## MULTIBUNCH INSTABILITIES IN SUBSYSTEMS OF 0.5 AND 1.0 TEV LINEAR COLLIDERS\*

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#### ABSTRACT

The acceleration of multiple bunches per RF fill of the linac accelerating structures is an important feature of the SLAC design for a next-generation linear collider, in order to use the RF as efficiently as possible and to obtain a higher luminosity. In this paper, we give simulation results on the control of multibunch instabilities in the subsystems of a 1.0 Tev linear collider (TLC) and an "intermediate" linear collider (ILC) of about 0.5 TeV center of mass energy.

#### INTRODUCTION

The acceleration of multiple bunches per RF fill of the linac accelerating structures is an important feature of the SLAC design for a very high energy linear collider (0.5 to 1.0 Tev). The reasons for multibunching are to use the RF as efficiently as possible and to obtain a higher luminosity. However, the problems of cumulative beam breakup in the linacs and coupled bunch instabilities in the damping rings must be addressed. In both cases, damped acceleration cavities may be a useful cure.<sup>1)</sup> In such cavities, the transverse dipole wake is strongly damped by the use of axial slots through the irises of the RF structure coupled to radial waveguides. The Q of the fundamental transverse mode can be made as low as about 10 in this way, without significant adverse effect on the longitudinal accelerating mode, and the Q's of higher order modes are expected to be at least as low as the Q of the fundamental.

Tuning the frequency of the fundamental transverse dipole mode to allow placement of the bunches near zero crossings of the wake fields will probably also be necessary in the main linacs. The transverse dipole wake for the accelerating structure considered here is strongly dominated by its fundamental mode and has zero crossings that are approximately equally spaced. Therefore it is possible to place all the bunches in a train near zero crossings of the wake field, if the ratio of the frequency of the fundamental dipole mode to the frequency of the accelerating RF is tuned to satisfy:

$$\frac{1}{2} n \lambda_{W_{\perp}} = m \lambda_{rf} = \ell \quad , \tag{1}$$

where  $\ell$  is the bunch spacing, *m* and *n* are integers, and  $\lambda_{rf}$  and  $\lambda_{W_{\perp}}$  are the wavelengths of the RF and the fundamental dipole wake mode.

The theoretical models and details of the computer simulations used in our study of multi-bunch instabilities have been discussed in more detail elsewhere.<sup>2,3)</sup> The main emphasis of this paper will be on results for TLC and ILC parameter sets at the present stage of design.

### MULTIBUNCH BEAM DYNAMICS

The bunches are taken to be point macroparticles, with an equal charge of N electrons in each bunch. The spacing  $\ell$  between adjacent bunches is uniform and is an integral number of RF wavelengths in each subsystem.

The transverse dipole wake function is a sum of modes of the following form:

$$W_{\perp}(z) = \sum_{m} W_{m} \sin(K_{m} z) \exp\left(-\frac{K_{m} z}{2Q_{m}}\right) \quad , \qquad (2)$$

where z is the distance behind the exciting bunch.  $K_m$  is the wavenumber and  $Q_m$  is the quality factor of mode m, and the  $W'_m s$  are constant coefficients. The units of  $W_{\perp}(z)$ are V/Coul/m<sup>2</sup>.

#### 2.1 Transverse Dynamics in Linacs

The acceleration is assumed to be linear:  $\gamma = \gamma_0 + Gs$ , where  $\gamma$  is the energy divided by the rest energy and Gis a constant. We use the smooth-focusing approximation  $k(s) = 1/\beta(s)$  for the focusing function, where  $\beta(s)$  is the "average" betatron function at longitudinal position s. The focusing is assumed to vary as

Presented at the DPF Summer Study: Snowmass '88, High Energy Physics in the 1990's, Snowmass, Colorado, June 27–July 15, 1988

<sup>\*</sup> Work supported by the Department of Energy, contract DE-AC03-76SF00515.

