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ROLLFIX — AN ADIABATIC ROLL TRANSITION FOR THE SLC ARCS*

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ABSTRACT

The SLC Arcs were rolled at achromat boundaries to follow the terrain of the SLAC site. This makes the linear optics sensitive to systematic gradient errors, from which severe cross-plane coupling effects may arise. As a partial correction, a smoother roll transition was introduced which relieves much of this sensitivity. We present an evaluation of this scheme and report on the observed improvements.

INTRODUCTION

The two Arcs of the Stanford Linear Collider (SLC) are designed to bend the electrons and positrons around and into collision, without significant emittance dilution.[1] To minimize emittance growth from synchrotron radiation in the bend field, the focusing must be strong and compact and, therefore, use combined function magnets. To assure achromatic imaging, the magnets must also include sextupole components, and be grouped into sets (achromats) appropriately symmetrized for the suppression of optical aberrations.[2] In addition, for economical reasons, the two Arcs were designed to follow the terrain of the SLAC site (see Fig. 1), and thus include vertical deflections. For maximum compactness, these deflections were produced by rolling the magnets around their axis by up to 10° at achromat boundaries (see Fig. 2). To provide an overall cancellation of the induced cross-plane coupling, the rolls were grouped in pairs separated by one or several achromats, corresponding in the ideal system to an identity transfer matrix with 6 $n\pi$ phase-advance.

However, these long-range cancellations resulted in a limited bandwidth for the optical transfer, which had relatively stringent tolerances to systematic focusing errors. This fact was realized in several stages, before and during the initial beam tests. At first, a stringent tolerance to systematic horizontal displacement errors, which in the combined function magnets generate systematic focusing errors, was noticed through computer simulations. For example, see Ref. 3.

During the first beam tests, it was observed that the transfer of betatron oscillations and of the dispersion function across rolled achromat boundaries could be associated with a large magnification. A detailed calculation of the magnification associated with this transfer can be found in Ref. 4. In the initial commissioning, this magnification was observed to be as large as a factor of three over the whole length of the Arc because the systematic errors exceeded the specified design tolerances of 0.002.[5]

Although, for the case of equal input emittances (corresponding to the SLC design specification), such growth is in principle recoverable downstream, in the Final Focus, where a set of skew corrections are installed,[6] its magnitude required initial correction within the Arcs. Also, as was later found, the projected transverse emittance must in fact be close to the nominal design value at the exit to the Arcs, in order to minimize detector backgrounds induced by beam-tails striking the smaller (normalized) aperture in the Final Focus region.[7] Two basic cures were therefore devised. The first consisted of adjusting the

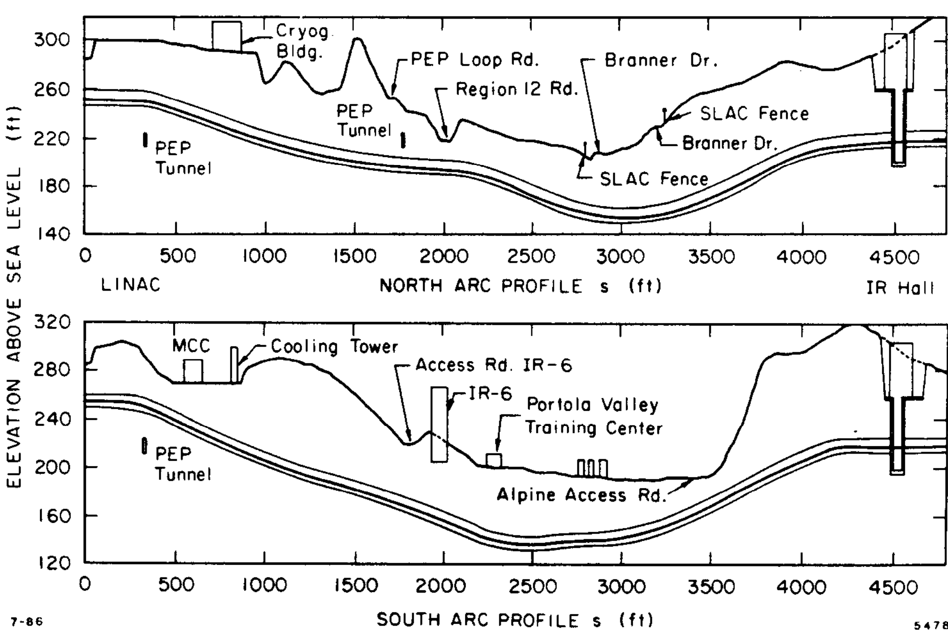

Fig. 1. Vertical profiles of the SLC Arcs.

