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EMBEDDING AND BREAKING OF GAUGE SYMMETRIES*

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ABSTRACT

The effective potential of "weak" interactions of a gauge group R is studied in the framework of models whose global symmetry group G is dynamically broken down to its subgroup H. The properties of its critical points on the coset manifold $R\backslash G/H$ and their dependence on the embeddings $R \subset G, H \subset G$ are investigated. All critical points with a nontrivial residual gauge symmetry $H_w = R \cap H$ are identified in the series of models $SU(n)\backslash SU(N)/SO(N),\ n < N \leq 6$ and $SU(n)\backslash SU(2N)/Sp(N),\ n < 2N \leq 6$ with $n = 2, 3$. Examples of degenerate hessians i.e. potentials with flat directions both neutral and charged under H_w are presented and their physical implications are briefly discussed.

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This paper concerns the embeddings of gauge symmetries into dynamically broken global symmetries in gauge field theory models without elementary scalars. It suggests a general prescription to identify and presents models to demonstrate the embeddings which are necessary for the realization of specific breaking patterns of gauge symmetries.

I will begin with a brief review of the general framework underlying the results which will be established subsequently; for more details the reader is referred to the original literature cited in Ref. 1.

The local gauge interactions are introduced into the field theory by means of an embedding of the corresponding gauge group R into its global symmetry group G. The group G is assumed to be dynamically broken down to its subgroup $H \subset G$. The embedding $R(H) \subset G$ is determined by two sets of complimentary conditions which may be characterized as kinematical or dynamical in nature reflecting respectively mathematical or physical aspect of the problem.

The first set specifies the choice of groups and representations in question. In particular the $R(H)$ - representation content of the G - representation, i.e., the decomposition $\underline{\mathrm{n}}_G = \sum_\alpha \oplus \underline{\mathrm{n}}^{(\alpha)}_{R(H)}$ is an important condition as it determines the embedded group $R(H)$ up to an inner automorphism of $G : R \to g_r R g_r^{-1} (H \to g_h H g_h^{-1})$, $g_{r(h)} \in G$.[2]

The second set is of intrinsically dynamical origin being often referred to as alignment conditions.[3,4] It is based on a minimum energy principle which removes the mathematical freedom in the choice of the element $g = g_r g_h^{-1}$. The potential energy (density) $V(g)$ defined as a function of the group element g is induced by gauge interactions to the leading order of the fine structure constant α_R. It is given by an expectation of R - invariant product of two Nöether currents of the gauge group gRg^{-1} in a H - invariant vacuum state. Obviously such expression is a function of the coset element $1 \in G/H$ only i.e. $V(g) = V(1)$ for $g = lh, h \in H$. Furthermore by standard Wigner-Eckhardt arguments its 1-dependence may be always reduces as follows:

$$V(1) = C_\alpha r^{ab} R^M_{(\alpha)a}(1) R^N_{(\alpha)b}(1) g^{(\alpha)}_{MN}, \quad 1 \in G/H \tag{1}$$

where the unknown constants $\{C_\alpha\}$ represent contributions of various irreducible representations $\{\underline{\mathrm{n}}^{(\alpha)}_H\}$ whereas the matrix elements $R^M_{(\alpha)a}$ are defined as follows:

$$1 R_a 1^{-1} = R^M_{(\alpha)a}(1) X^{(\alpha)}_M \tag{2a}$$

$$\{R_a\} = AlgR, \quad \{X^{(\alpha)}_M \in \underline{\mathrm{n}}^{(\alpha)}_H\} = AlgG \tag{2b}$$

The matrices $r \equiv (r_{ab})(r^{ab} \equiv r^{-1}_{ab})$ and $g^{(\alpha)} = (g^{(\alpha)}_{MN})$ in (1) represent Killing forms of R and G groups respectively; the latter $g^{(\alpha)}$ is restricted to the subspace $\underline{\mathrm{n}}^{(\alpha)}_H$.

The factorized form (1) holds under rather general conditions. Indeed it is sufficient to demand the isotropy group H to be compact. This in its turn implies that the coset manifold G/H is reductive i.e. the complement $K = \{K_m\}$ of the algebra $H = \{H_i\}(H \oplus K = AlgG)$ is a representation of $H([H, K] = K)$. However the discussion will be restricted to the subclass of reductive manifolds i.e. compact symmetric manifolds G/H for which $[K, K] = H$ holds in addition to $[H, K] = K$. Furthermore it will be assumed that the group H is simple and its coset representation is irreducible. Then only two representations $\underline{\mathrm{n}}^{(\alpha)}_H, \alpha = h, k$ are involved in Eq. (1) and therefore within a irrelevant additive constant the potential function may be reduced to the form (c.f. Refs. 5,6).

$$V(1) = Cr^{ab} R_a^m(1) R_b^n(1) g_{mn}, \quad C > 0 \tag{3}$$

with

$$R_a^m(1) = Tr(1 R_a 1^{-1} K^m) \tag{4a}$$

$$g_{mn} = -Tr(K_m K_n), \quad r_{ab} = -Tr(R_a R_b) \tag{4b}$$

and a positive constant $C = C_{(k)} - C_{(h)}$.

