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Strong Interaction Corrections to the Decay $K \to \pi\nu\bar{\nu}$ for Large m_t ⋆

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ABSTRACT

We calculate the strong interaction corrections to the decay $K \to \pi\nu\bar{\nu}$ when m_t is large. The branching ratios for $K^\pm \to \pi^\pm\nu\bar{\nu}$ and $K_L^0 \to \pi^0\nu\bar{\nu}$ and their sensitivity to these corrections are discussed.

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One of the paths for searching for physics beyond the Standard Model is by probing low energy processes which are sensitive to the effects of high mass, virtual particles. This inspires much of the present round of rare K decay experiments. Of particular interest are processes that are forbidden in lowest order, such as neutral current processes which change quark flavors, but which can occur through one-loop Feynman diagrams.

Decays such as $K_L \to \pi^0 \ell^+ \ell^-$ and $K \to \pi \nu \bar{\nu}$ are of this type. The change in quark flavor, from an s (in the K) to a d (in the π), occurs in the Standard Model through diagrams involving one or more loops. There is a large window between the present experimental limits on the branching ratios for these processes and the corresponding standard model predictions. Within that window there is the possibility of a branching ratio arising due to new high mass particles in the loop. Even if these processes are finally observed at roughly the expected level, they provide information on the parameters of the Standard Model and, in the case of $K \to \pi \nu \bar{\nu}$, a rate which depends on the number of light neutrino species.

We have recently re-evaluated the Quantum Chromodynamic (QCD) corrections to the short-distance amplitude for the process $K_L \to \pi^0 \ell^+ \ell^-$ in the light of the present situation, where the mass of the t quark is comparable to, or greater than, the mass of the W boson.[1] In Standard Model predictions for $K \to \pi \nu \bar{\nu}$, QCD corrections are often neglected or, to the same end result, stated to be small.[2] When included, they are sometimes treated as an overall multiplicative factor for the whole amplitude, even though it arises from a sum of pieces due to c and t quarks in the loop. An exception is the work of Ellis and Hagelin[3] where QCD corrected top-quark contributions are given in the case where the mass of the top quark in the loop is much smaller or comparable to that of the W.

In this paper we re-evaluate the QCD corrections to the short-distance amplitude for $K \to \pi \nu \bar{\nu}$ when $m_t \sim M_W$. We give an analytic form for the corrections to the leading logarithmic pieces and discuss the ambiguities in non-leading terms. Quantitatively, the rate for $K^+ \to \pi^+ \nu \bar{\nu}$ is decreased by 15 to 30%, a result which is in fact numerically similar to that[4] of applying Ref. 3, even though the detailed expressions are different.

Feynman diagrams for the process $K \to \pi \nu_\ell \bar{\nu}_\ell$ are shown in Figure 1. They are similar to the diagrams for the short-distance contributions to the process $K \to \pi \ell^+ \ell^-$, except for the absence of the "electromagnetic penguin" and the appearance of different lepton lines. The latter induces a dependence on the mass of the charged lepton in the loop and a different weighting of the diagrams.

At a hadronic scale μ below the charm mass and appropriate for K decays, we write an effective Hamiltonian for $\Delta S = 1$ processes[5]

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_i C_i(\mu^2) Q_i \ + \ h.\ c.\ , \tag{1}$$

where the effective four-quark operators Q_1 to Q_6 are the same as in Ref. 5. The $V - A$ character of the gauge boson coupling to neutrinos allows only the operator

$$Q_\nu = \frac{e^2}{4\pi} (\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha)(\bar{\nu}_\ell \gamma^\mu (1 - \gamma_5) \nu_\ell) \tag{2}$$

to appear to lowest order in electroweak interactions to represent the short-distance contributions to $K \to \pi \nu \bar{\nu}$ in the summation.