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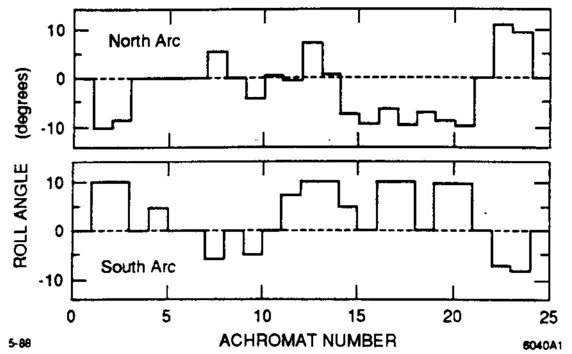


Fig. 2. Roll angle about beam axis versus achromat number for North-and South Arcs.

focusing gradients, based on measurements of the phase-advance in each achromat,[8] to their nominal values. Such adjustments helped bring the growth in the betatron transfer down to within a factor of about two, but further reductions by this procedure were limited by measurement errors and by the lack of fully separate controls in each plane and in each achromat.[8]

A second cure consisted of splitting each roll over several magnets on each side of the boundaries, to yield smoother transitions having a greater tolerance to phase-advance errors, and to nearly suppress the coupling of horizontal lattice dispersion into the vertical plane. The adjustments of the phase-advance had brought the system close to specification, and had minimized the coupling to the point where it could be handled relatively well in the Final Focus. It was however felt important for future operability to implement this passive *rollfix* cure, which makes the system significantly more error-tolerant, particularly for equal or close to equal initial emittances in both planes, as was noted above.

In addition, the nearly suppressed vertical dispersion is expected to reduce synchrotron radiation induced emittance growth in the vertical plane. The scheme could be installed without major disruption to the beam-line, and resulted in some observed improvements. The reference trajectory could be kept unperturbed through small vertical displacements of the magnets involved.

In this paper, after introducing an approximate measure for the cross-plane coupling in the betatron transfer, we evaluate the sensitivity to errors in the initial and modified designs, and characterize the predicted improvements. We then report on observed improvements.

APPROXIMATE CHARACTERIZATION OF CROSS-PLANE COUPLING

The practical consequences of cross-plane coupling are different in a beam-line than in a circular machine. In a circular machine, the motion is stable only for tunes such that sum-resonances (corresponding to $p\nu_x + q\nu_y = n$), have negligible effects. Residual coupling arises in this case exclusively from difference-resonances (corresponding to $p\nu_x - q\nu_y = n$), which can be shown to result in stable beating between the two projected transverse emittances.[9] In a beam-line, distortions from cross-plane coupling can correspond both to growing and decaying solutions.[10,11] The two projected emittances can in this case both grow. It has been shown that the severity of such growth can be characterized by the determinant of the off-diagonal two-by-two submatrix C of the general four-by-four transfer matrix:[10]

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad . \qquad (1)$$

This can be seen from calculating the projected emittances ϵ_x and ϵ_y onto each plane of a four dimensional phase-space transfered through a fully coupled system, and by using the six symplectic conditions imposed by Hamiltonian Mechanics.[9] With the simplifying assumption of upright phase-ellipses and of beams initially uncoupled in both planes, one can show that the following inequality holds:[10]

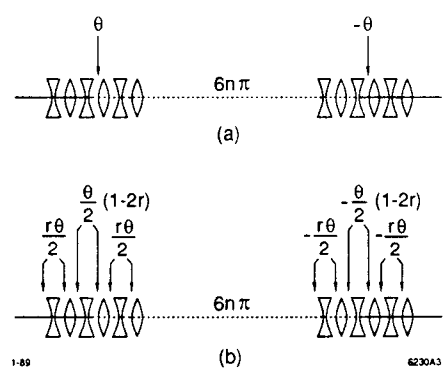
$$\epsilon_{x,y} \geq \epsilon_{x,y}(0)|1 - \det C| + \epsilon_{y,x}(0)|\det C| \quad . \qquad (2)$$

In most cases, the inequality sign in Eq. (2) can be replaced by an equality. This has been observed in several computer simulations. An analytical attempt to show this is described in the Appendix.