I will proceed to the mathematical formulation of the embedding conditions. To this end the potential function (3) will be expressed in a standard geometric language. This is immediately achieved by recognizing that the matrix elements $R_a^m(1)$ are directly related to the Killing vectors $\xi_M^\mu(1)$ and vielbeins $e_m^\mu(1)$. Parametrizing the coset element $1 = 1(x)$ by local cooridinates $x = \{x^\mu\}$ e.g. via $1(x) = \exp(x^\mu K_\mu)$ one finds

$$V(x) = r^{ab} \xi_a^\mu(x) \xi_b^\nu(x) g_{\mu\nu}(x) \equiv r^{ab}(\xi_a, \xi_b) \tag{5}$$

$$\xi_a^\mu(x) = e_m^\mu(x) R_a^m(x) \tag{6a}$$

$$g_{\mu\nu}(x) = e_\mu^m(x) e_\nu^n(x) g_{mn} \tag{6b}$$

Note that the constant factor C of Eq (3) has been suppressed.

Now the variational problem of the potential function can be addressed. In light of the Ansatz (5), it is convenient to consider variations induced by infinitesimal group motions (diffeomorphisms). Indeed the first variation is simply described by the Lie derivative L_M as $L_M g_{\mu\nu} = 0, \quad L_M \xi_N^\mu = \xi_{[MN]}^\mu$ etc. The second variation may be realized by means of the Laplas-Beltrami operator $\Delta_R = r^{ab} L_a L_b$. Hence one easily arrives at the following theorem.

Theorem.

If the potential function (5) has a critical point (extremum) at the origin $x = 0$ i.e.

$$V_m(0) \equiv \partial V(x)/\partial x_m|_{x=0} = L_m V(x)|_{x=0} = 2r^{ab}(\xi_a, \xi_{[mb]})|_{x=0} \tag{7}$$

then its hessian (mass matrix) at the critical point is given by

$$V_{mn}(0) \equiv \partial^2 V(x)/\partial x_m \partial x_n|_{x=0} \tag{8a}$$

$$V_{mn}(0) = L_m L_n V(x)|_{x=0} = \{\Delta_R(\xi_m, \xi_n) - (\Delta_R \xi_m, \xi_n) + (m \leftrightarrow n)\}_{x=0} \tag{8b}$$

Clearly by redefinition of R-generators i.e., $R_a(x_c) = 1(x_c) R_a 1^\dagger(x_c) \to R_a$ the critical point $x = x_c \neq 0$ can be translated to the origin. Therefore, it will be henceforth assumed $x_c = 0$. Thus the conditions (7,8) can be used to determine critical configurations $\{R_a\}$ directly. The task is facilitated by the following straightforward deductions.

Corollary 1.

The unbroken (H_a) and broken (K_a) components of critical gauge generators $R_a = H_a + K_a, H_a \in H, K_a \in K$ obey the constraints

$$V_m = -2r^{ab} Tr\{[H_a, R_b] K_m\} = 0 \tag{9a}$$

or alternatively

$$r^{ab}[H_a, K_b] = 0 \tag{9b}$$

Corollary 2.

Let the symmetry of the critical point $x = 0$ be $H_c = H_w \otimes H_g$ with the unbroken (residual) gauge subgroup $H_w = R \cap H$ and the global symmetry $H_g \subset H$ defined by $[H_g, R] = 0$ Then the coset generators K_m which are not neutral under H_c i.e. $[K_m, H_c] \neq 0$ satisfy the condition (9) identically.

Corollary 3.

The decomposition $\xi_m(x) = f_{mA}\xi_A(x)$ of the H-multiplet $\{\xi_m(x)\}$ in terms of R-multiplets $\{\xi_A(x)|\Delta_R\xi_A = -C_A\xi_A\}$ reduces the hessian (8) to the form.

$$V_{mn} = f_{mB}f_{nC}C_A\{(\delta_{AB} + \delta_{AC}) - 2(BC \mid A)\}(\xi_B, \xi_C)_{x=0} \tag{10}$$

where the coefficients C_A and $(BC \mid A)$ stand for the second Casimir and Clebsh-Gordan coefficients of the group R.