At a scale M_W, before QCD corrections are introduced, the coefficient C_ν receives a contribution involving a quark $q = u, c, t$ from both the "Z penguin" and the "Box" diagrams. When the coefficient is chosen to match the free quark result,[6] one obtains:[7]

$$\widetilde{C}_{\nu,q}^{(Z)}(M_W^2) = \frac{1}{\sin^2 \theta_W} \frac{x_q}{16\pi} \left[\frac{(x_q - 6)(x_q - 1) + (3x_q + 2) ln(x_q)}{(x_q - 1)^2} \right] , \tag{3a}$$

$$\widetilde{C}_{\nu,q}^{(Box)}(M_W^2) = \frac{1}{\sin^2 \theta_W} \frac{x_q}{2\pi} \left[\frac{x_q - 1 - ln(x_q)}{(x_q - 1)^2} \right] , \tag{3b}$$

where $x_q = m_q^2 / M_W^2$. The tilde over the coefficient indicates that the Kobayashi-

Maskawa (KM) factor has been removed from it:

$$C_\nu(M_W^2) = \sum_{q=u,c,t} \frac{V_{qs}^* V_{qd}}{V_{us}^* V_{ud}} \, \widetilde{C}_{\nu,q}(M_W^2) \, . \tag{4}$$

For small x_q, the leading terms in Eqs. (3a) and (3b) come from the logarithm, with its coefficient in $\widetilde{C}_{\nu,q}^{(Box)}$ being four times larger and of opposite sign to that in $\widetilde{C}_{\nu,q}^{(Z)}$. The contribution of the u quark is neglected (because of an overall factor of x_u), leaving c and t quark contributions which are comparable in the amplitude for $K^\pm \to \pi^\pm \nu\bar{\nu}$ when the respective Kobayashi-Maskawa factors are included.

The leading logarithmic QCD corrections are applied to the term proportional to $ln(x_q)$, which arises here from an integration from the scale m_q^2 to that of the weak interaction, M_W^2. With the introduction of QCD, the integrand gets an additional scale dependence reflecting those of the four-fermion interaction and the running of masses, so that[8]

$$\int_{m_q^2}^{M_W^2} \frac{dq^2}{q^2} \xrightarrow[QCD]{} \int_{m_q^2}^{M_W^2} \frac{dq^2}{q^2} C(q^2) = \eta_q \int_{m_q^2}^{M_W^2} \frac{dq^2}{q^2} \, .$$

Since by assumption the t quark has a mass comparable to the W, its contribution has no large logarithms and in addition comes from a region where α_s is small. Therefore the QCD corrections to $\widetilde{C}_{\nu,t}$ are neglected. Consequently, the only significant QCD corrections of interest here are those to the charm contribution. Following Ref. 8 and breaking up the region of integration into segments that correspond to a given number of operative quark flavors, or equivalently, following the general renormalization group discussion of Ref. 5, we find for the "Box"

$$\eta_c^{(Box)} = \left(\frac{12\pi}{ln(M_W^2/m_c^2)}\right)\left(\frac{\left(K_{c/b}^{1/25} - 1\right)}{\alpha_s(m_c^2)} + \frac{K_{c/b}^{-24/25}\left(1 - K_{b/W}^{-1/23}\right)}{\alpha_s(m_b^2)}\right) \, , \tag{5}$$

with $K_{b/W} = \alpha_s(m_b^2)/\alpha_s(M_W^2)$ and $K_{c/b} = \alpha_s(m_c^2)/\alpha_s(m_b^2)$ in effective five and four quark theories, respectively.

For the "Z penguin" the corresponding QCD correction factor is:

$$\eta_c^{(Z)} = \left(\frac{12\pi}{ln(M_W^2/m_c^2)}\right) \times$$
$$\left(\left[\frac{2}{7} K_{b/W}^{-6/23} K_{c/b}^{-6/25} \frac{(K_{c/b}^{7/25} - 1)}{\alpha_s(m_c^2)} - \frac{1}{11} K_{b/W}^{12/23} K_{c/b}^{12/25} \frac{(1 - K_{c/b}^{-11/25})}{\alpha_s(m_c^2)}\right] \right. \quad (6)$$
$$\left. + K_{c/b}^{-24/25} \left[\frac{2}{5} K_{b/W}^{-6/23} \frac{(K_{b/W}^{5/23} - 1)}{\alpha_s(m_b^2)} - \frac{1}{13} K_{b/W}^{12/23} \frac{(1 - K_{b/W}^{-13/23})}{\alpha_s(m_b^2)}\right] \right) .$$

