## A STUDY OF $K^{-} \pi^{+}$SCATTERING IN THE REACTION $K^{-} p \rightarrow K^{-} \pi^{+} n$ AT $11 \mathrm{GeV} / c^{*}$

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#### Abstract

Results from a high statistics study of the reaction $K^{-} p \rightarrow K^{-} \pi^{+} n$ are presented. These results are based on data obtained with an $11 \mathrm{GeV} / \mathrm{c}$ beam using the LASS spectrometer at SLAC. The mass dependence of the spherical harmonic moments provides clear evidence for the production of the complete leading orbitally excited $K^{*}$ series up through $J^{P}=5^{-}$. These moments are used to perform an energy independent partial wave analysis of the $K^{-} \pi^{+}$system from threshold to $2.6 \mathrm{GeV} / \mathrm{c}^{2}$ using a $t$-dependent parametrization of the production amplitudes. The amplitudes corroborate the leading $K^{*}(892), K_{2}^{*}(1430), K_{3}^{*}(1780), K_{4}^{*}(2060)$, and $K_{5}^{*}(2380)$ resonances observed directly in the moments, and also provide new evidence for underlying states. The $0^{+}$amplitude contains the $K_{0}^{*}(1350)$ and a second $0^{+} K_{0}^{*}(1950)$ at higher mass. The $1^{-} K^{*}(1790)$ seen in earlier two and threebody analyses is confirmed, and evidence is provided for a suppressed $K^{-} \pi^{+}$decay mode of a second $1^{-}$state, the $K^{*}(1410)$, which has been seen in earlier three-body analyses.


## 1. Introduction

In this paper, we present an analysis of high statistics data on the reaction

$$
\begin{equation*}
K^{-} p \rightarrow K^{-} \pi^{+} n \tag{1.1}
\end{equation*}
$$

at $11 \mathrm{GeV} / \mathrm{c}$ incident momentum taken with the LASS spectrometer at SLAC. A $t$-dependent parametrization is used to isolate the $\pi$ exchange contribution to this reaction which then allows energy independent $K^{-} \pi^{+}$scattering amplitudes to be obtained from threshold to $2.6 \mathrm{GeV} / \mathrm{c}^{2}$. Many of these amplitudes display clear resonant structures which are fitted to simple mass dependent parametrizations to obtain measurements for the masses and widths of the observed $K^{*}$ resonances.

For a variety of reasons, the strange mesons are an excellent place to try to understand a pure $q \bar{q}$ spectrum. The states have overt flavor so there is no isoscalarisovector mixing and no confusion with pure glueball states, and the experimental widths tend to be somewhat less than the equivalent non-strange light quark states. Previous studies of reaction (1.1) have been extremely fruitful in the study of the strange meson spectrum. This reaction is topologically very simple, contains only natural spin-parity meson decays, and has a large cross-section which is dominated by $\pi$ exchange at small values of momentum transfer ( $t^{\prime}=t-t_{\text {min }}$ ) [1]. These features have allowed observations of the leading $K^{*}$ states to be made directly in the spherical harmonic moments. Previous experiments [2] have observed the three lowest-mass orbitally excited states-the spin-parity $1^{-} K^{*}(892)$, the $2^{+} K_{2}^{*}(1430)$, and the $3^{-} K_{3}^{*}(1780)$-and provided evidence for the next Lexcitation, the $4^{+} K_{4}^{*}(2060)$ [3]. A moments analysis of the present data has allowed new measurements for these resonances and provided evidence of a new $5^{-}$state,
the $K_{5}^{*}(2380)$ [4]. Prior partial wave analyses have confirmed the observations of the first four leading $K^{*}$ states, and have provided clear evidence for underlying resonances in the S-wave, the $K_{0}^{*}(1350)$, and the P-wave, the $K^{*}(1790)$, and more limited evidence for a higher mass S-wave state around $1.9 \mathrm{GeV} / \mathrm{c}^{2}$ [5-7]. The data sample discussed in this paper, which contains approximately four times more events than the largest earlier experiment, coupled with the excellent acceptance of the LASS spectrometer, allows the confirmation of these observations, and furthers the search for the additional $K^{*}$ states expected in $q \bar{q}$ potential models [8].

## 2. The Experiment

### 2.1 The Spectrometer

The experiment was performed in the LASS spectrometer at SLAC. Details of the experiment are given elsewhere [9], and are only briefly reviewed here. The LASS spectrometer is shown schematically in fig. 1. The $R F$ separated $11 \mathrm{GeV} / \mathrm{c} K^{ \pm}$ beam has a typical $K / \pi$ purity of $50 / 1$ or better before final tagging by two threshold Čerenkov counters. Scintillation counter hodoscopes and proportional wire chambers are located in the beam to measure the momentum, position, and angle of the incident kaon. The angle and position resolution of the beam are 0.3 mrad and 0.3 mm , respectively, while the momentum resolution is $0.4 \%$.

LASS is built around two large magnets. The first magnet is a superconducting solenoid with a 22.4 kG magnetic field parallel to the beam direction. This magnet is followed by a $30 \mathrm{kG} \cdot \mathrm{m}$ dipole magnet with a vertical field. The solenoid is effective in measuring the large angle, low momentum particles produced in an
interaction. High momentum particles close to the beam line are momentum analyzed in the dipole spectrometer. This combination provides essentially uniform geometrical acceptance with good resolution over the entire $4 \pi$ solid angle for nearly all reactions.

The $K^{-}$beam is incident on a 84.6 cm long liquid $H_{2}$ target situated on the axis of the solenoid. It is surrounded by six layers of cylindrical proportional wire chambers (PWCs) that read out anode wire hits and the pulse heights on the cathodes, so that each chamber provides three-dimensional space points for tracks at a common radius. The region of the solenoid following the target contains three PWCs with analog cathode readout, three sets of conventional anode readout PWCs with three planes per chamber, and three segmented foil chambers with digital cathode readout. The central regions of the cathode readout chambers are deadened to reduce ambiguities in the high flux beam region. These regions are covered by three sets of small PWC chambers with each chamber consisting of three anode planes with 1 mm wire spacing and conventional digital anode readout.

The dipole spectrometer following the solenoid uses seven magnetostrictive (MS) chambers, three upstream of the dipole and four downstream, to provide most of the coordinate information. Each MS chamber has two spark gaps and provides four coordinate measurements. These chambers are supplemented by four PWC planes upstream, and one PWC plane and two segmented scintillation counter hodoscopes downstream, which provide fast in-time corroboration of charged particles.

