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Chiral Corrections to the Anomalous 2γ Decays of π^o, η and η'

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ABSTRACT

To any order in chiral perturbation theory, the anomalous Wess-Zumino term is shown to generate only chirally invariant counterterms. Explicit examples of $O(p^6)$ terms generated by one-loop graphs are given, some of which are relevant to the two-photon decays of π^0, η and η'.

MOTIVATION

To lowest order, chiral symmetry forbids the process $\pi^0 \rightarrow 2\gamma$, but including the chiral anomaly one obtains $\Gamma^{anom.}_{\pi^0 \rightarrow 2\gamma} \approx 7.7$ eV at $m_\pi^2 = 0$ in close agreement with early measurements. However, recent two-photon experiments at e^+e^- colliders find $\Gamma^{exp.}_{\pi^0 \rightarrow 2\gamma} = (7.3 \pm 0.2)$ eV.[1] The discrepancy is of the order expected in SU(2) current algebra (CA), indicating the need for meson-loop corrections. The need for a refined calculation is even more apparent in the η meson: In the SU(3) limit, $\Gamma^{anom.}_{\eta \rightarrow 2\gamma} = (1/3)(m_\eta/m_\pi)^3 \cdot \Gamma_{\pi^0 \rightarrow 2\gamma} \approx 170$ eV in sharp contrast with $\Gamma^{exp.}_{\eta \rightarrow 2\gamma} = (524 \pm 31)$ eV from 2γ experiments[1](older measurements based on the Primakoff effect are controversial). Note the analogy with the $\eta \rightarrow 3\pi$ puzzle. CA cannot predict the two-photon width of η' without further assumptions; experiment finds $\Gamma^{exp.}_{\eta' \rightarrow 2\gamma} = (4.25 \pm 0.19)$ keV.[1]

In recent years, these questions have been mainly studied in the context of chiral perturbation theory (ChPT).[2,3] ChPT systematically expands the low-energy effective action in powers of p^2 and m_{quark}, taking into account meson loops. The effects of the chiral anomaly are captured by the Wess-Zumino term.[4] When sufficiently precise data become available, the chiral anomaly can be tested in a similarly stringent way as chiral symmetry has been verified in the non-anomalous sector.[5] Below, I report on some aspects of ongoing theoretical work towards this goal.

NON-RENORMALIZATION OF THE WESS-ZUMINO TERM

Adler and Bardeen showed that the chiral anomaly in a theory with fermions like QCD is given by the basic triangle graph, higher-order graphs renormalizing only the masses and couplings of the theory.[6] An analogous theorem holds in the effective meson theory: *Quantum corrections induced by the Wess-Zumino term require only chirally invariant counterterms to any finite order in* ChPT.[3]

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Disregarding for the moment the need for symmetry-preserving regularization, this is easily deduced in the path-integral formulation: The action is

$$S = S_{inv.} + S_{WZ} = \int d^4x\left(\mathcal{L}_{inv.}(U,V,A,...) + \mathcal{L}_{WZ}(U,V,A)\right) \tag{1}$$

where V, A are external vector and axial-vector fields, respectively, and U is the exponential of the meson fields, $U(x) = \exp(i\frac{\phi^a(x)}{f}\frac{\lambda_a}{2})$. Under a chiral rotation β, $S_{inv.}$ is invariant and $\delta_\beta S_{WZ} = \int d^4x \text{ Tr } \left(\beta(x)\Omega(V(x),A(x))\right)$ is independent of $U(x)$. The generating functional Z of connected Green functions varies as

$$\begin{aligned}\delta_\beta Z[V,A,...] &= -ie^{-iZ}(\delta_\beta e^{iZ}) = -ie^{-iZ}\delta_\beta \int [dU] e^{iS_{inv.}[U,V,A,...]+iS_{WZ}[U,V,A,...]}\\ &= -ie^{-iZ}\int [dU](i\delta_\beta S_{WZ})e^{iS_{inv.}+iS_{WZ}} = \int d^4x \text{ Tr } \left(\beta(x)\Omega(V,A)\right).\end{aligned} \tag{2}$$

The regularized version of the above argument expands the action around the classical extremum realized by some $U_0(x)$ and makes use of the fact that $\Delta S_{WZ}[U,U_0,V,A] \equiv S_{WZ}[U,V,A] - S_{WZ}[U_0,V,A]$ can be shown to be a local four-dimensional chiral invariant. Building graphs with the usual propagators and vertices as well as vertices from ΔS_{WZ}, only chirally invariant expressions are obtained in an appropriate regularization scheme, such as operator regularization.[7] A technical problem intimately related to the expansion of the action around the classical solution $U_0(x)$ is the appearance of divergences that cannot be absorbed simply by adding a corresponding graph with a higher-order vertex replacing the loop. Instead, several divergent graphs together are renormalized by a set of counterterms that correspond, in effect, to a modification of the classical equations of motion for $U_0(x)$.[3]

