \tag{35}$$

where the color factor is introduced to I_{AM}. By integrating out the two delta functions and taking the $T\to\infty$ limit, the new potential V' becomes

$$\begin{aligned} V' = \lim_{T\to\infty} & -\frac{1}{T}\ln e^{-\epsilon(r_x)T} \\ & \times\left\{1+\frac{1}{3}\alpha_s^2\frac{(m_1m_2)^{3/2}}{\pi^5}\exp[i(m_1-m_2)T]\ \frac{\sin[(m_2-m_1)T]}{m_2-m_1}\right. \\ & \left.\times\int d^3\vec{y_1}d^3\vec{y_2}K_{-3/2}(m_2r_y)K_{-3/2}(m_1r_x)\frac{\vec{r_x}\cdot\vec{r_y}}{(r_xr_y)^{3/2}}f(r_{11},r_{12},\Delta)\right\} \quad . \end{aligned} \tag{36}$$

If $m_1 \neq m_2$, and assuming that the corrected term is small with respect to unity, we have

$$V' = \epsilon(r_x) \quad , \tag{37}$$

so that there is no additional contribution. This is due to the fact that there is no linear T dependence in the second correction term. When $m_1 = m_2 = m$, V' is given by

$$V' = \epsilon(r_x) - \frac{1}{3}\alpha_s^2 \frac{m^3}{\pi^5} \int d^3\vec{y_1} d^3\vec{y_2} K_{-3/2}(mr_x) K_{-3/2}(mr_y) \frac{\vec{r_x}\cdot\vec{r_y}}{(r_x r_y)^{3/2}} f(r_{11}, r_{12}, \Delta), \tag{38}$$

where the integrations are to be carried out over suitable states with non-zero r_{11}.

Now we return to the case of the annihilation in a fixed state with the same initial and final quark mass. In this case, the first term in Eq. (21) corresponding to the first graph of Fig. 2 must be changed from that of $r_x \neq r_y$ case. When $r_x = r_y$, the two gluons of the first graph of Fig. 2 propagate only along the time axis, so that there are no contributions from physical transverse degrees of freedom. For longitudinal gluon propagator, there exist several suggestions to write down the form in space-time coordinates. In temporal gauge, Frenkel[15] has suggested that the longitudinal propagator is given by

$$iD_L^{ij,ab}(x,y) = -\frac{i}{2}\delta^{ab}\epsilon(x^0 - y^0)(x^0 - y^0)\frac{\partial^i\partial^j}{\nabla^2}\delta(\vec{x} - \vec{y}) \quad . \tag{39}$$

Later studies of the boundary effects[16] resulted in an additional term and the propagator was changed into the form[17]

$$iD_L^{ij,ab}(x,y) = -\frac{i}{2}\delta^{ab}\left[\epsilon(x^0 - y^0)(x^0 - y^0) + (x^0 + y^0)\right]\frac{\partial^i\partial^j}{\nabla^2}\delta(\vec{x} - \vec{y}) \quad . \tag{40}$$

If we use the propagator in Eq. (40), the first term in Eq. (21) becomes

$$\epsilon_{\mu\nu i} r_y^i \epsilon_{\alpha\beta j} r_x^j D_{\mu\beta}(y_4, x_4) D_{\nu\alpha}(y_3, x_3) = -\frac{2}{9}\vec{r_x}\cdot\vec{r_y} y_4^0 y_3^0 \delta(\vec{y_4} - \vec{x_4})\delta(\vec{y_3} - \vec{x_3}) \quad , \tag{41}$$

when $y_4^0 > x_4^0$, and $y_3^0 > x_3^0$, so that I_A becomes for $m_1 = m_2 = m$, and $r_x = r_y = r$