Particle identification is provided by a segmented, atmospheric pressure, gas threshold Cerenkov counter located at the downstream end of the solenoid that provides $K / \pi$ discrimination for tracks above a momentum of $2.6 \mathrm{GeV} / \mathrm{c}$, followed
by a time-of-flight (TOF) scintillation counter array with 24 wedge shaped segments that provide identification for lower momentum particles. A pressurized, gas threshold Cerenkov counter located at the downstream end of the dipole spectrometer provides additional particle separation information for high momentum tracks. In addition, $d E / d x$ information obtained from the cathode pulse heights from the cylindrical chambers surrounding the target provides $\pi / p$ separation below a momentum of $\sim 0.6 \mathrm{GeV} / \mathrm{c}$.

The event trigger for the experiment requires a well measured incident kaon that interacts in the target, and at least two charged secondaries in the event. The number of charged secondaries emerging from the target is determined by counting the number of hits in the two inner cylindrical PWC chambers and in the first two small PWC chambers that follow the target. Conceptually, the combination of these devices forms a pair of closed cylinders around the target. Hardware attached to the anodes of these chambers determines the number of clustered hits in each of these planes, which are combined in various ways to determine a lower limit on the number of emerging tracks. The event trigger is efficient and clean; approximately $85 \%$ of the event triggers recorded are good interactions in the target.

### 2.2 The Monte Carlo

The geometrical acceptance of the LASS spectrometer covers most of the $4 \pi$ solid angle but Monte Carlo acceptance corrections are still performed to account for non-uniformities in the spectrometer geometry and trigger, decay and absorption of particles, resolution and efficiency of detectors, and the cuts applied in defining the data sample. The Monte Carlo simulation propagates particles through the spectrometcr taking proper account of energy loss, multiple scattering, nuclear
absorption, and weak decay. When a particle intersects a tracking detector, the measured efficiency and resolution for each plane are used to create "raw" chamber coordinates. After applying a simulation of the hardware trigger logic, these coordinates, along with simulated information for the particle identification devices and the hodoscopes, are subjected to the complete reconstruction chain and written to tape as Monte Carlo events. The same fitting and event selection programs are then applied to the Monte Carlo events and to the real data.

The experiment contains $\sim 135$ million raw events of which $\sim 22$ million were taken with a $K^{+}$beam. These events were reconstructed off-line at Nagoya University, using a FACOM M-200 at the physics laboratory and an M-382 at the computer center, and at SLAC, using the $168 / \mathrm{E}$ microprocessors designed and built for this purpose [10]. The reconstruction program first recognizes and reconstructs the particle trajectories. Then it finds the location of primary and secondary vertices, and assigns the events to candidate topologies based on their configuration. The resulting information is written onto data summary tapes (DSTs) for later event selection and processing. The sensitivity of the data sample studied here along with its systematic error is $3.846 \pm 0.104$ events/nb.

From the DSTs, events with an interaction vertex containing two oppositely charged prongs are selected as candidates for reaction (1.1). Events which satisfy $K^{-} p$ elastic kinematics are discarded. Since we are only interested in foward $K^{-} \pi^{+}$production, the sum of the charged track momenta is required to be greater than $9.0 \mathrm{GeV} / \mathrm{c}$ to suppress the number of non $K^{-} \pi^{+} n$ events passed through the fitting procedure which follows. The fitting procedure includes a geometrical fit
that constrains the tracks to form a primary vertex, followed by a kinematical fit that constrains the missing neutron mass when $K^{-}$and $\pi^{+}$masses are assigned to the charged secondaries, and provides improved resolutions on track momenta and event variables.

Backgrounds arising as reflections from the narrow $K_{s}^{0}, \phi$, and $\Lambda$ states are removed by applying cuts to invariant masses when the charged prongs are given $\pi^{-} \pi^{+}, K^{-} K^{+}$, and $\pi^{-} p$ masses, respectively. Particle identification information from the TOF counter and the two Cerenkov counters is also used to reduce the level of background from misassigned protons or $\pi^{-} s$. One large background to reaction (1.1) is the diffractive reaction

$$
\begin{equation*}
K^{-} p \rightarrow K^{-} \pi^{0} p \tag{2.1}
\end{equation*}
$$

where the proton is misassigned as a $\pi^{+}$. Since the meson system is produced very strongly foward, this reaction typically has a very slow proton which can be removed by cutting on the $d E / d x$ information from the cylindrical chambers for momenta below $0.6 \mathrm{GeV} / \mathrm{c}$. The small residual contamination from events with proton momenta greater than $0.6 \mathrm{GeV} / \mathrm{c}$ tends to reflect substantially below the neutron missing mass peak when treated as $K^{-} \pi^{+} n$ and is removed during selection of neutron events as discussed below.

When a small angle, high momentum track is measured only in the solenoid spectrometer, as when it hits the upstream face of the dipole magnet, its momentum error is large and has a strong correlation with its momentum and angle. The parameters for events which contain such tracks, particularly the missing mass to the neutral particle, are rather poorly measured, so it is most convenient to remove
them from the data sample by rejecting events having tracks with polar angles less than 70 mrad or momenta greater than $9 \mathrm{GeV} / \mathrm{c}$. This cut does not produce holes in the angular acceptance of the $K^{-} \pi^{+}$system since this region of phase space is covered with moderate efficiency by the dipole spectrometer.

Neutrons recoiling against the $K^{-} \pi^{+}$system are selected using the calculated missing mass squared ( $M M^{2}$ ) recoiling against the visible two prongs. Figure 2 shows the $M M^{2}(K \pi)$ distribution for the small momentum transfer region, $\left|t^{\prime}\right| \leq$ $0.2(\mathrm{GeV} / \mathrm{c})^{2}$. There is a clear neutron missing mass peak with tails from other processes also visible both below and above the peak. The events above the peak centered in a bump around $1.5\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{2}$ come mainly from the reaction $K^{-} p \rightarrow$ $K^{-} \pi^{+} \Delta^{0}$. The events lying in the lower tail of the neutron peak are mainly the residual contribution of events from the reaction $K^{-} p \rightarrow K^{-} \pi^{0} p$ after the $d E / d x$ cut described above. These events reflect into the high $K^{-} \pi^{+}$invariant mass region and come from events with a $K^{-}$measured in the forward dipole spectrometer. To remove these backgrounds, such events are required to lie within the missing mass window, $0.5 \leq M M^{2}(K \pi) \leq 1.2\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{2}$, while for other configurations where the $K^{-} \pi^{\circ} p$ contribution is absent, the lower missing mass limit is set to zero. The results of the analysis of the $K^{-} \pi^{+}$system are insensitive to the details of this selection procedure.