$$k(s) = \left(\frac{\gamma_0}{\gamma(s)}\right)^{1/2} k_0 \quad , \tag{3}$$

unless stated otherwise. Taking acceleration into account, the equation of motion for the transverse offset  $x_n$  of bunch n is:

$$\gamma(s)x_n'' + \gamma'(s)x_n' + \gamma(s)k_n^2(s)x_n = F_n(s) \quad . \tag{4}$$

where the driving term due to the wake is:

$$F_{n}(s) \equiv \frac{Ne^{2}}{mc^{2}} \sum_{j=1}^{n-1} W_{\perp}[(n-j)\ell] x_{j}(s) \quad .$$
 (5)

Here primes denote derivatives with respect to s, and m is the rest mass of the electron. The transverse offsets are taken to be all in one plane. The WKB solutions of the homogeneous equation are

$$a_{n}^{\pm}(s) = x_{n}^{\pm}(0) \left[ \frac{\gamma_{0} k_{n}(0)}{\gamma(s) k_{n}(s)} \right]^{1/2} \exp\left[ \pm i \int_{0}^{s} k_{n}(s') ds' \right]$$
  
$$= x_{n}^{\pm}(0) \left[ \frac{\gamma_{0}}{\gamma(s)} \right]^{1/4} \exp\left[ \pm i \int_{0}^{s} k_{n}(s') ds' \right] , \qquad (6)$$

where we have used Eq. (3) for the variation of  $k_n$  with  $\gamma$ . Assuming the acceleration is adiabatic and the blowup of the transverse offset is slow compared to a betatron oscillation, it is straightforward to show (see Ref. 2 or 3 for details) that the general solution to the inhomogeneous equation for bunch n is

$$x_n(s) = a_n^+ x_n^+(s) + a_n^- x_n^-(s) + \int_0^s G_n(s, s') F_n(s') ds' \quad , \ (7)$$

where  $a_n^+$  and  $a_n^-$  are arbitrary constants, the Green function is:

$$G_n(s,s') = [\gamma(s)\gamma(s')k_n(s)k_n(s')]^{-1/2}\sin\psi_n(s,s') \quad , \quad (8)$$

and

$$\psi_n(s,s') \equiv \int_{s'}^{s} k_n(s'') ds'' \tag{9}$$

is the phase advance for bunch n. If we take the "positive phase" WKB solution

$$x_1(s) = x_1(0) \left(\frac{\gamma_0}{\gamma(s)}\right)^{1/4} \exp\left[\psi_1(s,0)\right] ,$$
 (10)

as the motion for the first bunch, assume  $a_n = 0$  for all n > 1, and drop a term with rapidly oscillating integrand, the solution for the transverse motion of bunch n may be written:

$$\begin{aligned} x_{n}(s) &= \left[ x_{n}(0) + \frac{Ne^{2}}{2i\gamma_{0}mc^{2}k_{n}(0)} \int_{0}^{s} \left( \frac{\gamma_{0}}{\gamma(s')} \right)^{1/4} \exp[-i\psi_{n}(s',0)] \right. \\ &\times \sum_{j=1}^{n-1} W_{\perp}[(n-j)\ell]x_{j}(s')ds' \right] \\ &\times \left( \frac{\gamma_{0}}{\gamma(s)} \right)^{1/4} \exp[+i\psi_{n}(s,0)] \end{aligned}$$

$$(11)$$

We note that this result is equivalent (see Refs. 2 and 3) to that obtained ignoring acceleration, from the equations of motion

$$x_n'' + k_n^2 x_n = \frac{Ne^2}{E} \sum_{j=1}^{n-1} W_{\perp}[(n-j)\ell] x_j(s) \quad , \qquad (12)$$

with solution

$$x_{n}(s) = \left[x_{n}(0) + \frac{Ne^{2}}{2iEk_{n}} \int_{0}^{s} e^{-ik_{n}s'} \times \sum_{j=1}^{n-1} W_{\perp}[(n-j)\ell]x_{j}(s')ds'\right] e^{ik_{n}s} , \qquad (13)$$

provided that we interpret E and the  $k_n$  to be the energy and the focusing functions at the beginning of the linac, and s to be not the true distance along the accelerator, but rather an "effective distance"  $\psi(s)/k_0$ , where  $\psi(s)$  is the phase advance in the actual distance s along the linac and  $k_0$  is the focusing function at the beginning of the linac:

$$s_{eff} = \frac{\psi(s)}{k_0} = \frac{1}{k_0} \int_0^s k(s) ds$$

$$= \frac{2\gamma_0^{1/2}}{G} \left[ \gamma^{1/2} - \gamma_0^{1/2} \right] \quad . \tag{14}$$