$$
\begin{aligned}
I_A = & \frac{8m^3}{\pi}\alpha_s^2 e^{-im(y_1^0+y_2^0-x_1^0-x_2^0)} \times \frac{1}{r}[K_{-\frac{3}{2}}(mr)]^2 \\
& \times \int dy_4^0 dx_4^0 e^{2im(y_4^0-x_4^0)} \left\{ \frac{2}{9}(y_4^0)^2 \delta(\vec{y_1} - \vec{x_1})\delta(\vec{y_2} - \vec{x_2}) \right. \\
& \left. + \frac{1}{8\pi^4}\frac{1}{[(y_4^0 - x_4^0)^2 - r^2 - i\varepsilon]^2} \right\} \\
& \times \int [dA_\mu] Tr\left[P\exp\left(ig\oint_{C_y} dz_\mu A^\mu(z)\right)\right] Tr\left[P\exp\left(ig\oint_{C_x} dz_\mu A^\mu(z)\right)\right] e^{iS_{YM}(A)},
\end{aligned}
\tag{42}
$$

where $(y_4^0)^2$ in the first term of time integral must be changed into $(x_4^0)^2$ when $y_4^0 < x_4^0$, and $y_3^0 < x_3^0$. The first part of the time integral becomes for the interval given by Eq. (28)

$$
\begin{aligned}
& -\frac{2}{9}\delta(\vec{y_1} - \vec{x_1})\delta(\vec{y_2} - \vec{x_2})\left\{ \int_0^{T-\Delta} dx_4^0 \int_{x_4^0}^{x_4^0+\Delta} dy_4^0 e^{2im(y_4^0-x_4^0)}(y_4^0)^2 \right. \\
& \left. + \int_\Delta^T dx_4^0 \int_{x_4^0-\Delta}^{x_4^0} dy_4^0 e^{2im(y_4^0-x_4^0)}(x_4^0)^2 \right\} \\
& \cong -\frac{2}{9}\delta(\vec{y_1} - \vec{x_1})\delta(\vec{y_2} - \vec{x_2})\frac{T^3}{3m}\sin 2m\Delta \quad ,
\end{aligned}
\tag{43}
$$

as $T \to \infty$. If we take the limit as in Eq. (36), the modified potential diverges. It is apparent that the divergence comes from the boundary terms of Eq. (40), so it seems better to use the form of Eq. (39). In fact, the annihilation contributions are related only to gluon propagators which are separated from the time boundaries.

If we use the propagator in Eq. (39), we have

$$
\begin{aligned}
&\epsilon_{\mu\nu i} r_y^i \epsilon_{\alpha\beta j} r_x^j D_{\mu\beta}(y_4, x_4) D_{\nu\alpha}(y_3, x_3) \\
&= -\frac{\vec{r_x}\cdot\vec{r_y}}{18}(y_4^0 - x_4^0)(y_3^0 - x_3^0)\delta(\vec{y_4}-\vec{x_4})\delta(\vec{y_3}-\vec{x_3}) \quad ,
\end{aligned} \tag{44}
$$

so that the time integral for this term becomes

$$
\begin{aligned}
&-\frac{1}{18}\left\{\int_0^{T-\Delta} dx_4^0 \int_{x_4^0}^{x_4^0+\Delta} dy_4^0 + \int_{\Delta}^{T} dx_4^0 \int_{x_4^0-\Delta}^{x_4^0} dy_4^0\right\} e^{2im(y_4^0-x_4^0)}(y_4^0-x_4^0)^2 \\
&\times \delta(\vec{y_1}-\vec{x_1})\delta(\vec{y_2}-\vec{x_2}) \\
&= -\frac{T-\Delta}{18}\left(\frac{\Delta^2}{m}\sin 2m\Delta + \frac{\Delta}{m^2}\cos 2m\Delta - \frac{1}{2m^3}\sin 2m\Delta\right)\delta(\vec{y_1}-\vec{x_1})\delta(\vec{y_2}-\vec{x_2}).
\end{aligned} \tag{45}
$$