The detailed shape of the $M M^{2}$ distribution is a function of the measurement resolution of the spectrometer and is determined by the Monte Carlo simulation described above. A fit to the distribution of fig. 2 using the predicted distribution for the neutron and $\Delta^{0}$ events and a Gaussian shape to represent the $K^{-} \pi^{0} p$ events describes the data distribution quite well as shown by the solid line in the figure.

The background contamination in the neutron sample is shown by the other curves in the figure and is estimated to be $6 \%$.

## 3. General Features of the Data

The $K^{-} \pi^{+} n$ data sample contains 730,000 events with $\left|t^{\prime}\right|<1.0(\mathrm{GeV} / \mathrm{c})^{2}$. The $K^{-} \pi^{+}$invariant mass distribution is shown as the outer histogram in fig. 3. In addition to peaks corresponding to the $K^{*}$ (892) and $K_{2}^{*}(1430)$ resonances, a bump around $1.8 \mathrm{GeV} / \mathrm{c}^{2}$ can be seen which arises from several $K^{-} \pi^{+}$states, including the leading $K_{3}^{*}(1780)$. However, there is little direct evidence in this plot for additional structure in the high mass region where higher spin resonances would be expected.

In addition to the forward production of $K^{*}$ mesons, there is also substantial production of $N^{*}$ systems in the region of low $n \pi^{+}$invariant mass, as is shown in fig. 4, which reflect into the high $K^{-} \pi^{+}$mass region. Structures of moderate width at $n \pi^{+}$masses around $1.5 \mathrm{GeV} / \mathrm{c}^{2}$ and $1.65 \mathrm{GeV} / \mathrm{c}^{2}$ resulting from diffractive $N^{*}$ production can be seen above a broad structure which contains the sum of additional $N^{*}$ production along with the reflection of $K^{-} \pi^{+}$diffractive scattering from the high $K^{-} \pi^{+}$mass region. No clear $\Delta^{0}$ signal is observed. The strong sharp peak at about $1.2 \mathrm{GeV} / \mathrm{c}^{2}$ comes from the reaction $K^{-} p \rightarrow \pi^{-} \Sigma^{+}$which survives the particle identification cuts described above since the $\pi^{-}$is sufficiently fast to exceed $K$ threshold in the Cerenkov counter. Since these events in the $n \pi^{+}$resonance region reflect into the $K^{-} \pi^{+}$system in a complex way, we have chosen to remove events with a $n \pi^{+}$mass below $1.7 \mathrm{GeV} / \mathrm{c}$ from the subsequent analysis. This cut removes all surviving background from reaction (2.1) as well. However, since it introduces serious non-uniformities into the acceptance at high
$K^{-} \pi^{+}$mass which must be corrected, the data were also analyzed with tighter cuts to ensure that the results did not depend significantly on the precise position of this cut.

The $K^{-} \pi^{+}$mass distribution for the resulting sample, which contains 385,000 events, is shown as the shaded histogram in fig. 3. The removal of the $N^{*}$ region has little effect on the visible structure. Though the lower mass leading states remain obvious, there is still no clear evidence for resonances at higher masses, primarily because the number of overlapping $K^{*}$ resonances at higher mass becomes sufficiently large that the invariant mass projection is rather featureless.

Information about these resonances can be extracted from the decay angular distribution. The structure caused by these resonances can be seen directly in the scatter plots of figs. 5 (a) and (b), which show the $K^{-} \pi^{+}$invariant mass against the cosine of the helicity angle in the $t$-channel helicity (Gottfried-Jackson) frame for events with $\left|t^{\prime}\right| \leq 0.2(\mathrm{GeV} / \mathrm{c})^{2}$, and $0.2<\left|t^{\prime}\right| \leq 1.0(\mathrm{GeV} / \mathrm{c})^{2}$, respectively. Clear structures can be seen in both figures corresponding to the $K^{*}(892)$ and the $K_{2}^{*}(1430)$. However, there are large differences in the angular structure due to the dominance of $\pi$ exchange in fig. $5(\mathrm{a})$, as opposed to the increasing dominance of the heavier $\rho$ and $A_{2}$ exchanges in the event sample of fig. 5(b). In fig. 5(a), for example, there is substantial forward and backward peaking of the $K_{2}^{*}(1430)$. There is also an apparent curvature of the $K_{2}^{*}(1430)$ band in mass which comes from the interference between the leading state and its S-wave triplet partner, the $K_{0}^{*}$ (1350), which peaks around $1.4 \mathrm{GeV} / \mathrm{c}^{2}$. At higher masses, the interference structure grows more complex. Around $1.8 \mathrm{GeV} / \mathrm{c}^{2}$, for example, there is a concentration of events at $\cos \theta_{G J} \sim-0.5$ and a depletion at $\cos \theta_{G J} \sim+0.5$ and -1 , which results from
interference between the leading $K_{3}^{*}(1780)$ and underlying states, while around 2.1 $\mathrm{GeV} / \mathrm{c}^{2}$ there is a rather sharp backward structure. The apparent absence of similar interfering structures in fig. 5(b) demonstrates the strong dependence of resonance cross sections on the production mechanism. In any case, the complexity of these interference patterns demonstrates the importance of performing a partial wave analysis at the pion pole in order to fully understand the resonance structure. The final data sample used for analysis contains 151,000 events in the $K^{-} \pi^{+}$mass region below $2.6 \mathrm{GeV} / \mathrm{c}^{2}$ with $\left|t^{\prime}\right| \leq 0.2(\mathrm{GeV} / \mathrm{c})^{2}$.

The angular structure of the acceptance corrected $K^{-} \pi^{+}$data can be observed directly in terms of the coefficients $\left(\mathrm{t}_{L}^{M}\right)$ of the spherical harmonics in the $t$-channel helicity frame. The production angular distribution $I_{\text {prod }}$ of the $K^{-} \pi^{+}$system can be expanded as

$$
\begin{equation*}
I_{p r o d}\left(m_{K \pi}, t^{\prime}, \Omega\right)=\frac{1}{\sqrt{4 \pi}} \sum_{L, M \geq 0} \mathrm{t}_{L}^{M}\left(m_{K \pi}, t^{\prime}\right)\left(2-\delta_{M 0}\right) \operatorname{Re}\left[Y_{L M}(\Omega)\right] \tag{3.1}
\end{equation*}
$$

where $\Omega$ stands for $\left(\cos \theta_{G J}, \varphi_{T Y}\right)$, and $\delta_{M M^{\prime}}$ is Krönecker's delta. The normalization condition is such that $t_{0}^{0}$ corresponds to the acceptance corrected number of events. The production distribution $I_{\text {prod }}$ is obtained from the observed distribution $I_{o b s}$ corrected for the spectrometer acceptance $A$,

$$
\begin{equation*}
I_{o b s}\left(m_{K \pi}, t^{\prime}, \Omega\right)=A\left(m_{K \pi}, t^{\prime}, \Omega\right) I_{p r o d}\left(m_{K \pi}, t^{\prime}, \Omega\right) \tag{3.2}
\end{equation*}
$$

In practice, the acceptance function $A$ is also expanded in spherical harmonic moments for use in the fit. The acceptance corrected moments are obtained by the extended maximum likelihood method. The number of moments required to fit the
$K^{-} \pi^{+}$angular distribution is found to increase with $K^{-} \pi^{+}$mass. In general, in order to minimize the correlation between moments which arises from non-uniformities in the acceptance, a minimum set of moments $M \leq M_{\max }, L \leq L_{m a x}$ is used in each mass region, where $M_{m a x}$ and $L_{m a x}$ are the smallest maximum values required by the data. Unnecessary higher moments, if included, do not alter the quality of the fit, but the correlations do substantially increase the error bars on all moments.