COUNTERTERMS OF ORDER p^6 AND PHENOMENOLOGY

The lowest-order chiral Lagrangean is $O(p^2)$, and the corresponding one-loop graphs and their counterterms are $O(p^4)$.[5] The Wess-Zumino action being itself $O(p^4)$, a corresponding calculation in the anomalous sector is $O(p^6)$. It is of theoretical and phenomenological interest to determine which of the chirally invariant terms of $O(p^6)$ with an ϵ symbol are actually needed for absorbing divergences. These calculations are somewhat tedious but nearing completion. Some examples of divergent terms are

$$\begin{aligned}&\frac{c}{2}\epsilon^{\mu\nu\rho\sigma} \text{ Tr } \big(D_\tau U D_\mu U^\dagger D^\tau U D_\nu U^\dagger D_\rho U D_\sigma U^\dagger\\ &\qquad + D^\tau(D_\tau U D_\mu U^\dagger - D_\mu U D_\tau U^\dagger) U D_\nu U^\dagger D_\rho U D_\sigma U^\dagger\big),\end{aligned} \tag{3}$$

$$c\epsilon^{\mu\nu\rho\sigma} \text{ Tr } \left(\{D_\nu U U^\dagger, F^L_{\rho\sigma} + U F^R_{\rho\sigma} U^\dagger\}(D^\tau F^L_{\mu\tau} + U(D^\tau F^R_{\mu\tau})U^\dagger)\right), \tag{4}$$

$$
\begin{aligned}
-\frac{c}{2}\epsilon^{\mu\nu\rho\sigma}\ \mathrm{Tr}\ \Big(& (F^L_{\mu\tau} - UF^R_{\mu\tau}U^\dagger) * \\
& *\big(- D_\nu U(U^\dagger F^L_{\rho\sigma}U + F^R_{\rho\sigma})D^\tau U^\dagger + D^\tau U(U^\dagger F^L_{\rho\sigma}U + F^R_{\rho\sigma})D_\nu U^\dagger \\
& + D^\tau U D_\nu U^\dagger (F^L_{\rho\sigma} + UF^R_{\rho\sigma}U^\dagger) - (F^L_{\rho\sigma} + UF^R_{\rho\sigma}U^\dagger) D_\nu U D^\tau U^\dagger\big)\Big),
\end{aligned}
\tag{5}
$$

where $c \equiv (-256i\pi^4 f^2\epsilon)^{-1}$, $\epsilon \longrightarrow 0$ is a regularization parameter, f the pion decay constant, $D_\mu U \equiv \partial_\mu U + V^L_\mu U - UV^R_\mu$, $V^{R,L} \equiv V \pm A$, and $F^{R,L}_{\mu\nu}$ are the corresponding field strengths. For the full list of terms see Ref. 3.

A comprehensive phenomenological analysis of the 2γ channel of pseudoscalar mesons has been given by other authors.[2] The main conclusions are that loop corrections are finite for on-shell photons but divergent for off-shell photons. In both cases the $O(p^6)$ effective Lagrangean contributes to the meson-mass dependence of the decay rate. Unfortunately, however, present experimental accuracy does not allow to reliably determine these higher-order terms because the effects are too small for π^0 and, for η, masked by mixing with η'. I plan to extend the analysis to other processes in the anomalous sector in order to identify the experiments with the greatest potential for determining the new low-energy parameters.

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REFERENCES

1. J. Olsson, *Nucl. Phys. B (Proc. Suppl.)* **3** (1988) 613.
2. J.F. Donoghue, B.R. Holstein and Y.C. Lin, *Phys. Rev. Lett.* **55** (1985) 2766; Erratum (to be published);
   J. Bijnens, A. Bramon and F. Cornet, preprint MPI-PAE/PTh 30/88 (August 1988);
   J.F. Donoghue and D. Wyler, Zürich University preprint (September 1988).
3. D. Issler, in preparation.
4. J. Wess and B. Zumino, *Phys. Lett.* **B37** (1969) 95;
   E. Witten, *Nucl. Phys.* **B223** (1983) 422.
5. J. Gasser and H. Leutwyler, *Ann. Phys.* **158** (1984) 142; *Nucl. Phys.* **B250** (1985) 465, 517, 539.
6. S.L. Adler and W.A. Bardeen, *Phys. Rev.* **182** (1969) 1517.
7. D.G.C. McKeon and T.N. Sherry, *Phys. Rev. Lett.* **59** (1987) 532.