Note that if  $\gamma(L) \gg \gamma_0$  at s = L, the end of the linac, the effective length of the linac is approximately

$$L_{eff} = 2 \left(\frac{\gamma_0}{\gamma}\right)^{1/2} L \quad . \tag{15}$$

Solutions for the transverse blowup of each bunch as a function of s may be obtained by numerical integration of the above equations for  $x_n(s)$ , and we shall present results in later sections for parameter sets of interest.

Finally, we note that in cases where the wake field is strongly damped, a bunch will only see a significant wake from the immediately preceding bunch, and a simple "daisy chain" model gives excellent agreement with the simulation results. Using the effective length approximation the equations of motion become:

$$x_1'' + k^2 x_1 = 0$$
  

$$x_n'' + k^2 x_n = \frac{N e^2 W_{\perp}(l)}{E} x_{n-1} \qquad (n > 1) \quad .$$
(16)

Assuming  $x_n(s) = a_n(s)e^{iks}$ , the solution for the envelope function  $a_n(s)$  for initial conditions  $a_n(0) = 1$  is

$$a_n(s) = \sum_{j=0}^{n-1} \frac{(-i\sigma s)^j}{j!} \quad . \tag{17}$$

Since this is the first *n* terms of the Taylor series expansion of  $\exp(-i\sigma_0 s)$ , it is apparent that the criterion for little or no blow-up in the linac under these conditions is  $|\sigma L| < 1$ , that is:

$$\frac{Ne^2 |W_{\perp}(l)|L}{2kE} < 1 \quad , \tag{18}$$

where L is the effective length of the linac, and k and E are the values of focusing function and energy at the beginning of the linac.

In the main linacs of a TeV collider with damped acceleration cavities, we can have  $|\sigma L| \sim 1$ . Thus for *n* sufficiently large,  $a_n(s)$  is approximately  $\exp(-i\sigma s)$ , and there is almost no blowup of the  $n^{th}$  bunch.

## 2.2 Transverse Dynamics in Damping Rings

We assume the damping ring impedance is dominated by the contribution of the RF cavities and that damped acceleration cavities are used. If the Q of the transverse modes is sufficiently low that turn-to-turn wakes are negligible, then the equations of transverse motion are similar to those for a linac, except for the presence of a coherent damping term due to tune spread and/or head-tail damping, which we parameterize in a constant  $\alpha_{cd}$  (units m<sup>-1</sup>):

$$x_n'' + 2\alpha_{cd}x_n' + k^2 x_n(s) = \frac{Ne^2}{E} \sum_{j=1}^{n-1} W[(n-j)\ell] x_j(s) \quad . \tag{19}$$

We divide the motion on one turn through the ring into two parts: (1) the motion through the RF cavities, where the effect of the wake fields is modelled by a localized kick, and (2) a mapping around the rest of the ring, with specified average beta function and phase advance. Let us define  $x_n^-(x_n^+)$  and  $x_n'^-(x_n'^+)$  to be the transverse offset and angular divergence of bunch n just before(after) the RF cavities (assumed to be clumped together). Then, assuming that  $\beta' = 0$  at the location of the cavities,

$$\begin{pmatrix} x_n^-\\ x_n'^- \end{pmatrix} = e^{-\alpha_{cd}C} \begin{pmatrix} \cos\mu & \beta\sin\mu\\ -\frac{1}{\beta}\sin\mu & \cos\mu \end{pmatrix} \begin{pmatrix} x_n^{old} +\\ x_n'^{old} + \end{pmatrix}$$
(20)