The moments obtained are shown in figs. 6, 7, and 8. Below a $K^{-} \pi^{+}$mass of $1.88 \mathrm{GeV} / \mathrm{c}^{2}$, the data are fit in mass bins which vary from 10 to $40 \mathrm{MeV} / \mathrm{c}^{2}$. Above $1.88 \mathrm{GeV} / \mathrm{c}^{2}$ overlapping bins with widths between 80 and $160 \mathrm{MeV} / \mathrm{c}^{2}$ are used to keep sufficient statistics in each bin. Moments with $\mathrm{M}>1$ or $\mathrm{L}>10$ are not required to fit the data in the high mass region and are therefore not included in the fit to determine the moments with $\mathrm{L} \leq 10$ which are displayed. However, when included, the $L=11$ and $L=12$ moments (also shown in figs. 6 and 7) are structureless and consistent with zero indicating the the maximum amplitude required by the data is the H -wave. Due to the dominance of $\pi$ exchange in the data, the largest moments have $M=0$ and resonances with spin $J$ appear in moments with $L$ up to $2 J$. The first 5 leading orbitally excited $K^{*}$ states (the $K^{*}(892), K_{2}^{*}(1430)$, $K_{3}^{*}(1780), K_{4}^{*}(2060)$, and $\left.K_{5}^{*}(2380)\right)$ can be clearly seen in the $\mathrm{t}_{2}^{0}, \mathrm{t}_{4}^{0}, \mathrm{t}_{6}^{0}, \mathrm{t}_{8}^{0}$, and $\mathrm{t}_{10}^{0}$ moments, respectively. Each state dominates the highest moment required in the relevant mass region, clearly demonstrating its spin-parity. A previous analysis has discussed the evidence for these states and obtained estimates for their masses and widths using simple mass dependent fits to these moments [4].

## 4. The Partial Wave Analysis

The $K^{-} \pi^{+}$angular moments show the leading resonances directly. However, in order to reveal the underlying structure, it is necessary to extract the partial wave amplitudes of the data. Over 25 years ago, Chew and Low suggested that it would be possible to extract the physical $\pi \pi$ scattering amplitude in a reaction dominated at small $t$ by $\pi$ exchange, via an extrapolation to the pion pole [11]. Many extensions and refinements of this classic technique have been applied since to the study of the $\pi \pi$ and $K \pi$ amplitudes. One of the most powerful methods was developed by Estabrooks and Martin, and applied to data from both the $\pi \pi$ and $K \pi$ final states $[1,5,12]$. Since this model has been described in detail elsewhere, we will only give a short description here, sufficient to be self-contained and to fix the notation. The method proceeds in two steps. First, in section 4.1, we discuss the isolation of the $\pi$ exchange contribution to the production amplitudes in the framework of an exchange model. Then, in section 4.2, the exchange parametrization developed will be applied to extract the $K \pi$ scattering amplitudes from the $K \pi$ moments presented in section 3.

### 4.1 The $K \pi$ Production Model

The model describes the production mechanism in terms of strongly exchange degenerate $\pi-B$ and $\rho-A_{2}$ Regge exchanges and Regge "cuts" that have simple structure in the $t$-channel helicity frame [1]. It was first applied to a study of data in the $K^{*}(892)$ and $K_{2}^{*}(1430)$ resonance regions, and then used in partial wave analyses of the $K \pi$ mass region up to $2.2 \mathrm{GeV} / \mathrm{c}^{2}$, and has been shown to provide a good description of the $t$-dependence of the data $[5,7]$. To develop the model parameters for the $t$-dependence of the data as a function of $K^{-} \pi^{+}$mass, relatively
wide intervals in $K^{-} \pi^{+}$mass are chosen in order to have sufficient statistics. For each $K^{-} \pi^{+}$mass bin, the $t$-dependent parametrization of the naturality amplitudes $L_{\lambda}^{ \pm \star}$ for production of a $K^{-} \pi^{+}$state of mass $M$, center-of-mass momentum $q$, angular momentum $L, t$-channel helicity $\lambda$, and natural ( + ) and unnatural ( - ) parity exchange is as follows:

$$
\begin{align*}
L_{0}= & \frac{\sqrt{-t}}{m_{\pi}^{2}-t} \cdot G_{K \pi}^{L}(M, t) \\
L_{1}^{-}= & \sqrt{L(L+1) / 2} \cdot G_{K \pi}^{L}(M, t) \cdot \gamma_{C}(M) \cdot e^{b_{C}(M) \cdot\left(t-m_{\pi}^{2}\right)} \\
L_{1}^{+}= & \sqrt{L(L+1) / 2} \cdot G_{K \pi}^{L}(M, t) \cdot\left[\gamma_{C}(M) \cdot e^{b_{C}(M) \cdot\left(t-m_{\pi}^{2}\right)}\right.  \tag{4.1}\\
& \left.-2 i \gamma_{A}(M) \cdot\left|t^{\prime}\right| \cdot e^{b_{A}(M) \cdot\left(t-m_{\pi}^{2}\right)}\right] \\
L_{\lambda}^{ \pm}= & 0 \quad \lambda \geq 2
\end{align*}
$$

where $G_{K \pi}^{L}$ is related to the $K^{-} \pi^{+}$elastic scattering amplitude $a_{L}$ by

$$
\begin{equation*}
G_{K \pi}^{L}(M, t)=N \cdot \frac{M}{\sqrt{q}} \cdot a_{L}(M) \cdot e^{b_{L}(M) \cdot\left(t-m_{\pi}^{2}\right)} \tag{4.2}
\end{equation*}
$$

The parameters $\gamma_{A}, \gamma_{C}, b_{A}, b_{C}$, and $b_{L}$ are determined by fitting the data in each mass interval while $N$ is an overall normalization constant of the experiment. In principle, the $b_{L}$ are independent parameters for each $L$ value. However, the $b_{L}$ are not well determined separately for $L \geq 3$, so are defined to be equal (i.e., $b_{F}=b_{G}=b_{H}$ ). In a few cases the amplitude is too small in a given mass interval to allow the value of $b_{L}$ to be determined there. In this case the value is determined by assuming continuity in mass.