Corollary 4.

A necessary conditon for the embedding $R \subset G$ to be physical is that the potential function $V(x)$ has a relative minimum at the critical point $x = 0$ i.e. it possesses a positive hessian $V_{mn} > 0$ along with $V_m(0) = 0$.

At this point it is appropriate to dwell upon some special features of the hessians in question. In general, hessians can be degenerate with a spectrum consisting of positive, negative as well as zero eighenvalues. However, mathematical discussions (e.g. in Morse theory) are usually restricted to the functions with non-degenerate hessians as they are known to be dense in the class of smooth functions. Nevertheless in what follows I will not be bound by such restrictions; moreover, the search of potential functions with degenerate hessians will be a major thrust of the discussion.

A physical significance of the degeneracy may be appreciated by delineating prominent characteristics of the generators $\{X_{\underline{m}} = K_{\underline{m}}(\mathrm{mod} H)\}$ representing zero modes. Obviously R-singlet generators $\{X_{\underline{m}} | [X_{\underline{m}}, R_a] = 0\}$ have a trivial nature and it is sufficient to focus upon R-nonsinglets $\{X_{\underline{m}} | [X_{\underline{m}}, R_{\underline{a}}] \neq 0\}$. They can be classified according to the properties $\{K^{(g)}\} = \{K_a\}, [K^{(n)}_{\underline{m}}, H_w] = 0$ and $[K^{(c)}_{\underline{m}}, H_w] \neq 0$ describing respectively gauge, neutral and charged zero modes. An insight into the intricate origin of these modes may be gleaned by identifying positive (H_{mn}) and negative $(-K_{mn})$ components of the hessian[5]; One easily infers from the definition (8a) that

$$V_{mn}(0) = H_{mn} - K_{mn} \tag{11}$$

$$Z_{mn} = 2r^{ab} Tr\{[Z_a, K_m][Z_b, K_n]\}, \quad Z = H, K \tag{12}$$

Evidently all zero modes $\{K_{\underline{m}}\}$ arise as a result of a subtle conspiracy of broken (K_a) and unbroken (H_a) components of R-generators leading to an exact cancellation of their contributions i.e. $K_{\underline{mn}} = H_{\underline{mn}}$. However, the origin of the conspiracy underlying gauge zero modes is fundamentally distinct from that of charged/neutral zero modes. In the former case it is a manifestation of the universal property i.e. R-gauge invariance; the potential function $V(x)$ is altogether independent of the gauge coset parameters $\{x^{(g)}_{\underline{m}}\}$ (as well as R-singlet ones) or more precisely in the vicinity of the critical point it is defined on the left-right coset $R \setminus G/H$. On the other hand for charged/neutral zero modes $\{x^{(c)}_{\underline{m}}, x^{(n)}_{\underline{m}}\}$ such conspiracy is an expression of flatness of the potential in certain directions enforced by particular features of the coset manifold in question. Furthermore, it is important to realize that the potential $V(x)$ restricted to its flat directions

$\phi = \{x_{\underline{\mathrm{m}}}^{(c)}, x_{\underline{\mathrm{m}}}^{(n)}\}$ is, in general, a non-trivial function $V(\phi)$ of ϕ:

$$V(x) = V(0) + (1/2)V_{mn}x_m x_n + 0(x^2) \tag{13a}$$

$$V(\phi) = V(0) + 0(\phi^2), \quad \phi = (x_{\underline{\mathrm{m}}}^{(n)}, x_{\underline{\mathrm{m}}}^{(c)}) \tag{13b}$$

The general features described above will be demonstrated on a series of models of the class introduced in Refs. 7,8:

$$\begin{aligned} R \setminus G/H_s &= SU(n) \setminus SU(M)/SO(M) \\ &\equiv K_s(n \mid M), n < M = 2N, 2N+1 \end{aligned} \tag{14a}$$

$$R \setminus G/H_a = SU(n) \setminus SU(M)/Sp(N) \equiv K_a(n \mid N), n < M = 2N \tag{14b}$$

The embeddings both $H \subset G$ and $R \subset G$ will be closely examined with a special attention to the resulting intersection $H_w = R \cap H$ i.e. residual gauge symmetry. For this purpose the choice of the Weyl basis for $U(M)$ generators $\{X_M\}$ is most suitable:

$$X: \quad (X_j^i)_n^m = \delta_n^i \delta_j^m, \quad i,j = (0), \pm 1, ..., \pm N, \quad M = 2N(2N+1) \tag{15}$$