In the $T \to \infty$ limit, I_A becomes

$$
\begin{aligned}
I_A \cong& \frac{2}{3}\cdot\frac{8m^3}{\pi}\alpha_s^2 e^{-2imT} \times \frac{1}{r}[K_{-\frac{3}{2}}(mr)]^2 W \\
&\times \left[\frac{T}{18}\left(\frac{\Delta^2}{m}\sin 2m\Delta + \frac{\Delta}{m^2}\cos 2m\Delta - \frac{1}{2m^3}\sin 2m\Delta\right)\delta(\vec{y_1}-\vec{x_1})\delta(\vec{y_2}-\vec{x_2})\right. \\
&+ \frac{T}{2r^3}\left(\cos 2mr\left\{ci\left[2m(r+\Delta)\right] - ci\left[2m(r-\Delta)\right]\right\}\right. \\
&\left.\left. + \sin 2mr\left\{si\left[2m(r+\Delta)\right] - si\left[2m(r-\Delta)\right]\right\}\right)\right] \quad ,
\end{aligned} \tag{46}
$$

where color factor of 2/3 is included. By comparing I_A with I_{NR} as in Eq. (35), we have the new potential V' as

$$V' = \lim_{T\to\infty} -\frac{1}{T} \ln e^{-\epsilon(r)T} \Bigg\{ 1 + \frac{4m^3}{3\pi}\alpha_s^2 T$$

$$\times \left[\frac{1}{9r}(K_{-\frac{3}{2}}(mr))^2 \left(\frac{\Delta^2}{m} \sin 2m\Delta + \frac{\Delta}{m^2} \cos 2m\Delta - \frac{1}{2m^3} \sin 2m\Delta \right) \right.$$

$$+ \int d^3\vec{y_1} d^3\vec{y_2} \frac{1}{r^4} \left(K_{-\frac{3}{2}}(mr) \right)^2 \Big(\cos 2mr\{ci[2m(r+\Delta)] - ci[2m(r-\Delta)]\}$$

$$\left. + \sin 2mr\{si[2m(r+\Delta)] - si[2m(r-\Delta)]\} \Big) \right]$$

$$= \epsilon(r) - \frac{4m^3}{3\pi}\alpha_s^2 \left[\frac{1}{9r}[K_{-\frac{3}{2}}(mr)]^2 \left(\frac{\Delta^2}{m} \sin 2m\Delta + \frac{\Delta}{m^2} \cos 2m\Delta - \frac{1}{2m^3} \sin 2m\Delta \right) \right.$$

$$+ \int d^3\vec{y_1} d^3\vec{y_2} \frac{1}{r^4} [K_{-\frac{3}{2}}(mr)]^2 \Bigg(\cos 2mr\{ci[2m(r+\Delta)] - ci[2m(r-\Delta)]\}$$

$$\left. + \sin 2mr\{si[2m(r+\Delta)] - si[2m(r-\Delta)]\} \Bigg) \right] \quad . \tag{47}$$

The integrations are to be carried out with fixed r, so that they represent the summation over various angular states. In order to estimate the annihilation contribution in a given state we must assume an appropriate form of $\epsilon(r)$, and then calculate the spectra with the corrected terms included. The correction terms are dependent on the r, quark mass m, and the time interval parameter Δ, and show oscillatory behavior with respect to these variables. The oscillatory behavior provides a possible mechanism for the positive mass shift of η.

DISCUSSIONS

In this paper, we defined the annihilation graphs as those with free gluon propagators without any associated Wilson loop factor. If we introduce additional

Wilson loop factor to the intermediate gluons, the contribution becomes a mixing between quarkonium states through glueball states. Since we cannot solve the bound state problems of glueballs, it is better to distinguish the pure annihilation graphs from those of glueballs. Of course, it is an important problem to calculate the glueball contributions.