[^1]The $K^{-} \pi^{+}$scattering amplitudes are the sum of an isospin $\frac{1}{2}$ component and a $\frac{3}{2}$ component,

$$
\begin{equation*}
a_{L}=a_{L}^{I=\frac{1}{2}}+\frac{1}{2} a_{L}^{I=\frac{3}{2}} \tag{4.3}
\end{equation*}
$$

where an overall isospin Clebsch-Gordan coefficient of $\frac{2}{3}$ is absorbed in the normalization constant. The $K^{-} \pi^{+}$amplitudes are conventionally defined as

$$
\begin{equation*}
\frac{a_{L}^{I}}{\sqrt{2 L+1}}=\epsilon_{L}^{I} \sin \delta_{L}^{I} e^{i \delta_{L}^{I}} \tag{4.4}
\end{equation*}
$$

The requirement of elastic unitarity is equivalent to setting $\epsilon_{L}^{I}=1$, in which case the amplitude is determined by a single parameter $\delta_{L}^{I}$. The amplitude can also be written as

$$
\begin{equation*}
a_{L}=\left|a_{L}\right| e^{i \phi_{L}} \tag{4.5}
\end{equation*}
$$

where $\left|a_{L}\right|$ is the magnitude and $\phi_{L}$ is the phase of the spin- $L$ wave. The normalization constant $N$ is determined by requiring the $K^{*}(892)$ to be a purely elastic resonance. As a check of this value, we have also calculated $N$ from the sensitivity of the experiment and the Chew-Low equation [11]. This gives a value about $8 \%$ larger than that obtained from the previous method.

To apply the parametrization of eqn. 4.1 to the data, the amplitudes are transformed into angular moments $\mathrm{t}_{\mathrm{L}, \mathrm{th}}^{M}$ and fit to the data moments using a chi-square minimization technique with

$$
\begin{equation*}
\chi_{P W A}^{2}=\sum_{L M, L^{\prime} M^{\prime}, i}\left[\left(\mathrm{t}_{L, t h}^{M}-\mathrm{t}_{L}^{M}\right) E_{L M, L^{\prime} M^{\prime}}^{-1}\left(\mathrm{t}_{L^{\prime}, t h}^{M^{\prime}}-\mathrm{t}_{L^{\prime}}^{M^{\prime}}\right)\right]_{i} \tag{4.6}
\end{equation*}
$$

where $E_{L M, L^{\prime} M^{\prime}}$ is the error matrix from the determination of the moments for the mass bin $i$, and the $\gamma s$ and $b s$ discussed above are the fit parameters. To insure
compatibility with the mass sweep which follows, we impose elastic unitarity on the S-wave below $1.29 \mathrm{GeV} / \mathrm{c}^{2}$ and on the P -wave below $1.17 \mathrm{GeV} / \mathrm{c}^{2}$, as discussed in section 4.2. The parametrization of eqn. 4.1 provides a good description of the $t^{\prime}$ dependence of the $K^{-} \pi^{+}$moments up to $\left|t^{\prime}\right|=0.3(\mathrm{GeV} / \mathrm{c})^{2}$ in all $K^{-} \pi^{+}$mass regions, as can be seen in fig. 9 , which compares the data with the results of the fit for three mass regions; $0.87 \leq M_{K \pi} \leq 0.89 ; 1.42 \leq M_{K \pi} \leq 1.48 ; 1.80 \leq M_{K \pi} \leq 1.90$ $\mathrm{GeV} / \mathrm{c}^{2}$. The values of the parameters $b_{L}, \gamma_{C}, \gamma_{A}, b_{C}$, and $b_{A}$ describing the $t$ dependence of the model which are obtained in these fits are shown plotted as a function of $K^{-} \pi^{+}$mass in fig. 10. The slope parameters, $b_{L}$, are fairly constant for high $K^{-} \pi^{+}$mass but show some structure at lower mass, while $\gamma_{A}$ and $\gamma_{C}$ are large at lower masses but approach zero as the mass increases. This implies that the $\rho-A_{2}$ and "cut" contributions are significant at lower masses but become less important at high mass, which agrees well with results from earlier analyses $[1,5$, $6,7]$. The curves in fig. 10 represent fits to the data using simple spline smoothing functions. The use of these smoothed parameters will be discussed in section 4.2.

### 4.2 Determination of the Partial Wave Amplitudes

The best determination of the $K^{-} \pi^{+}$partial waves as a function of $M_{K \pi}$ is obtained using the experimental $K^{-} \pi^{+}$moments for the $\pi$ exchange dominated momentum transfer region with $\left|t^{\prime}\right| \leq 0.2(\mathrm{GeV} / \mathrm{c})^{2}$ which are described in section 3. For consistency, the moments used in this calculation are calculated using $L_{m a x}=2 J_{\max }$, where $J_{\max }$ is the maximum angular momentum of the amplitudes used in the fit. Having obtained the $t$-dependent parameters of the model, as described above, we then use the smoothed values of these parameters to integrate the amplitudes of eqn. 4.1 over the $t^{\prime}$ region corresponding to the data moments, and
determine the $K^{-} \pi^{+}$amplitudes $a_{L}$ from a $\chi^{2}$ minimization to the data moments. It has been demonstrated both here and in earlier analyses that the correlation between small changes in the values of the production parameters and the values obtained for the $K^{-} \pi^{+}$amplitudes is small, and so the results are insensitive to the smoothing of the parameters of the model which describe its $t$-dependence [5, 7].

In general, the data determine only the magnitudes and relative phases between the amplitudes. An overall phase cannot be determined. In the low mass region, the $S$ and $P$-waves are known to be elastic so that the imposition of elastic unitarity is sufficent to fix the overall phase. Imposing this constraint requires knowledge of the $I=\frac{3}{2}$ components of the amplitudes which have been well measured in this mass region in an earlier experiment [5]. The elasticity constraint is imposed on the P-wave below $1.17 \mathrm{GeV} / \mathrm{c}^{2}$ and the S -wave below $1.29 \mathrm{GeV} / \mathrm{c}^{2}$. In the $K^{-} \pi^{+}$mass region above $1.29 \mathrm{GeV} / \mathrm{c}^{2}$, where elastic unitarity no longer holds, we require that the phase of the wave containing the leading $K^{*}$ resonance in each region be set to the Breit-Wigner phase, allowing small rotations away from the predicted BreitWigner phase in order to assure smooth phase variations. Between 1.0 and 1.29 $\mathrm{GeV} / \mathrm{c}^{2}$, the data require a small D-wave amplitude which is consistent with the leading edge of the $K_{2}^{*}(1430)$. It is most conveniently fixed to the resonance fit value to assure continuity. The fits to the leading amplitudes which are used to determine the overall phase are described in the following section.