Indeed the H_z-generators can be now simply identified by means of an involutive automorphism $e_z \in G$:

$$H: \quad {}^zH_j^i = X_j^i - e_z^\dagger \tilde{X}_j^i e_z, \quad eH = -\tilde{H}e, \tag{16a}$$

$$K: \quad {}^zK_j^i = X_j^i + e_z^\dagger \tilde{X}_j^i e_z, \quad eK = \tilde{K}e. \tag{16b}$$

Here the tilde indicates transposed matrices and

$$(e_z)_{mn} = \varepsilon^z_m \delta_{m+n}, \quad z = s, a, \quad \varepsilon^s_m = 1, \quad \varepsilon^a_m = \mathrm{sign}(m) \tag{17}$$

An important property of the matrix $[(e_z)_{mn}]$ is that it defines ${}^zH$ invariant (anti)symmetric bilinear form for the fundamental representation ${}^z\underline{\mathrm{n}}_H, z = s(a)$. The following lemma is a direct consequence of this fact.

Lemma 1

- A necessary and sufficient condition for the embedding $R \subset {}^zH$ with the content ${}^z\underline{\mathrm{n}}_H = \sum_\alpha \oplus \underline{\mathrm{n}}_R^{(\alpha)}$ to exist is to have a R invariant (anti)symmetric bilinear form on the space ${}^z\underline{\mathrm{n}}_H, \quad z = s(a)$.

Hence one is immediately led to a simple and yet very effective prescription to identify breaking patterns.

Lemma 2

- A necessary condition for the breaking $R \to H_w = R \cap H$ with the content $\underline{\mathrm{M}}_G = \underline{\mathrm{n}}_H = \sum_\alpha \oplus \underline{\mathrm{n}}_w^{(\alpha)}$ $\;(\underline{\mathrm{M}}_G = \underline{\mathrm{n}}_H = \sum_\alpha \oplus \underline{\mathrm{n}}_R^{(\alpha)})$ to occur is the existence (absence) of a bilinear form which is both $H_w(R)$ and H invariant.

The familiar example of the group $R = SU(2)$ provides a simple demonstration of the first lemma. Any irreducible representation of a (half) - integer spin admits an (anti)symmetric bilinear form. Thus it can be embedded into $SO(Sp)$ - groups but cannot be embedded into $Sp(SO)$ - groups. Evidently a reducible representation containing both integer and half-integer spins cannot be accommodated by either SO or Sp groups. However one should bear in mind that conjugate representations of $SU(2)$ are equivalent. Therefore a reducible representation with two identical spins admits both symmetric and anti-symmetric bilinear forms and its embedding into either SO or Sp is permitted.

Now the results described in the Table should not be difficult to verify. All possible embeddings of $R = SU(n), \quad n = 2,3$ into $G = SU(M), M \leq 6$ have been examined and critical points of the potential function of corresponding models in the series (14) have been identified. A crucial feature of all cases considered is the special choice of the R content which by Lemma 2 obstructs the embedding $R \subset H$ apriori. The content of the Table, in general, is not intended to be exhaustive. However, the listing of critical points with $U_w(1)$ or $SU_w(2)$ gauge symmetries is complete. A comprehensive discussion of these results will be presented elsewhere. Here the discussion will be restricted to the potentials with flat directions.

The models of interest are as follows:

$$K_a(2 \mid 6), \quad \underline{6}_G = \underline{3}_R + 3 \times \underline{1}_R, \quad H_c = SU_g(2) \otimes U_w(1) \tag{18a}$$

$$K_s(2 \mid 6), \quad \underline{6}_G = 2 \times \underline{2}_R + \underline{2}^*_R, \quad H_c = U_g(1) \otimes U_w(1) \tag{18b}$$

$$K_s(2 \mid 6), \quad \underline{6}_G = 3 \times \underline{2}_R, \quad H_c = S0_g(3) \otimes U_w(1) \tag{18c}$$

$$K_s(3 \mid 6), \quad \underline{6}_G = 2 \times \underline{3}_R, \quad H_c = U_g(1) \otimes S0_w(3) \tag{18d}$$