Numerical values of the charm contribution, before and after QCD corrections, can be found for various values of Λ_{QCD} in the Table. The values there correspond to $\eta_c^{(Box)} = 0.61$ and $\eta_c^{(Z)} = 0.31$ when $\Lambda_{QCD} = 150$ MeV. Especially for the "Z penguin," the QCD corrections are large. However, since the leading logarithm, $ln(M_W^2/m_c^2)$, enters the amplitude for $K \to \pi\nu\bar{\nu}$ in the ratio of 4 to -1 (of "Box" to "Z penguin"), the effective QCD correction factor to the leading logarithm in the overall charm contribution is [4(0.61) - 1(0.31)]/[4 - 1] = 0.71 .

There remains the question of how to treat the QCD corrections to the non-leading terms. In general, the coefficients $\widetilde{C}_{\nu,q}$ may contain different (non-leading) renormalization-scheme dependent terms of the form: $(constant) \times x_q$. Being a physical quantity, the net amplitude can not change in going from one scheme to another, as there are compensating changes in the matrix elements of the other operators. Without a higher order QCD calculation of the anomalous dimensions and the matrix elements, a scheme dependence remains in the QCD corrections to the non-leading terms in the coefficients.

If we take the non-leading terms in the charm contribution from Eqs. (3), then it doesn't make much difference what is done as far as QCD corrections to them. The next-to-leading terms are in the ratio of -4 to +3 and cancel against each other, as can be seen by comparing the (no QCD) leading logarithm portion with the full contribution of charm in the Table. As QCD corrections reduce the coefficient of the leading logarithm, the non-leading terms become relatively more important if no correction is applied to them. Even in this case, there is only a 10% difference in the total charm contribution (compare the sixth and third row of Table 1) if the non-leading terms are included, although the effects are very much bigger in the component pieces, especially $\widetilde{C}_{\nu,c}^{(Z)}$. Applying QCD corrections characteristic of the scale m_c to these next-to-leading terms, in the spirit of Ref. 8, reduces their magnitude and makes them even less significant (row seven of the Table). The lesson is that there is a sizable difference in the charm contribution due to QCD corrections to the leading logarithm, but only small differences induced from changing the value of Λ_{QCD} or from handling the QCD corrections to the non-leading terms in different ways.

With the QCD corrections in hand, we can apply them to the amplitudes for the processes of interest. The branching ratio (per neutrino flavor) for $K^{\pm} \to \pi^{\pm}\nu_\ell\bar{\nu}_\ell$ can be related to that for K_{e3} decay to yield,

$$\begin{aligned} B(K^{\pm} \to \pi^{\pm}\nu_\ell\bar{\nu}_\ell) &= 2\,|V_{ud}|^2\,\alpha^2\,|C_\nu|^2\,B(K^+ \to \pi^0 e^+ \nu_e) \\ &= 5.1 \times 10^{-6}\,|V_{ud}|^2\,|C_\nu|^2 \end{aligned} , \tag{7}$$

where

$$C_\nu \approx -\widetilde{C}_{\nu,c} + \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}\,\widetilde{C}_{\nu,t}\ . \tag{8}$$

We have performed a full numerical search over Kobayashi-Maskawa parameter space to obtain maximum and minimum values of the branching ratio as a function of m_t, which is shown in Figure 2. We allow the magnitude of the Kobayashi-Maskawa matrix elements to lie in the ranges directly allowed by experiment.[9] In addition we impose the constraints of requiring that the short-distance (box diagram) contribution to the respective mass matrices account for ϵ in the K^0 system and mixing in the B_d^0 system.[10] Our results without QCD corrections (dashed curve) are very close to those of Nir.[11]

An upper limit[11] on the rate occurs when m_c is as large as allowed (1.7 GeV here), and we replace $V_{ts}^* V_{td}/V_{us}^* V_{ud}$ in Eq. (8) by minus its maximum magnitude, allowing complete constructive interference between the charm and top contributions. Unitarity of the KM matrix gives $|V_{td}| < 0.024$, while for $m_t \geq 120$ GeV, a more stringent upper limit on $|V_{td}|$ occurs from $B_d - \bar{B}_d$ mixing. The upper bound on the rate so derived holds for three generations of quarks irrespective of whether CP violation arises from the Kobayashi-Maskawa matrix. In fact, adding the ϵ constraint lowers the maximum rate by at most a few percent. On the other hand the minimum of $B(K^{\pm} \to \pi^{\pm}\nu_\ell\bar{\nu}_\ell)$ for a given m_t occurs both when m_c is as small as allowed (1.3 GeV here) and the potentially constructive interference between the charm and top contributions tends to be as small as possible.