In a pure $0^{-} 0^{-}$scattering process, there are $2^{J_{\max }}$ discrete ambiguous solutions to the partial wave decomposition of the data. These discrete ambiguities are most readily apparent in the behavior of the imaginary part of the amplitude zeros ( $Z_{i}$ ) described by Barrelet [13]. Requiring the solutions to be smooth, it is possible
to switch from one solution to another only when a Barrelet zero approaches the real axis. Figure 11 shows the real and imaginary parts of the Barrelet zeros for one particular solution (solution A). The Wigner condition [14], combined with the existence of the leading resonances, requires all of the $Z_{i}$ to begin with negative imaginary parts; combined with the unitarity constraint, this leads to a unique solution in the low mass region below $1.29 \mathrm{GeV} / \mathrm{c}^{2}$. Thus, ambiguities do not arise until $\operatorname{Im}\left(Z_{3}\right)$ approaches zero at $1.84 \mathrm{GeV} / \mathrm{c}^{2}$. There is a unique solution below $1.84 \mathrm{GeV} / \mathrm{c}^{2}$ and two distinct branches above, corresponding to the different signs of $\operatorname{Im}\left(Z_{3}\right)$. We call thesc solutions $\mathbf{A}$ and B , using the same labeling convention defined by earlier experiments $[5,6]$ as shown in table $I$.

Table I
Labeling convention for the ambiguous solutions in terms of the signs of the imaginary parts of the Barrelet zeros; below $1.84 \mathrm{GeV} / \mathrm{c}^{2}$, the solution is unique.

|  | $\operatorname{Im}\left(Z_{i}\right)$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Solution | $Z_{1}$ | $Z_{2}$ | $Z_{3}$ | $Z_{4}$, | $Z_{5}$ | Mass $\left(\mathrm{GeV} / \mathrm{c}^{2}\right)$ |
| A | + | - | - |  |  |  |
| B | + | - | - |  |  | $\mathrm{M}_{\mathrm{K} \pi} \leq 1.84$ |
| A | + | - | + | - | - | $1.84<\mathrm{M}_{\mathrm{K} \pi-}<1.98$ |
| B | + | - | - | - | - |  |
| A | - | - | + | - | - |  |
| B | - | - | - | - | - | $1.98<\mathrm{M}_{\mathrm{K} \pi}$ |

In the region between 1.9 and $2.0 \mathrm{GeV} / \mathrm{c}^{2} \operatorname{Im}\left(Z_{1}\right)$ approaches zero, and $\operatorname{Im}\left(Z_{2}\right)$ first approaches zero around $2.2 \mathrm{GeV} / \mathrm{c}^{2}$ so that, in principle, additional ambigu-
ities arise in the high mass region. However, since both of these amplitude zeros are consistent with being purely real everywhere above their first approach to the real axis, the ambiguous solutions that arise are not physically distinct. Thus, all ambiguous solutions can be classified as either type A or B. This behavior of the Barrelet zero ambiguities is essentially identical to that found in the earlier high statistics analysis which studied the $K^{-} \pi^{+}$mass spectrum up to $2.2 \mathrm{GeV} / \mathrm{c}^{2}[6]$.

The $K^{-} \pi^{+}$partial wave magnitudes and phases for solutions A and B are shown in figs. 12 and 13 respectively. The results are consistent with previous measurements, but are of higher statistical significance, particularly in the mass region above the $K_{2}^{*}(1430)[5,6]$. This is the first analysis to probe the mass region above $2.2 \mathrm{GeV} / \mathrm{c}^{2}$. The leading orbitally excited states can be clearly seen as bumps in the partial wave magnitudes in all waves from P through H , and this behavior is independent of the solution since the waves are determined mainly by the highest moment in each mass bin. However, the behavior of the underlying waves in the high mass region does depend on the solution type.

The partial wave amplitudes expressed as Argand diagrams are shown in fig. 14. Both solutions show the leading $J^{P}=1^{-}, 2^{+}, 3^{-}, 4^{+}$, and $5^{-}$resonances as counterclockwise loops in the appropriate Argand diagrams. As discussed above, the total S-wave, which is a sum of the different isospin parts, departs significantly from the unitary circle in the low mass region. However, the $I=\frac{1}{2}$ component, shown by the dotted line in fig. 14, is consistent with unitarity. It exhibits a slow circular motion in the region from 0.8 to $1.3 \mathrm{GeV} / \mathrm{c}^{2}$, followed by a rapid loop around 1.4 $\mathrm{GeV} / \mathrm{c}^{2}$. Clear loops can also be seen in the S -wave in the $1.9 \mathrm{GeV} / \mathrm{c}^{2}$ mass region, and in the P -wave around $1.7 \mathrm{GeV} / \mathrm{c}^{2}$.

## 5. The Results of the Partial Wave Analysis

In the following sections, we discuss the important features of each partial wave amplitude and extract parameters for the resonances. In all cases, except when specifically indicated otherwise, these parameters are derived from fits using a relativistic Breit-Wigner parametrization for a spin- $L$ resonance of the form

$$
\begin{equation*}
B W=\frac{\sqrt{2 L+1} \quad \epsilon-M_{R} \Gamma}{\left(M_{R}^{2}-M^{2}\right)-i M_{R} \Gamma} \tag{5.1}
\end{equation*}
$$

where $M_{R}$ is the resonance mass, $\epsilon$ is the $K^{-} \pi^{+}$elasticity, $M$ is the invariant mass of the $K^{-} \pi^{+}$system, and $\Gamma$ is the mass dependent width given by

$$
\begin{equation*}
\Gamma(M)=\left(\frac{q}{q_{R}}\right)^{2 L+1}\left(\frac{M_{R}}{M}\right) \frac{D_{L}\left(q_{R} r\right)}{D_{L}(q r)} \Gamma_{R} \tag{5.2}
\end{equation*}
$$

where $q$ is the momentum in the $K^{-} \pi^{+}$center of mass, $\Gamma_{R}$ is the width of the resonance, $q_{R}$ is $q$ evaluated at the resonance mass, $r$ is the interaction radius, and $D_{L}$ is the barrier factor for a spin- $L$ wave defined by Blatt and Weisskopf [15]. In addition to the statistical errors on the fit parameters, additional uncertainties also arise when different mass ranges or background forms are used in the fits and when different normalization procedures are used in the model. Estimates for the size of these model dependent variations are shown as the systematic errors in the tables.