From Eq. (2), we see that the most severe coupling effects arise if: $\det C < 0$, or if: $\det C > 1$. In this case, the phase-space projections will grow in both planes, irrespective of the initial emittance values. On the other hand, coupling effects with $0 < \det C < 1$ are severe only if the initial emittance values are very assymetric (and if it is desirable to preserve such an assymetry). In this case, the coupling will tend to equalize the two emittance projections. In the SLC, where the initial emittances are close to equal, coupling effects with $0 < \det C < 1$ are benign.

DESCRIPTION OF ORIGINAL AND TAPERED ROLL TRANSITIONS[13]

Figures 3(a) and (b) show the principle of the original and tapered roll transition pairs. The first transition has a total angle *θ*. It is matched by a second transition, with a total angle *−θ*, located an integer number of achromats downstream. In this way, all cross-plane coupling effects cancel after the second transition, if the achromats in between are perfect, and correspond to an identity transfer matrix.



Figs. 3(a) and (b). Principle of original (a) and tapered (b) roll transition pairs. In the original set-up (a), pairs of rolls were concentrated at achromat boundaries separated by 6 nπ betatron phase-advance. In the tapered solution (b), each roll was split about five magnets to yield a smoother transition. The optimum value for the ratio r of the total roll of the first cell to the total roll of the transition is near r = 0.28.

In the original transition [see Fig. 3(a)], the full roll is concentrated at the achromat boundary. In the tapered transition [see Fig. 3(b)], the rolls are distributed across three cells around the boundary. As we will show, this tapering suppresses the most damaging component of the cross-plane coupling induced in the betatron transfer. Within each cell, the rolls are split equally across the defocussing magnet. This was found to nearly cancel the coupling of horizontal lattice dispersion into the vertical plane, and is due to the fact that the vertical phase-advance across a defocusing magnet is only about 22°, and because the angular horizontal lattice dispersion has opposite sign at the entrance to focusing and to defocusing magnets. We define by r the ratio of the roll of the first cell to the total roll of the transition. In the original proposal, $r = 0.38$ was used, by analogy with the coefficients for a matched trajectory bump. It was later found that $r = 0.28$ gives a slightly better results.[14]

TOLERANCES WITH ORIGINAL AND TAPERED ROLL TRANSITIONS

Following the above description of cross-plane coupling effects, we use the magnitude and the sign of $\det C$ to characterize the severity of the cross-plane coupling effects which arise from errors in the Arc lattice.

The original design was especially sensitive to the total deviations $\Delta\mu_{x,y}$ from the nominal phase-advance between transition pairs. For two original roll transitions, each of angle θ, and separated by a regular FODO lattice with an integer number of betatron periods [as in Fig. 3(a)], it can be shown that after the second transition:

$$\det\ C = \sin^2(2\theta)\left[\sin^2\left(\frac{\Delta\mu_x - \Delta\mu_y}{2}\right) - \alpha^2 \sin\Delta\mu_x \sin\Delta\mu_y\right] , \tag{3}$$

where α is the usual Twiss parameter at the transition (in the Arc, $\alpha = 2.65$).

For phase-advance errors with the same sign in each plane, $\det C < 0$, while for phase-advance errors with opposite sign, $\det C > 0$. The magnitudes of $\det C$, computed with a simulation[15] to confirm Eq. (5), are shown in Fig. 4 as a function of $(1/2)(\Delta\mu_x \pm \Delta\mu_y)$, for a typical transition with $\theta = 10^0$. As expected, the maximum value for $|\det C|$, which is close to one, is reached for $(1/2)(\Delta\mu_x \pm \Delta\mu_y) = \pm 90^0$. The same quantity is shown in Fig. 5 for the tapered transition. As can be seen, the onset of a negative $\det C$ is nearly suppressed by the tapered transition, and the onset of a positive $\det C$ is reduced by a factor of two.