Here  $x_n^{old}$  + and  $x_n^{old}$  + are the transverse offset and angular divergence just after passing through the RF cavities on the preceding turn, C is the circumference of the ring, and  $\mu$ 

is the coherent phase advance around the ring. One may also show (see Ref. 3):

$$\begin{pmatrix} x_n^+\\ x_n'^+ \end{pmatrix} = \begin{pmatrix} x_n^-\\ x_n'^- + \frac{Ne^2}{E} L_{rf} \sum_{j=1}^{n-1} W[(n-j)\ell] x_j^- \end{pmatrix},$$
(21)

where  $L_{rf}$  is the total length of the RF cavities and W(z) is the wake per unit length in the cavities. The results presented later on damping ring transverse instabilities are obtained using a tracking program based on these equations.

# PARAMETERS OF COLLIDER SUB-SYSTEMS

We assume a train of ten bunches is to be accelerated on each RF fill, and that the interbunch spacing within the train is 21.0 cm (i.e., two wavelengths at S-band or twelve RF wavelengths at 17.1 GHz).

Table 1. Parameters for TLC.

Subsystem	Energy (GeV)	Particles per bunch
Injection accelerators	$0.18 \rightarrow 1.8$	$2 \times 10^{10}$
Damping rings	1.8	$2 \times 10^{10}$
Preaccelerators	$1.8 \rightarrow 18.0$	$1.8 \times 10^{10}$
Main linacs	$18.0 \rightarrow 500.$	$1.67 \times 10^{10}$

Table 2. Parameters for ILC.

Subsystem	Energy (GeV)	Particles per bunch
Injection accelerators	$0.18 \rightarrow 1.8$	$1 \times 10^{10}$
Damping rings	1.8	$1 \times 10^{10}$
Preaccelerators	$1.8 \rightarrow 18.0$	$9 \times 10^{9}$
Main linacs	$18.0 \rightarrow 250.$	$8.33 \times 10^{9}$

Other parameters of individual collider subsystems are given in Table 1 for the TLC design, and in Table 2 for the ILC design.

We shall discuss results for each of these subsystems in the TLC design, and for the main linacs of an ILC design. Since the parameters for other subsystems of the ILC will be assumed the same as those for the TLC except for a lower charge per bunch, it is clear that instabilities can be controlled in these subsystems of the ILC if they can be controlled in the corresponding subsystems of the TLC.

# **RESULTS FOR MAIN LINACS IN THE TLC**

We assume a main linac accelerating frequency of 17.1 GHz, and a disk-loaded accelerating structure with

Table 3. Parameters for main linacs at 17.1 GHz.

Number of bunches	10
Number of particles per bunch	$1.67 \times 10^{10}$
Bunch spacing $\ell$	21.0 cm
Initial energy of linac	18 GeV
Final energy of linac	500 GeV
Linac length	3000 m
Initial beta function	$3.2 \text{ m} \\ (k_0 = 0.3125m^{-1})$

cell length 5.83 mm, internal cell radius 7.47 mm, and relatively large iris radius of 3.47 mm.

The parameter set used is shown in Table 3. Each linac accelerates 10 bunches per RF fill, to an energy of 0.5 TeV. The spacing of 12 RF wavelengths between bunches (at the assumed RF frequency) is chosen in order to match the energy extracted by the bunch train to the energy input from the RF. This gives very nearly the same acceleration for every bunch in the bunch train.<sup>4)</sup>

The RF wavelength at 17.1 GHz is 1.75 cm, and the wavelength of the fundamental mode of the unmodified transverse dipole wake for our cavity design is 1.36 cm. If the frequency of the fundamental mode is shifted slightly, so that its wavelength is 1.31 cm, then Eq. (1) is satisfied with n=32, and we have

$$\frac{\lambda_{rf}}{\lambda_{W_{\perp}}} = \frac{4}{3} \quad . \tag{22}$$

When this relation is satisfied, the frequency of the fundamental transverse mode is 477.85  $m^{-1}$ , which we shall denote by  $K_{W_0}$ .