### 5.1 The S-wave Amplitude

In the region below $1.3 \mathrm{GeV} / \mathrm{c}^{2}$, the magnitude and phase of the S-wave amplitude rise slowly and are compatible with unitarity. The first inelastic two-body threshold, the $K \eta$, is at about $1.05 \mathrm{GeV} / \mathrm{c}^{2}$, but this channcl appears to be very
weakly coupled to the S-wave $K^{*}$ system, as is predicted by $\operatorname{SU}(3)$. The phase reaches $90^{\circ}$ at $1.34 \mathrm{GeV} / \mathrm{c}^{2}$. Above this mass, the motion becomes more rapid. The magnitude peaks just below $1.4 \mathrm{GeV} / \mathrm{c}^{2}$, and then drops precipitously to nearly zero at $1.7 \mathrm{GeV} / \mathrm{c}^{2}$, while the phase varies rapidly in this same mass region. Above $1.85 \mathrm{GeV} / \mathrm{c}^{2}$, the two solutions have different detailed structure, but both show a peak in the magnitude around $1.9 \mathrm{GeV} / \mathrm{c}^{2}$ with rapidly varying phase motion. The Argand diagrams also display rapid circular motion in these mass regions, which support the assertion that there are two resonant structures in the $K^{-} \pi^{+}$S-wave, one at about $1.4 \mathrm{GeV} / \mathrm{c}^{2}$ and the other around $1.9 \mathrm{GeV} / \mathrm{c}^{2}$. The structure around $1.4 \mathrm{GeV} / \mathrm{c}^{2}$ has been seen by many earlier experiments [2], while the observation of the second structure around $1.9 \mathrm{GeV} / \mathrm{c}^{2}$ confirms evidence provided by an earlier experiment in LASS [6]. Determination of the resonance parameters for the lower mass object is complicated by the large elastic phase shift in the low mass region and the proximity of the $K \eta^{\prime}$ threshold. Historically, the mass value has been associated with the point where $\delta_{S}^{\frac{1}{2}}$ reaches $90^{\circ}$ [2] and, more recently, a unitary coupled-channel analysis of earlier data obtained a value which was consistent with this simple definition [16]. Other models have been proposed which also fit the data well, but give values for the mass which are about $0.1 \mathrm{GeV} / \mathrm{c}^{2}$ higher [17]. Thus, given the quality of the data, the determination of the S-wave parameters is limited at present by theoretical considerations.

To indicate the range of values allowed for the $K_{0}^{*}(1350)$ resonance, we quote parameter values from two models as shown in table II. In the first (Model I) the mass is taken as the point where the phase shift reaches $90^{\circ}$ while the width is approximately determined from the speed of the phase motion there. As a second method (Model II) the data are fit to a parametrization developed by Estabrooks
[17] that considers $K \eta^{\prime}$ to be the only important inelastic channel. The S-wave amplitude is parametrized as the sum of an inelastic Breit-Wigner resonance and a background term parametrized as an effective range,

$$
\begin{align*}
a_{S}= & B G+B W^{\prime} e^{i \phi} \\
& B G=\sin \delta_{B G} e^{i \delta_{B G}} ; \quad \cot \delta_{B G}=\frac{1}{a q}+\frac{1}{2} b q  \tag{5.1}\\
& B W^{\prime}=\frac{M_{R} \Gamma_{1}}{\left(M_{R}^{2}-M^{2}\right)-i M_{R}\left(\Gamma_{1}+\Gamma_{2}\right)} ; \Gamma_{i}=q_{i} \Gamma_{R, i} \quad \text { for } i=1,2
\end{align*}
$$

where the subscripts 1 and 2 refer to the $K^{-} \pi^{+}$and $K \eta^{\prime}$ channels respectively and the elasticity $\epsilon$ is the ratio of the $K \pi$ partial width to the total. In this parametrization, the unitarity constraint is imposed by requiring $\phi=2 \delta_{B G}$. The results of the fit are shown by the curve in fig. 15 and the parameters are shown in table II.

Table II
Resonance parameters for the $K_{0}^{*}$ (1350) determined using two different models as described in the text.

| $K_{0}^{*}(1350)$ |  |
| :---: | :---: |
| Model I |  |
| Mass $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | 1340 |
| Width $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | 350 |
| Model II |  |
| Mass $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $1429 \pm 4 \pm 5$ |
| Width $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $287 \pm 10 \pm 21$ |
| $\epsilon$ | $0.93 \pm 0.04 \pm 0.09$ |
| $\phi(\mathrm{degrees})$ | $-16.9 \pm 12.8 \pm 9.0$ |
| $\mathrm{a}(\mathrm{GeV} / \mathrm{c})^{-1}$ | $4.03 \pm 1.72 \pm 0.06$ |
| $\mathrm{~b}(\mathrm{GeV} / \mathrm{c})^{-1}$ | $1.29 \pm 0.63 \pm 0.67$ |
| $\chi^{2} / N D F$ | $52.4 / 45$ |

The indicated errors are statistical and systematic respectively.

To determine the parameters of the higher mass S-wave resonance around 1.9 $\mathrm{GeV} / \mathrm{c}^{2}$, the data are fit to a model containing a Breit-Wigner resonance as given in eq. 5.1 plus a simple polynomial term for the background,

$$
\begin{aligned}
& a_{S}=B G+B W e^{i \phi} \\
& \quad B G=\left(a_{0}+a_{1} M\right) e^{i\left(\varphi_{0}+\varphi_{1} M\right)}
\end{aligned}
$$

where $a_{0}, a_{1}, \varphi_{0}, \varphi_{1}$, and $\phi$ are parameters to be determined by the fit. The two solutions in this mass region are fit separately to the model. The results of the fits are shown as solid lines in fig. 16, and their parameters are given.in table III.

Table III

Resonance parameters for the $K_{0}^{*}(1950)$; the two ambiguous amplitudes are fitted separately using the Breit-Wigner parametrization described in the text.

| $K_{0}^{*}(1950)$ |  |
| :---: | :---: |
| Solution A |  |
| Mass $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $1934 \pm 8 \pm 20$ |
| Width $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $174 \pm 19 \pm 79$ |
| $\epsilon$ | $0.55 \pm 0.08 \pm 0.12$ |
| $\phi$ (degrees) | $48.3 \pm 5.6 \pm 15.5$ |
| $\chi^{2} / N D F$ | $4.2 / 8$ |
| Solution B |  |
| Mass $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $1955 \pm 10 \pm 8$ |
| Width $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $228 \pm 34 \pm 22$ |
| $\epsilon$ | $0.48 \pm 0.02 \pm 0.02$ |
| $\phi($ degrees $)$ | $49.1 \pm 4.5 \pm 1.1$ |
| $\chi^{2} / N D F$ | $5.4 / 8$ |

The indicated errors are statistical and systematic respectively.