The analysis should be extended beyond the calculation of their hessians and the following expression of the potential (5) is more suitable for this purpose

$$2V(x) = N_R + r^{ab} Tr[e^\dagger \tilde{R}_a e U^\dagger(x) R_b U(x)] \tag{19}$$

where

$$N_R = r^{ab} r_{ab} \tag{20a}$$

$$U(x) = 1(x)e^{\dagger}\tilde{1}(x)e \tag{20b}$$

Thus the potential is defined as a bilinear function of matrix elements $\{U_{mn}\}$. In particular the representations of R-generators given in the Table reduce the potentials of the models (18) to the form:

$$2V_A(x) = 3 - (| U_{11} |^2 + | U_{1-2} |^2 + | U_{21} |^2) \tag{21a}$$

$$3V_B(x) = 6 - (| U_{11} + U_{22} |^2 + | U_{23} - U_{1-3} |^2 + | U_{32} + U_{-31} |^2) \tag{21b}$$

$$6V_C(x) = 12 - (| U_{12} - U_{21} |^2 + | U_{13} - U_{31} |^2 + | U_{23} - U_{32} |^2) \tag{21c}$$

$$2V_D(x) = 10 - (| U_{11} - U_{22} |^2 + | U_{13} - U_{32} |^2 + | U_{31} - U_{23} |^2) \tag{21d}$$

Obviously it is sufficient to introduce the parametrization

$$U(x) = 1^2(x), \quad 1(x) = \exp[(i/2)x_{ij}K^i_j] \tag{22}$$

to infer the hessians of the above potentials (c.f. Eq. (8))

$$V_A(x) = 1 + (| x_{1-3} |^2 + | x_{13} |^2) + 0(x^2) \tag{23a}$$

$$3V_B(x) = 2 + (| x_{11} - x_{22} |^2 + 4 | x_{12} |^2 + 4 | x_{1-2} |^2 + 2 | x_{1-1} |^2$$
$$+ 2 | x_{2-2} |^2 + 2 | x_{13} |^2 + 2 | x_{2-3} |^2) + 0(x^2) \tag{23b}$$

$$6V_C(x) = 12 - (| x_{12} - x_{21} |^2 + | x_{13} - x_{31} |^2 + | x_{23} - x_{32} |^2) + 0(x^2) \tag{23c}$$

$$2V_D(x) = 10 - (| x_{11} - x_{22} |^2 + | x_{13} - x_{32} |^2 + | x_{31} - x_{23} |^2) + 0(x^2) \tag{23d}$$

Hence one readily identifies charged and neutral zero modes by their H_c - content:

$$H_c = \{SU_g(2) \mid H_3^3/2, X_{-3}^3\} \otimes \{U_w(1) \mid H_1^1\}$$

$$\psi\sqrt{2} \equiv x_{12} + x_{2-1} \in \underline{1}_g^0 \otimes \underline{1}_w^\dagger, \quad \delta \equiv x_{11} - x_{22} \in \underline{1}_g^0 \otimes \underline{1}_w^0 \tag{24a}$$

$$(\chi^1, \chi^2) \equiv (x_{23}, x_{2-3}) \in \underline{2}_g \otimes \underline{1}_w^0$$

$$H_c = \{U_g(1) \mid H_1^1 + H_2^2\} \otimes \{U_w(1) \mid H_3^3 + H_2^2 - H_1^1)/2\}$$

$$\eta \equiv x_{23} \in \underline{1}_g^\dagger \otimes \underline{1}_w^0, \quad \chi \equiv x_{3-1} \in \underline{1}_g^\dagger \otimes \underline{1}_w^0 \tag{24b}$$

$$H_c = \{SO_g(3) \mid \varepsilon_{ijm} H_j^i, \quad m = 1, 2, 3\} \otimes \{U_w(1) \mid (H_1^1 + H_2^2 + H_3^3)/2\}$$

$$\eta_{ij} \equiv x_{i-j} - (1/3)\delta_{ij} x_{m-m} \in \underline{5}_g \otimes \underline{1}_w^\dagger, \quad i, j = 1, 2, 3 \tag{24c}$$

$$H_c = \{U_g(1) \mid (H_1 + H_2 + H_3)/2\} \otimes \{SO_w(3) \mid H_1^1 - H_2^2, H_3^1 - H_2^2\}$$

$$\chi_{ij} \equiv x_{i-j} - (1/3)\delta_{ij} x_{m-m} \in \underline{1}_g^\dagger \otimes \underline{5}_w, \quad i, j = 1, 2, 3 \tag{24d}$$

Now the evaluation of the potentials restricted to the flat directions (24a, ..., d) becomes straightforward. It is easily accomplished by means of the following parametrization of the coset element $1(x)$ (see Eq. (20b)).