In Fig. 1, we show "tuning curves" of the maximum transverse amplitude  $x_{max}$  in the bunch train as a function of the frequency of the fundamental transverse dipole mode, for values of Q = 30, 40, 50, and 60. The value of  $x_{max}$  is the maximum of the amplitudes reached by all bunches as they travel down the linac, normalized by dividing out the adiabatic damping factor  $\left(\frac{\gamma_0}{\gamma}\right)^{1/4}$ . The central frequency, at which  $\frac{\lambda_{rf}}{\lambda_{W_{\perp}}} = \frac{4}{3}$ , is 477.85  $m^{-1}$ . The range about the central frequency shown in the figure is only  $\pm 0.1\%$ , so the tolerance on tuning the frequency of the low-Q dipole mode with respect to the accelerating mode is rather tight compared to its bandwidth.

Note that at M bunch spacings behind a bunch, the wake field has damped by a factor of about

$$\exp\left(-\frac{K_{W_0}\ell M}{2Q}\right) \approx \exp\left(-\frac{50M}{Q}\right)$$
, (23)

and so a given bunch can feel a significant wake field from several bunches ahead of it, unless it is near a node of these wakes and/or the value of Q is well below 50. Figure 2 shows tuning curves for the smaller values Q = 15, 20, and



Fig. 1. Maximum transverse amplitude  $x_{max}$ (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of Q = 30, 40, 50, and 60, at 17.1 GHz accelerating frequency. The central frequency, where  $\lambda_{rf}/\lambda_{W_{\perp}} = 4/3$ , is 477.85 m<sup>-1</sup>. The spread shown about  $K_{W_0}$  is  $\pm 0.1\%$ .



Fig. 2. Maximum transverse amplitude  $x_{max}$ (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of Q = 15, 20, and 25, at 17.1 GHz accelerating frequency. The central value of the frequency, where  $\lambda_{rf}/\lambda_{W_{\perp}} = 4/3$ , is 477.85 m<sup>-1</sup>. The spread shown about  $K_{W_0}$  is  $\pm 1\%$ .

25. The spread around  $K_{W_0}$  in this figure is  $\pm 1\%$ , and as can be seen from the figure, for Q = 15, we have  $x_{max} < 2$  for frequencies within this range.

For low values of Q, the frequency of the fundamental transverse mode becomes less sharply defined; the full width at half-maximum of the resonance around the central frequency  $K_{W_0}$  is  $\Gamma \equiv K_{W_0}/Q$  (and the central frequency is shifted slightly from that of the undamped mode). So it is also of interest to compare the ratio R of the tuning tolerance for a given Q to the full width  $\Gamma$  of the resonance at that Q:

$$R \equiv \frac{\Delta K_{W_0}}{\Gamma} \quad . \tag{24}$$

Table 4. Tuning parameters for the fundamental transverse dipole mode for TLC main linacs.

Q	$\Delta K_{W_0}(\mathrm{m}^{-1})$	$\Gamma = \frac{K_{W_0}}{Q}(\mathrm{m}^{-1})$	$\frac{\Delta K_{W_0}}{K_{W_0}}$	$\frac{\Delta K_{W_0}}{\Gamma}$
15	>11.0	31.9	>2.3%	>34%
20	4.37	23.9	0.91%	18%
25	2.56	19.1	0.54%	13%
30	1.60	15.9	0.33%	10%
40	0.87	11.9	0.18%	7.3%
50	0.57	9.56	0.12%	6.0%
60	0.38	7.96	0.08%	4.8%

In Table 4, we show the full-width tuning tolerance  $\Delta K_{W_0}$  for the criterion  $x_{max} \leq 2$ , the full-width  $\Gamma$  of the resonance peak, the tuning tolerance expressed as a percentage of the undamped central frequency, and the ratio R. The parameters used and the values of Q tabulated are those used in Figs. 1 and 2. The lower values of Q, say up to 30 or so, seem to be the most desirable in that the tolerance on tuning is at least 10% of the bandwidth of the resonance; this should be straightforward to do. Such Q's have already been achieved in models (Ref. 1).