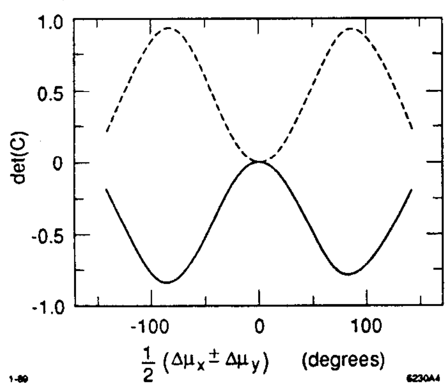


Fig. 4. $\det C$ *as a function of systematic phase-advance errors of equal sign (dashed line) and of opposite sign (solid line), between two original roll transitions.*

The actual roll distributions in the two Arcs are more complicated (see Fig. 2), but are superpositions of the basic ones shown in Fig. 3. We therefore expect the same overall features as for the simple examples examined above. To verify this, we show in Figs. 6 and 7, the same quantities as in Figs. 4 and 5, under the same conditions — but for the whole North Arc, as a function of the fractional phase-advance deviations $(1/2)[(\Delta\mu_x/\mu_x) \pm (\Delta\mu_y/\mu_y)]$. As can be seen in Fig. 6 (solid line), the tolerance to systematic phase-advance errors, for negligible coupling to occur, was about ± 0.006 in the original North Arc.

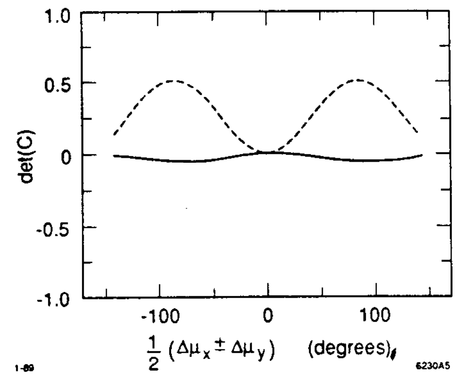


Fig. 5. $\det C$ *as a function of systematic phase-advance errors of equal sign (dashed line) and of opposite sign (solid line), between two tapered roll transitions.*

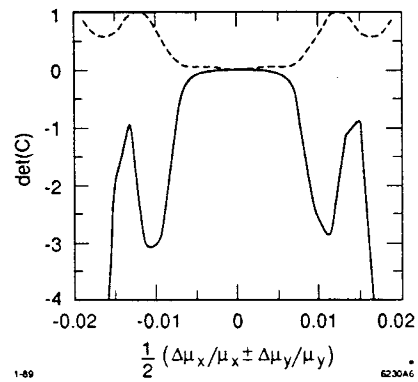


Fig. 6. $\det C$ *as a function of systematic phase-advance errors of equal sign (dashed line) and of opposite sign (solid line) in the entire North Arc with original roll transitions.*

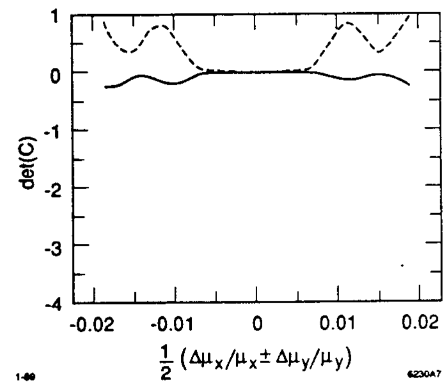
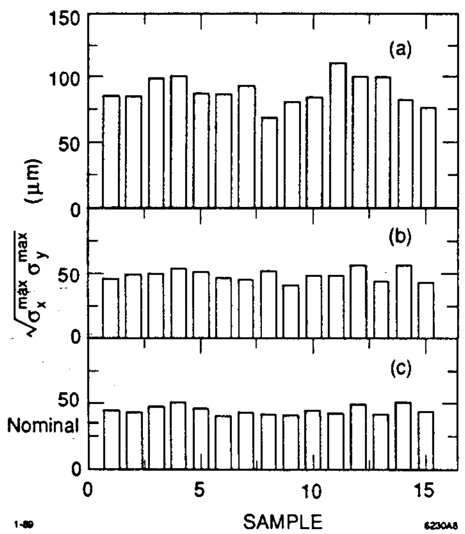


Fig. 7. $\det C$ *as a function of systematic phase-advance errors of equal sign (dashed line) and of opposite sign (solid line) in the entire North Arc with tapered roll transitions.*

In the modified North Arc, and in the case of equal emittances in both planes (for which cross-plane coupling with $0 < \det C < 1$ is benign), this particular tolerance is very broad and one expects the sensitivity to errors to be comparable to that of a flat Arc. For unequal x and y emittances, the improvements are, however, not expected to be as great (see Fig. 7 — dashed line).