$$1_A(x) \equiv 1(\delta)1(\psi)1(\chi/2) \tag{25a}$$

$$1_B(x) \equiv 1(\eta/2, \chi/2) \tag{25b}$$

$$1_C(x) \equiv 1(\eta_{ij}/2) \tag{25c}$$

$$1_D(x) \equiv 1(\chi_{ij}/2) \tag{25d}$$

Note that every factor $1(\phi)$on the right hand side of Eq. (25) may be represented as an exponential of a "off diagonal" matrix. Therefor the $1(\phi)$ is amenable to some standard manipulations i.e.

$$1(\phi) = \exp[i(\phi K + h.c.)] \equiv \exp\begin{bmatrix} 0 & B \\ -B^{\dagger} & 0 \end{bmatrix} = \begin{bmatrix} (1 - zz^{\dagger})^{1/2} & z \\ -z^{\dagger} & (1 - z^{\dagger}z)^{1/2} \end{bmatrix} \quad (26)$$

where

$$z = B \sin(B^{\dagger}B)^{1/2}/(B^{\dagger}B)^{1/2} \quad (27)$$

Returning to Eq. (21) with the parametrization of the matrix U given by Eqs. (20, 25-27) and performing some simple algebra one derives (c.f. Eq. (13b)):

$$2V_A(\phi) = 2 + \sin^2 \mid \psi \mid \sin^2 \mid \chi \mid = 2 + \mid \psi \mid^2 \mid \chi \mid^2 + 0(\phi^4) \quad (28a)$$

$$12V_B(\phi) = 8 + 16\sin^4 (1/2)(\mid \eta \mid^2 + \mid \chi \mid^2)^{1/2}$$

$$= 8 + (\mid \eta \mid^2 + \mid \chi \mid^2)^2 + 0(\phi^4) \quad (28b)$$

$$12V_C(\phi) = 24 - 4 \mid Tr\vec{t}\cos(\eta^{\dagger}\eta)^{1/2} \mid^2 = 24 - \mid Tr(\eta\vec{t}\eta^{\dagger}) \mid^2 + 0(\phi^4) \quad (28c)$$

$$4V_D(\phi) = 20 - 4 \mid Tr\vec{t}\cos(\chi^{\dagger}\chi)^{1/2} \mid^2 = 20 - \mid Tr(\chi\vec{t}\chi^{\dagger}) \mid^2 + 0(\phi^4) \quad (28d)$$

An actual difference in representations of the $SO(3)$ generators $\{t^a | Tr(t^a t^b) = 2\delta^{ab}\}$ in Eqs. (28c) and (28d) has been ignored as it can be compensated by an appropriate redefinition of the zero modes η or χ. Note that the evaluation of the potential (23a, 28a) is consistent with the previous calculations of the model (18a) carried out in somewhat different parameterization.[9]

The results (23, 28) lead to the main conclusion of the present article: There exist potentials with critical points (minima or maxima) described by a residual gauge symmetry $H_w = R \cap H \neq \emptyset$ and flat directions (degenerate hessians) either neutral (28b) or charged (28a,c,d) under H_w; These potentials restricted to the flat directions are nontrivial functions of the zero modes. For purposes of comparison one may recall flat potentials entertained previously. For example in field theory models with elementary scalars the flatness has been either postulated or enforced via supersymmetry;[10] On the other hand, in the framework of dynamical models with a semisimple gauge group R it has been achieved by a fine tuning of guage couplings.[11] Therefore the approach to flat potentials pursued in this work may be viewed as a natural development of the latter attempts.

Apparently, a physical significance of the above results predicated on the existence of a program which incorporates the flat potential as a necessary first step toward a dynamical realization of the Higgs potential. Such program has been advanced recently i.e. it has been proposed to construct models which possess the following features[9]:

1. Potential function (5) has a minimum which is described by a residual gauge symmetry and charged flat direction.

2. Potential function (5) restricted to its flat directions mimics a Higgs potential without mass terms.

3. Higher order corrections to the potential function (5) mimics Higgs mass terms.

Evidently the potential (28a) represents the only model which satisfies the first condition. Unfortunately none of the potentials (28) meets the second condition; The potential (28b) fails only because its flat directions are neutral under the residual gauge symmetry. A consideration of the third condition is beyond the scope of this discussion.

A further examination has revealed numerous examples of flat potentials however the realization of a bona-fide Higgs potential has proven to be a difficult task. An extensive discussion of these and related problems will be presented in forthcoming reports.