# **RESULTS FOR MAIN LINACS IN THE ILC**

It is also of interest to consider a design for an "intermediate" energy linear collider with half the center of mass energy. The parameters for the main linacs are as shown in Table 3 for the TLC, except that the final energy of each linac is 250 GeV and the number of particles per bunch is  $8.33 \times 10^9$ . The lower charge per bunch more than compensates for the fact that since the energy is lower the beam is less "stiff" (also, the initial focusing function is the same as in the TLC, but weakens less quickly with distance, according to Eq. (3)). The tuning curves for the ILC case are shown in Fig. 3.

The tuning parameters corresponding to Fig. 3 are shown in Table 5.

## RESULTS FOR THE PREACCELERATORS (TLC)

We turn now to the question of beam breakup in the preaccelerators of the linear collider. We assume the same type of large-irised, and possibly damped, accelerating structure as in the main linacs, but scaled to S-band.

The bunch spacing chosen for the main linacs, about 21.0 cm, is two S-band wavelengths, and other parameters are as shown in Table 6. In the preaccelerator the problem of beam breakup is much less acute than in the main



Fig. 3. Maximum transverse amplitude  $x_{max}$ (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of Q = 15, 20, 25, and 30, using the ILC design parameters. The central frequency, where  $\lambda_{rf}/\lambda_{W\perp} = 4/3$ , is  $477.85 \text{ m}^{-1}$ . The spread shown about  $K_{W_0}$  is  $\pm 1\%$ .

Table 5. Tuning parameters for the fundamental transverse dipole mode for ILC main linacs.

Q	$\Delta K_{W_0}(\mathrm{m}^{-1})$	$\Gamma = \frac{K_{w_0}}{Q}(\mathrm{m}^{-1})$	$\frac{\Delta K_{W_0}}{K_{W_0}}$	$\frac{\Delta K_{W_0}}{\Gamma}$
15	>15.0	31.9	>3%	>50%
20	7.20	23.9	1.5%	30%
25	3.73	19.1	0.78%	20%
30	2.51	15.9	0.53%	16%

Table 6. Parameters for preaccelerator.

RF accelerating frequency	2.856 GHz
Number of bunches	10
Number of particles per bunch	$1.8 \times 10^{10}$
Bunch spacing $\ell$	21.0 cm
Initial energy of preaccelerator	1.8 GeV
Final energy of preaccelerator	18 GeV
Acceleration gradient	50 MeV/m

linacs. It is not necessary to tune the frequency of the fundamental transverse dipole mode relative to the accelerating mode, so as to place all bunches near wake zero crossings. In our example the placement of the bunches is neither particularly favorable nor unfavorable.

Initial $\beta$ (meters)	Q	Max transverse blow-up factor
4.0	œ	1.22
2.0	$\infty$	1.05
1.0	$\infty$	1.01
4.0	40.0	1.05
2.0	40.0	1.01

Table 7. Transverse beam blowup at end of preaccelerators.

Results for several choices of Q and focusing function are given in Table 7. Note that we are assuming that the transverse focusing weakens with distance along the linac according to Eq. (3), and that one might choose to keep it more nearly constant.

# **RESULTS FOR THE INJECTION ACCELERATORS (TLC)**

Accelerators are also needed to take the bunches up to the damping ring energy of 1.8 GeV.

RF accelerating frequency	2.856 GHz
Number of bunches	10
Number of particles per bunch	$2 \times 10^{10}$
Bunch spacing $\ell$	21.0 cm
Initial energy	0.18 GeV
Final energy	1.8 GeV
Acceleration gradient	25 or 50 MeV/m

Table 8. Parameters for injection accelerator.

A representative parameter set is given in Table 8. We again assume an accelerating structure of the same type as in the main linacs, including damping if necessary, but scaled to S-band. Due to the lower energy, somewhat stronger focusing is needed here than is needed in the case of the preaccelerators.