In practice, the system can be perturbed by both random and systematic errors. Also, the tolerance to errors depends on the requirement put on the phase-space at the Arc exit. As was mentioned in the introduction, the cross-plane coupling distortions of the phase-space are correctable in the Final

Focus, but large distortions of the beam envelope — from any kind of error — can result in unacceptable background in the experiment, and must therefore be avoided. This leads us to define a tolerance in terms of the maximum deviation from the nominal size reached by the beam envelope at the end of the Arcs, as generated through the mixing of the distortions from both random and systematic errors. Extensive computer simulations[15,16,17] were performed to evaluate the sensitivity of the Arcs, with both random and systematic errors. It was found that in the case of equal emittances, the Arcs with modified roll transitions are about as sensitive to errors as a flat Arc without rolls. We illustrate this point with the result from one of these simulations in Figs. 8(a)–(c), where the geometric mean of the maximum growth of the horizontal and vertical monochromatic beam sizes at the end of the Arcs is calculated for the original North Arc, for the tapered North Arc, and for a hypothetical flat Arc.[15] The errors are the same in each case and correspond to systematic errors of 0.01 and to a sample of randomly distributed errors with a standard deviation of 0.005, in both the focusing and the defocusing magnets. As can be seen, the distortions are almost the same for the tapered Arc [case (b)] and for the flat Arc [case (c)]. Similar results were obtained for the South Arc.



Figs. 8(a),. (b) and (c). Maximum growth of the geometric mean of the monochromatic beam sizes in the vertical and horizontal planes at the end of the North Arc perturbed by systematic errors with the same sign in each plane, of 0.01, *and random errors, with a standard deviation of* 0.005. *The same sample of errors are used for comparing the North Arcs with the original roll transitions (a), with the tapered roll transitions (b), and without rolls (c). The nominal value is* 35 μm *in each plane.*

OPTIMIZATION OF TAPERED ROLL TRANSITION

A perfect match of the betatron transfer can be achieved by including at least five cells in the transition, but is not practical because the sign of the rolls must alternate in this case (presumably because the phase-advance across more than two cells becomes larger than π), and because the solution depends in this case very nonlinearly on the total roll of the transition.

The distribution of rolls indicated in Fig. 3(b), however, can be improved. An example of such an optimization is shown in Fig. 9, where the maximum positive and negative values of det C, occuring when the phase-advances between the pairs of roll transitions shown in Fig. 3 are perturbed to satisfy $(1/2)(\Delta\mu_x \pm \Delta\mu_y) = \pm 90^0$, is computed as a function of r. The dependance is fairly flat. The optimum value of $r = 0.28$ results in slight improvements in the overall performance.[14]

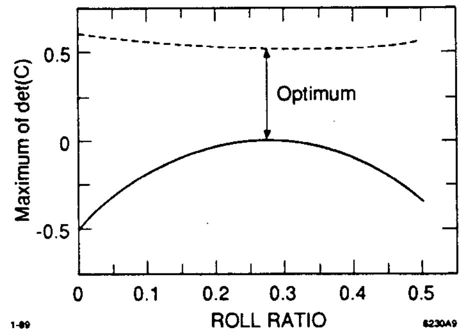
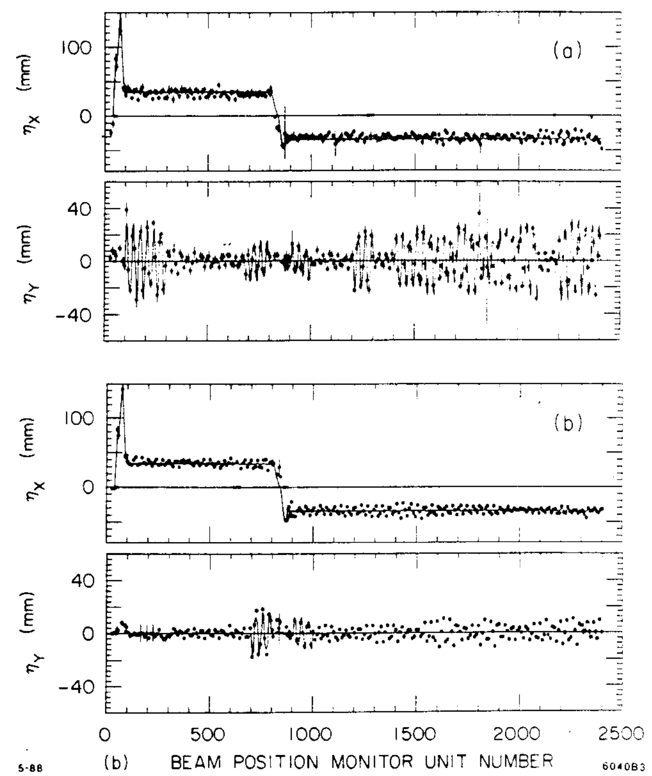