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TABLE

	$\{n_R^{(\alpha)}\}$	$H_c = H_g \otimes H_w$	R_0	R_+
$K_s(2\|3)$	$(\underline{2}+\underline{1})_{\mathrm{min}}$	$\emptyset$	$(X^0_0 - X^{-1}_{-1})/2$	$X^0_{-1}/\sqrt{2}$
$K_s(2\|3)$	$(\underline{2}+\underline{1})_{\mathrm{max}}$	$U_w(1)$	$H^1_1/2$	$X^1_{-1}/\sqrt{2}$
$K_s(2\|4)$	$(\underline{2}+2\times\underline{1})_{\mathrm{min}}$	$U_g(1)\otimes U_w(1)$	$H^1_1/2$	$X^1_{-1}/\sqrt{2}$
$K_s(2\|4)$	$(\underline{4})_{\mathrm{min}}$	$U_w(1)$	$(3H^2_2+H^1_1)/2$	$(\sqrt{3}H^2_1+2X^1_{-1})/\sqrt{2}$
$K_a(2\|4)$	$(\underline{3}+\underline{1})_{\mathrm{ext}}$	$U_w(1)$	H^1_1	$X^1_2 - X^2_{-1}$
$K_s(2\|5)$	$(\underline{2}+\underline{3})_{\mathrm{min}}$	$U_w(1)$	$H^2_2/2+H_1$	$X^2_{-2}/\sqrt{2}+H^1_0$
$K_s(2\|5)$	$(\underline{2}+\underline{3})_{\mathrm{max}}$	$\emptyset$	$H^2_2/2+(X^1_1-X^0_0)$	$X^2_{-2}/\sqrt{2}+(X^1_{-1}-X^{-1}_0)$
$K_s(2\|5)$	$(\underline{2}+\underline{3})_{\mathrm{ext}}$	$\emptyset$	$(X^1_1-X^0_0)/2+(X^{-1}_{-1}-X^2_2)$	$X^1_0/\sqrt{2}+(X^{-1}_{-2}-X^{-2}_2)$
$K_s(2\|5)$	$(\underline{4}+\underline{1})_{\mathrm{ext}}$	$U_w(1)$	$(3H^2_2+H^1_1)/2$	$(\sqrt{3}H^2_1+2X^1_{-1})/\sqrt{2}$
$K_a(2\|6)$	$(\underline{3}+3\times\underline{1})_{\mathrm{ext}}$	$SU_g(2)\otimes U_w(1)$	H^1_1	$X^1_2 - X^2_{-1}$
$K_a(2\|6)$	$(\underline{3}+\underline{2}+\underline{1})_{\mathrm{min}}$	$\emptyset$	$(X^3_3-X^{-1}_{-1})/2+(X^1_1-X^2_2)$	$X^3_{-1}/\sqrt{2}+(X^1_a-X^a_2)$
				$(\sqrt{2}z^a = z^{-2}+z^3)$
$K_a(2\|6)$	$(\underline{3}+\underline{2}+\underline{1})_{\mathrm{max}}$	$U_g(1)$	$(X^{-2}_{-2}-X^{-1}_{-1})/2+(X^2_2-X^1_1)$	$X^{-2}_{-1}/\sqrt{2}+(X^2_3-X^3_1)$
$K_a(2\|6)$	$(\underline{3}+\underline{2}+\underline{1})_{\mathrm{ext}}$	$U_w(1)$	$H^3_3/2+H^1_1$	$X^3_{-3}/\sqrt{2}+(X^1_2-X^2_{-1})$
$K_a(2\|6)$	$(5+1)_{\mathrm{ext}}$	$U_w(1)$	$2H^2_2+H^1_1$	$\sqrt{2}H^2_1+\sqrt{3}(X^1_3-X^3_{-1})$
$K_s(2\|6)$	$(2\times\underline{2}+\underline{2}^*)_{\mathrm{min}}$	$U_g(1)\otimes U_w(1)$	$(H^3_3+H^2_2-H^1_1)/2$	$(X^3_{-3}+H^2_1/\sqrt{2}$
$K_s(2\|6)$	$(3\times\underline{2})_{\mathrm{max}}$	$SO_g(3)\otimes U_w(1)$	$(H^3_3+H^2_2+H^1_1)/2$	$(X^1_{-1}+X^2_{-2}+X^3_{-3})/\sqrt{2}$
$K_s(2\|6)$	$(\underline{3}+\underline{2}+\underline{1})_{\mathrm{min}}$	$\emptyset$	$(X^3_3-X^a_a)/2+H^1_1$	$X^3_a/\sqrt{2}+(X^1_b-X^b_{-1})$
			$(\sqrt{2}z^a = z^2+z^{-2})$	$(\sqrt{2}z^b = -z^2+z^{-2})$
$K_s(2\|6)$	$(\underline{3}+\underline{2}+\underline{1})_{\mathrm{ext}}$	$U_w(1)$	$H^3_3/2+H^1_1$	$X^3_{-3}/\sqrt{2}+(X^1_a-X^a_{-1})$
				$(\sqrt{2}z^a = z^2+z^{-2})$
$K_s(2\|6)$	$(\underline{4}+\underline{2})_{\mathrm{min}}$	$\emptyset$	$(X^a_a-X^{-3}_{-3}+3H^2_2+X^b_b-X^{-1}_{-1})/2$	$X^a_{-3}+2X^b_{-1}+\sqrt{3}X^2_b-\sqrt{3}X^{-1}_{-2})/\sqrt{2}$
			$(3z^a = z^3+\sqrt{8}z^1)$	$(3z^b = -\sqrt{8}z^3+z^1)$
$K_s(2\|6)$	$(\underline{4}+\underline{2})_{\mathrm{ext}}$	$U_w(1)$	$(H^3_3+3H^2_2+H^1_1)/2$	$(X^3_{-3}+2X^1_{-1}+\sqrt{3}H^2_1)/\sqrt{2}$
$K_s(2\|6)$	$(\underline{6})_{\mathrm{min}}$	$U_w(1)$	$(5H^3_3+3H^2_2+H^1_1)/2$	$(\sqrt{5}H^3_2+2\sqrt{2}H^2_1-3X^1_{-1})\sqrt{2}$