When one compares to the branching ratio with QCD corrections (solid curve), there is a decrease in the minimum by $\approx 30\%$. The maximum, on the other hand, decreases by $\approx 25\%$ for smaller m_t and $\approx 15\%$ for larger values. Although the detailed formulas are different, this is numerically similar to the results of Refs. 3 and 4. From the preceding discussion this is to be expected in that, even though we take account of the change in operative quark flavors at the b-scale and m_t being comparable to M_W, the basic physics is the same and the magnitude of the QCD corrections is not sensitive to the details of the running of α_s.[12]

The decay $K_L^0 \to \pi^0\nu_\ell\bar{\nu}_\ell$ is CP violating and is of current interest.[13] Its branching ratio (per neutrino flavor) can again be related to that for K_{e3} decay:

$$B(K_L^0 \to \pi^0\nu_\ell\bar{\nu}_\ell) = 2.1 \times 10^{-5}\ |V_{ud}|^2\ |(\epsilon - i\xi)Re\ C_\nu + iIm\ C_\nu|^2\ . \qquad (9)$$

The term proportional to $Re\ C_\nu$ gives a negligible contribution and

$$Im\ C_\nu = Im\ \left(\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}\right)\left(\widetilde{C}_{\nu,t} - \widetilde{C}_{\nu,c}\right)\ , \qquad (10)$$

with the rephase invariant quantity $Im\ \left(V_{ts}^* V_{td}/V_{us}^* V_{ud}\right) \approx s_2 s_3 s_\delta$ in the original parametrization of Kobayashi and Maskawa.[14] Therefore

$$B(K_L^0 \to \pi^0\nu_\ell\bar{\nu}_\ell) \approx 2.1 \times 10^{-5}\left(s_2 s_3 s_\delta\right)^2\ |\widetilde{C}_{\nu,t} - \widetilde{C}_{\nu,c}|^2\ . \qquad (11)$$

The quantity $|\widetilde{C}_{\nu,t} - \widetilde{C}_{\nu,c}|^2$ is completely dominated by the top contribution and is shown in Figure 3. As $s_2 s_3 s_\delta$ is of order 10^{-3}, the branching ratio with three generations of neutrinos is of order 10^{-11}. The QCD corrections to the t quark contribution should be small, making this theoretically an ideal decay in which to study CP violation in the decay amplitude, although experimentally the problems are very formidable.[13]

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Table

The coefficient $\widetilde{C}_{\nu,c}$ for $m_c = 1.5$ GeV (Units of 10^{-4})

	$\widetilde{C}^{(Z)}_{\nu,c}$	$\widetilde{C}^{(Box)}_{\nu,c}$	$\widetilde{C}_{\nu,c}$
Leading Log Only			
No QCD	-4.7	18.9	14.2
$\Lambda_{QCD} = 100$ MeV	-1.7	12.1	10.4
$\Lambda_{QCD} = 150$ MeV	-1.5	11.5	10.0
$\Lambda_{QCD} = 250$ MeV	-1.1	10.5	9.4
Full Contribution			
No QCD	-3.0	16.6	13.6
QCD applied to leading log only ($\Lambda_{QCD} = 150$ MeV)	0.3	9.1	9.4
QCD applied to leading and non-leading terms ($\Lambda_{QCD} = 150$ MeV)	-1.0	10.6	9.6

FIGURE CAPTIONS

1) Feynman diagrams giving a short distance contribution to the process $K \to \pi\nu_\ell\bar{\nu}_\ell$: the "$Z$ penguin" and the "W box."

2) The maximum and minimum of the branching ratio (per neutrino flavor) for $K^\pm \to \pi^\pm\nu\bar{\nu}$ without (dashed curve) and with (solid curve) QCD corrections ($\Lambda_{QCD} = 150$ MeV).