Besides the glueball problem, there remain several points to be resolved further. First, we must determine the annihilation time scale Δ in order to calculate explicit values to be compared with experimental data. It seems arbitrary to introduce the parameter Δ; however, it is essential in the Wilson loop description of bound states to consider the annihilation time scale. Second, there is no mixing between states with different quark masses in our definition of annihilation graphs. The well-known example of mixing between η and η' must be treated only through glueball contributions. Another point concerns the form of the longitudinal gluon propagator. The introduction of a boundary term in Eq. (40) resulted in a divergence in our formalism as $T \to \infty$. Further studies of the forms of the gluon propagator in various gauges are necessary to confirm our results.

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FIGURE CAPTIONS

Fig. 1. Wilson loop with temporal length T and spatial length R.

Fig. 2. Annihilation diagrams in a Wilson loop formalism.

Fig. 3. Diagrams excluded by the definition of annihilation contributions.

(a) Diagram with large difference between r_x and r_y.

(b) Diagram with physical glueball contributions.

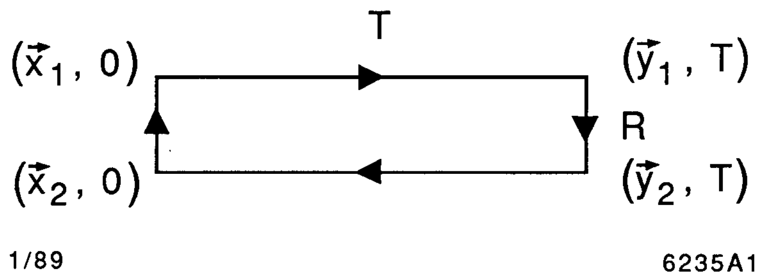

Fig. 1

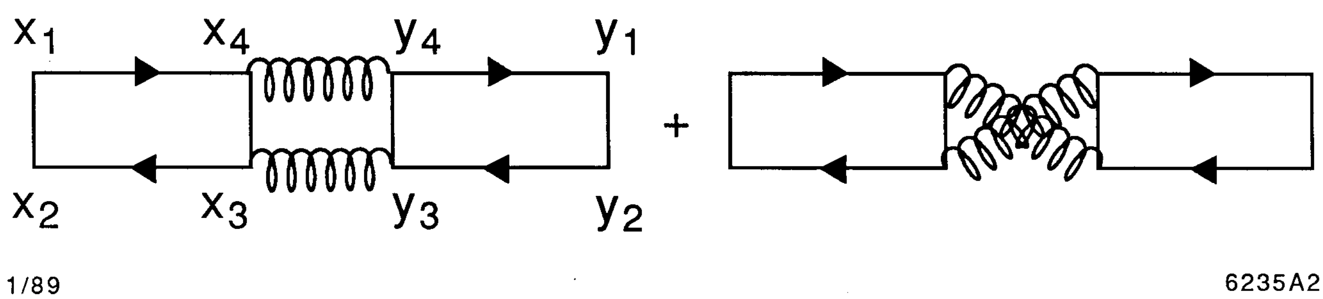

Fig. 2

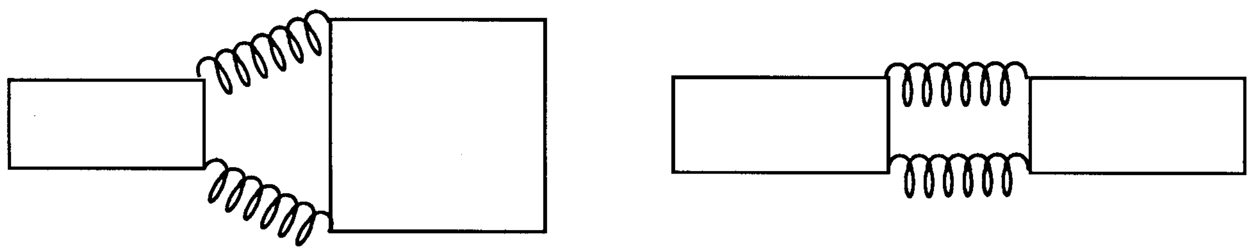

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Fig. 3