Results are shown in Table 9 for two values of acceleration gradient, again assuming that the focusing scales according to Eq. (3).

In Table 10, we show the results obtained assuming the focusing function does not vary with s.

## **RESULTS FOR DAMPING RINGS (TLC)**

We have carried out simulations for the damping ring parameters shown in Table 11, using two examples of RF cavity designs. One is the TLC main linac cavity described earlier, scaled to the damping ring frequency. The other is a scaled PEP cavity, which has nose cones and a relatively smaller iris. In both cases we assume the Q's of the transverse modes may be lowered by modifying the cavities to

Table 9.	Transverse	beam blowup
at end	of injection	accelerators.

Accel. gradient MeV/m	Initial $\beta$ (meters)	Q	Max transverse blowup factor
50.0	4.0	8	1.25
"	2.0	$\infty$	1.05
"	4.0	40	1.05
25.0	4.0	œ	2.40
n	2.0	œ	1.20
n	1.0	∞	1.06
'n	4.0	40	1.18
"	2.0	40	1.07

Table 10. Transverse beam blowup at end of Injection Accelerators with focusing assumed constant.

Accel. gradient MeV/m	β (meters)	Q	Max transverse blowup factor
50.0	4.0	8	1.06
25.0	4.0	$\infty$	1.32
n	2.0	œ	1.08
77	1.0	œ	1.02
n	4.0	40	1.08

Table 11. Damping ring parameters.

Number of bunch trains	10
Number of bunches per train	10
Number of particles per bunch	$2 \times 10^{10}$
Particle energy	1.8 GeV
Ring circumference	154 m
RF frequency	1.428 GHz
Bunch spacing within a train	21.0 cm

include axial slots coupled to radial waveguides, as in the main linacs.

For both cavities, the wavenumber  $K_1$  of the fundamental transverse mode is approximately 40 m<sup>-1</sup>. The number of *e*-foldings of this mode per turn around the circumference *C* of the ring is

$$\frac{K_1C}{2Q} \approx \frac{3000}{Q} \quad , \tag{25}$$

and the number of e-foldings between bunch trains (assuming ten trains symmetrically placed in the ring) is about

300/Q. Thus we expect turn-to-turn and train-to-train wake effects to be negligible for sufficiently low Q's.

Simulations were done with the tracking program discussed earlier. For Q = 30 and a coherent damping parameter  $\alpha_{cd}$  of at least  $10^{-6}$ m<sup>-1</sup>, none of the bunches ever get an amplitude larger than their initial injected amplitude, and the centroids of all the bunches have damped to small amplitudes after 50000 turns, due to the effect of  $\alpha_{cd}$ . Note that this value of  $\alpha_{cd}$  gives a coherent damping time of 3.33 msec, which corresponds to about 6500 turns. The amplitudes of the second and tenth bunches as a function of turn number are shown in Fig. 4, for the two cavities, for Q = 30 and  $\alpha_{cd} = 10^{-6}$ m<sup>-1</sup>.



Fig. 4. Offset of second and tenth bunch as a function of turn number, for the scaled PEP and scaled TLC damping ring cavities. The value of Q for the cavity is 30 and the coherent damping parameter  $\alpha_{cd}$  is  $10^{-6}m^{-1}$ .

## CONCLUSIONS AND ACKNOWLEDGMENTS

Control of transverse multibunch instabilities appears to be feasible in all the subsystems of a linear collider studied here, namely the main linacs, injector linacs, preaccelerators, and damping rings. In all these subsystems, damped acceleration cavities may be a useful tool, although in the preaccelerators and injection accelerators one can probably get by without them. In the main linacs, both damped cavities and placing the bunches near wake zero crossings are essential. In the damping rings, it is highly desirable to use damped cavities to decouple the bunch trains from each other as well as to reduce the transverse wakes within each train.

We thank the members of the linear collider working group at Snowmass for useful discussion and comments.

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