3) The quantity $|\widetilde{C}_{\nu,t} - \widetilde{C}_{\nu,c}|^2$, which enters the branching ratio for the CP violating decay $K_L \to \pi^0\nu_\ell\bar{\nu}_\ell$, as a function of m_t.

REFERENCES

1. C. O. Dib, I. Dunietz, and F. J. Gilman, SLAC preprints SLAC-PUB-4762, 1988 (to be published in Physics Letters) and SLAC-PUB-4818, 1988 (to be published in Physical Review D). Other recent work is found in J. Flynn and L. Randall, LBL preprint LBL-26310, 1988 (unpublished).

2. G. Belanger *et al.*, in *Proceedings of the 1988 Summer Study on High Energy Physics in the 1990s*, edited by S. Jensen (World Scientific, Singapore, 1989), p. 339, and references to previous work therein.

3. J. Ellis and J. Hagelin, Nucl. Phys. B217, 189 (1983).

4. J. S. Hagelin, Maharishi International University preprint MIU-THP-88/38, 1988 (unpublished); J. S. Hagelin and L. S. Littenberg, Maharishi International University preprint MIU-THP-89/039, 1989 (unpublished).

5. F. J. Gilman and M. B. Wise, Phys. Rev. D20, 2392 (1979).

6. T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981) and *erratum*, 65, 1772 (1981).

7. Eq. (3b) is strictly correct when $m_\ell^2 << m_q^2$. Before QCD corrections are applied, the non-negligible τ mass reduces the total charm coefficient $\widetilde{C}_{\nu,c}$ coming from the third generation of leptons by $\approx 20\%$.

8. We follow here a discussion analogous to that in V. A. Novikov *et al.*, Phys. Rev. D16, 223 (1977).

9. Particle Data Group, Phys. Lett. 204B, 107 (1988). The constraints are $s_{12} = |V_{us}| = 0.22$, $s_{23} = |V_{cb}| = 0.046 \pm 0.010$, and $0 \leq |V_{ub}/V_{cb}| \leq 0.20$.

10. We allow $1/3 \leq B_K \leq 1.5$, $m_c = 1.5 \pm 0.2$ GeV, $[B_B f_B^2]^{1/2} = 150 \pm 50$ MeV and impose standard QCD corrections to the mass matrix for given values of m_c and m_t. We use $|\epsilon| = 2.28 \times 10^{-3}$, $\tau_b = 1.18 \pm 0.14$ psec, and $x_d = 0.72 \pm 0.13$ from a combination of ARGUS (H. Albrecht *et al.*, Phys. Lett. 192B, 245 (1987)) and CLEO (M. Artuso *et al.*, Cornell preprint CLNS 89/889, 1989 (unpublished)) data.

11. Y. Nir, Nucl. Phys. B306, 14 (1988). See also J. Ellis, J. S. Hagelin, and S. Rudaz, Phys. Lett. 192B, 201 (1987); J. Ellis *et al.*, Nucl. Phys. B304, 205 (1988); J. S. Hagelin, Ref. 4; and J. S. Hagelin and L. S. Littenberg, Ref. 4, where QCD corrections were included.

12. For the same parameters and QCD corrections, Hagelin, Ref. 4, would have a branching ratio which is about 15% bigger than ours because he takes the electroweak coupling at the weak scale, which is appropriate if one takes M_W at that scale (as we do also). However, if one is going beyond lowest order in electroweak interactions, the relevant operators generally acquire non-trivial (electroweak) anomalous dimensions which should also be included and could produce effects of the same order.