TABLE CONTINUED

$K_a(3\|6)$	$(\underline{3}+3\times\underline{1})_{\rm min}$	$SU_g(2)\otimes SU_w(2)$	$H_1^1/2$ $(K_1^1-2X_2^2)/2\sqrt{3}$	$X^1_{-1}/\sqrt{2},\ X^1_2/\sqrt{2},\ X^2_{-1}/\sqrt{2}$
$K_a(3\|6)$	$(2\times\underline{3})_{\rm max}$	$U_g(1)\otimes S0_w(3)$	$(H_1^1-H_2^2)/2$ $(K_1^1+K_2^2-2K_3^3)/2\sqrt{3}$	$K_2^1/\sqrt{2},\ (X_3^1+X_{-3}^{-2})/\sqrt{2}$ $(X_2^3+X_{-1}^{-3})/\sqrt{2}$
$K_a(3\|6)$	$(2\times\underline{3})_{\rm ext}$	$U_g(1)\otimes SU_w(2)$	$(H_1^1-H_2^2)/2$ $(K_1^1+K_2^2-2K_3^3)/2\sqrt{3}$	$H_2^1/\sqrt{2},\ (X_3^1+X_{-3}^{-2})/\sqrt{2}$ $(X_2^3-X_{-1}^{-3})/\sqrt{2}$
$K_s(3\|6)$	$(\underline{3}+3\times\underline{1})_{\rm max}$	$SO_g(3)\otimes SO_w(3)$	$H_1^1/2,\ (K_1^1-2X_a^a)/2\sqrt{3}$ $(\sqrt{2}z^a=z^2+z^{-2})$	$X^1_{-1}/\sqrt{2},\ X^1_a/\sqrt{2},\ X^a_{-1}/\sqrt{2}$
$K_s(3\|6)$	$(2\times\underline{3})_{\rm max}$	$U_g(1)\otimes SO_w(3)$	$(H_1^1-H_2^2)/2$ $(K_1^1+K_2^2-2K_3^3)/2\sqrt{3}$	$K_2^1/\sqrt{2},\ (X_3^1+X_{-3}^{-2})/\sqrt{2}$ $(X_2^3+X_{-1}^{-3})/\sqrt{2}$
$K_s(3\|6)$	$(2\times\underline{3})_{\rm min}$	$U_g(1)\otimes SU_w(2)$	$(H_1^1-H_2^2)/2$ $(K_1^1+K_2^2-2K_3^3)/2\sqrt{3}$	$H_2^1/\sqrt{2},\ (X_3^1+X_{-3}^{-2})/\sqrt{2}$ $(X_2^3-X_{-1}^{-3})/\sqrt{2}$

Table - Critical points of the models defined in Eq. (14) are described by their

(*a*) R - content $\{n_R^{(\alpha)}\}$ of $R\subset G$ and nature as a relative minimum (min), maximum (max) or saddle (ext) point,

(*b*) Symmetry (H_c) with global (H_g) and risidual gauge (H_w) components,

(*c*) R - generators with zero (R_0) and positive (R_+) roots. The realization of the generators (15) is switched to the form $X_j^i=z^i\partial/\partial z^j$.