Fig. 9. Maximum value of det C *resulting from systematic errors with opposite sign (dashed line) and with the same sign (solid line) between the tapered transitions of Fig. 3(b), as a function of the roll ratio* r.



Figs. 10(a) and (b). Horizontal and vertical dispersion measured in the North Arc, before (a) and after (b) the installation of the tapered roll transitions. The vertical dispersion was essentially suppressed by the modification.

PERFORMANCE AND CONCLUDING REMARKS

The performance of the modification was particularly clear for the suppression of vertical lattice dispersion. This can be seen from the measurements in the North Arc, before and after the modification of the roll transitions [see Figs. 10(a) and (b)].

The minimization of the coupling in the betatron transfer which was achieved during the recommissioning of the Arcs after the installation of the modification cannot be attributed solely to this design change. It was also the result of the previous phase-advance adjustments,[8] of several other empirical adjustments and resteering performed at turn-on.

The modification did in addition improve the overall performance of the Arc, by reducing, as expected, the need for feeding back on the optics to cancel variations from steering or other changes. However, later, expectations on the quality of the phase-space at the exit to the Arcs were also enhanced, in particular from the requirement to minimize beam tail induced backgrounds in the detector.[7] Because of this it became necessary to implement further and more precise corrections.[11]

More recently, it has been possible to determine the transfer matrix along the Arc beam-line, by fitting betatron oscillations launched at several input phases.[18] Such calculations have shown that presently $\det C \simeq 0.2$ at the exit to the North Arc, leading to small coupling effects in the case of equal emittances.

APPENDIX

It is possible to calculate the projected emittances exactly with the simplifying assumption of upright phase-ellipses and of beams initially uncoupled in both planes, by folding the contributions to each emittance projection from the two planes. Because the phase-ellipses are not in general upright, the derived expression is an upper bound of the cases with phase-ellipses of arbitrary orientation. One obtains:

$$\epsilon_{x,y} = \left[\epsilon_{x,y}(0)|1-\det C| + \epsilon_{y,x}(0)|\det C|\right]\sqrt{1+\mathcal{F}} \quad , \qquad (4)$$

where $\epsilon_x(0)$ and $\epsilon_y(0)$ are the initial values, and where the factor:

$$\mathcal{F} = \frac{\epsilon_x(0)\epsilon_y(0)|\det C||1-\det C|}{\epsilon_{x,y}(0)|1-\det C| + \epsilon_{y,x}(0)|\det C|}\left(\frac{\lambda_x^2}{\lambda_y^2} + \frac{\lambda_y^2}{\lambda_x^2} - 2\right) \qquad (5)$$

describes the mixing which results from folding the phase-ellipses if they are not similar. Such dissimilarity arises from upright quadrupole perturbations to the lattice, both through random and, in the presence of rolls, systematic errors. The parameters $\lambda_{x,y}$ describe the magnitude of the mismatch which result in each plane. These parameters are defined in normalized phase-space as the ratio of the radius of the circle in which a distorted phase-ellipse is inscribed to that of the smaller circle which corresponds to the matched case.[12] If there are no upright quadrupole errors ($\lambda_{x,y} = 1$) or if the two phase-ellipses are similar ($\lambda_x = \lambda_y$), then the mixing term $\mathcal{F} = 0$. For mismatches with $\lambda_{x,y} \lesssim 2$, corresponding to upright quadrupole errors of up to 1%, then $\sqrt{1+\mathcal{F}} \lesssim 2.5$. For such cases, the inequality sign in the expression given in Eq. (2) can be replaced by an equality sign, for approximate calculations.

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