13. L. Littenberg, Brookhaven preprint BNL 42227, 1989 (unpublished).

14. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

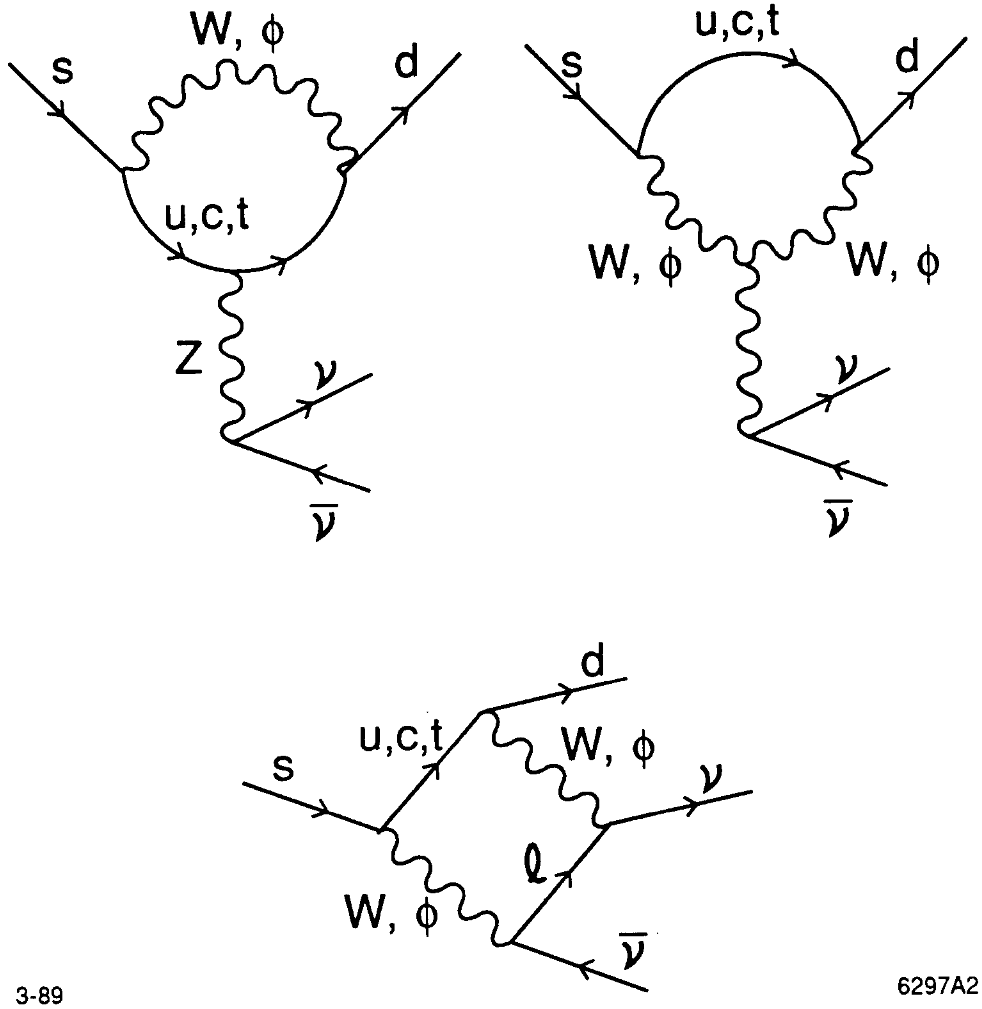

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Fig. 1

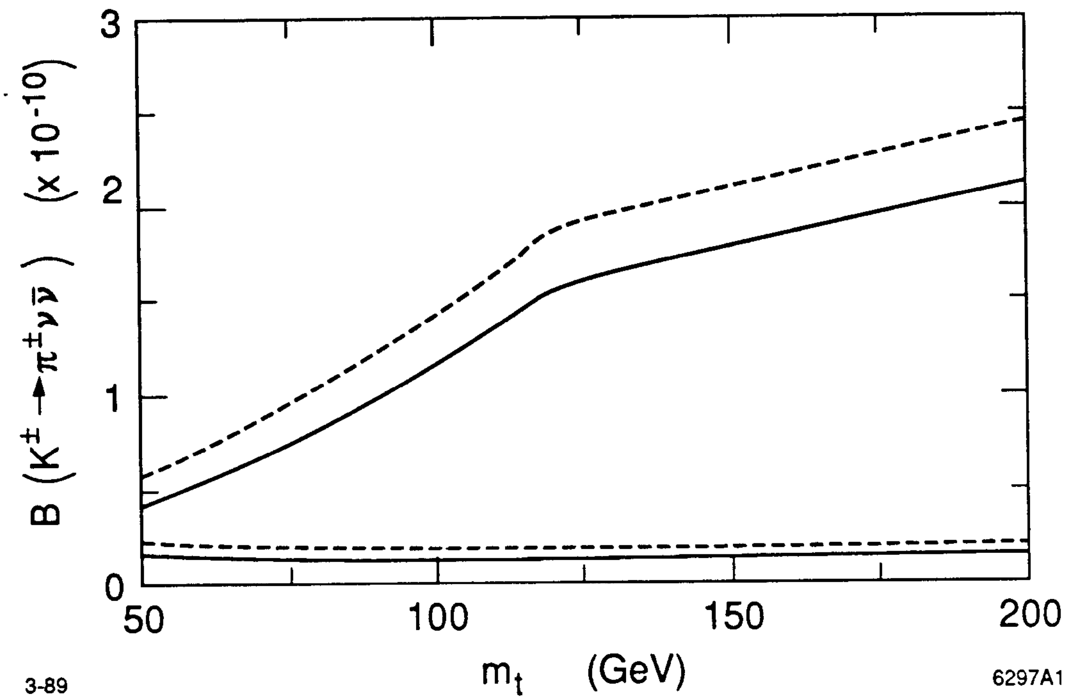
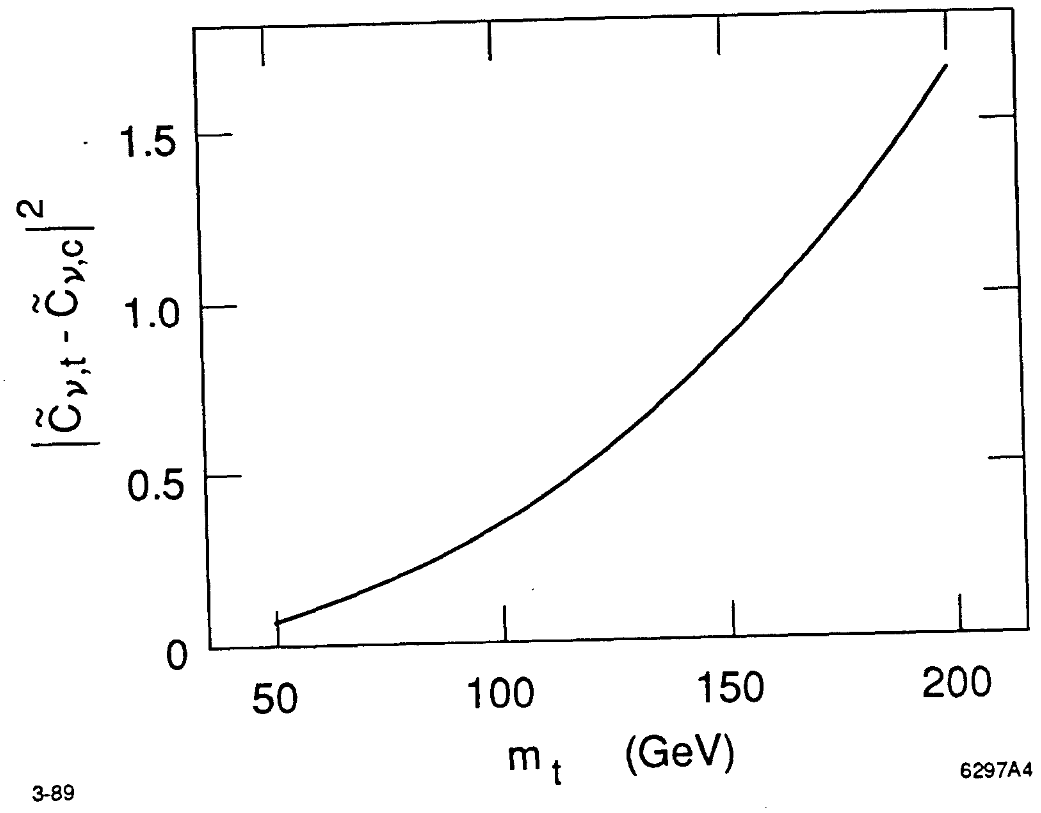


Fig. 2



1.5
1.0
0.5
0
$|\tilde{C}_{\nu,t} - \tilde{C}_{\nu,c}|^2$
50
100
150
200
m_t (GeV)
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Fig. 3