### 5.2 The P-Wave Amplitude

In the P -wave, as shown in fig. 17 , the well known $K^{*}(892)$ is evident from the classic Breit-Wigner resonance phase motion below $1.0 \mathrm{GeV} / \mathrm{c}^{2}$. A fit to the P-wave phase in this region to a Breit-Wigner form (5.1) with no background gives the resonance parameters for the $K^{*}(892)$ as noted in table IV.

Table IV

Resonance parameters for the $K^{*}(892)$.

| $K^{*}(892)$ |  |
| :---: | :---: |
| Mass $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $895.9 \pm 0.5 \pm 0.2$ |
| Width $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $50.8 \pm 0.8 \pm 0.9$ |
| $\epsilon$ | 1.0 (Fixed) |
| $\mathrm{r}(\mathrm{GeV} / \mathrm{c})^{-1}$ | $3.4 \pm 0.6 \pm 0.3$ |
| $\chi^{2} / N D F$ | $16.9 / 12$ |

The indicated errors are statistical and systematic respectively.
Between about 1.2 and $1.55 \mathrm{GeV} / \mathrm{c}^{2}$, the P-wave magnitude is small (about 0.15) and relatively flat. However there is a small, but statistically significant, increase in magnitude around $1.4 \mathrm{GeV} / \mathrm{c}^{2}$ which is accompanied by substantial structure in the phase. Above $1.55 \mathrm{GeV} / \mathrm{c}^{2}$, the magnitude increases to a peak around 1.74 $\mathrm{GeV} / \mathrm{c}^{2}$ and then decreases. The peak is accompanied by rapid phase motion which leads to clear resonance-like behavior in the Argand diagram. This higher mass structure is very similar to that observed by Estabrooks et al. [5] in 2 out of 4 ambiguous solutions, and by Aston et al. [6] in an unambiguous solution, and is interpreted as the observation of a P-wave resonance, the $K^{*}(1790)$, decaying into $K^{-} \pi^{+}$. A $1^{-} K^{*}$ resonance decaying into $\bar{K}^{0} \pi^{-} \pi^{+}$has also been observed
in this mass region, and is most simply interpreted as another decay mode of this same state $[18,19]$. These three-body analyses have also presented evidence for a $K^{*}(892) \pi$ decay mode of a second $1^{-} K^{*}$ resonance at around $1.42 \mathrm{GeV} / \mathrm{c}^{2}$, where the $K^{-} \pi^{+}$magnitude is very small.

The $K^{-} \pi^{+}$data in the regions of the two prominent resonances can be well represented separately by Breit-Wigner forms, with an additional background term in the high mass region. However, these fits do not reproduce the behavior of the amplitude in the $1.4 \mathrm{GeV} / \mathrm{c}^{2}$ region. Therefore, given the evidence for two $1^{-} K^{*}$ states in the region between 1.3 and $1.8 \mathrm{GeV} / \mathrm{c}^{2}$ decaying into $K^{0} \pi^{-} \pi^{+}$, and the excellent statistics of this experiment, we perform a fit of the P -wave for the entire mass region below $1.84 \mathrm{GeV} / \mathrm{c}^{2}$ with all three of these states to quantify the evidence for a $K^{-} \pi^{+}$decay mode of the state in the $1.42 \mathrm{GeV} / \mathrm{c}^{2}$ region. The model has the form,

$$
\begin{aligned}
a_{P}=B G & +\sum_{i} B W_{i} e^{i \phi_{i}} \\
B G & =\left(a_{0}+a_{1} M\right) e^{i\left(\varphi_{0}+\varphi_{1} M\right)} \\
B W_{i} & =\frac{\sqrt{2 L+1} \epsilon_{i} M_{R, i} \Gamma_{i}}{\left(M_{R, i}^{2}-M^{2}\right)-i M_{R, i} \Gamma_{i}}
\end{aligned}
$$

where the notation is as described for eq. 5.1.
The results of the fit are shown by the curves in fig. 17 and the parameters are summarized in table V. The dashed line indicates the result of the fit when only the two large resonances, the $K^{*}(892)$ and the $K^{*}(1790)$ are included. Though the data are well represented below 1.2 and above $1.6 \mathrm{GeV} / \mathrm{c}^{2}$, the amplitude is poorly reproduced around $1.4 \mathrm{GeV} / \mathrm{c}^{2}$. The solid line shows the results when a third resonance around $1.4 \mathrm{GeV} / \mathrm{c}^{2}$ is included in the fit. The data are now well represented throughout the entire mass range. The significance of the third resonance
in this model is $>6$ standard deviations, and the parameters for this resonance are compatible with those found in the three-body experiments. These resonance parameters are correlated with the background parameters, and can change significantly when different background forms are used. Several different background parametrizations have been used, in addition to the one described, and the model dependent variations in the parameters that are obtained are taken into account in the systematic errors indicated in table V .

Table V
P-wave resonance parameters for the $K^{*}(1410)$ and $K^{*}(1790)$ as determined in the model described in the text.

| $K^{*}(1410)$ |  |
| :---: | :---: |
| Mass $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $1380 \pm 21 \pm 19$ |
| Width $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $176 \pm 52 \pm 22$ |
| $\epsilon$ | $0.066 \pm 0.010 \pm 0.008$ |
| $\phi(\mathrm{degrees})$ | $22.9 \pm 20.6 \pm 16.9$ |
| $r\left(\mathrm{GeV} / \mathrm{c}^{-1}\right)$ | 2.0 (Fixed) |
| $K^{*}(1790)$ |  |
| Mass $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $1677 \pm 10 \pm 32$ |
| Width $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | $205 \pm 16 \pm 34$ |
| $\epsilon$ | $0.388 \pm 0.014 \pm 0.022$ |
| $\phi(\mathrm{degrees})$ | $-2.5 \pm 8.6 \pm 8.3$ |
| $r\left(\mathrm{GeV} / \mathrm{c}^{-1}\right)$ | 2.0 (Fixed) |
| $\chi^{2} / N D F$ | $75.4 / 51$ |

The indicated errors are statistical and systematic respectively.

The observation of the $K^{*}(1410)$ with a strongly suppressed $K^{-} \pi^{+}$coupling is corroborated by the production dependence of the three-body amplitudes which indicates that the $1^{-}$amplitudes in the $K^{*}(1790)$ region are steep as is expected from $\pi$ exchange, while the slope of the $1^{-}$amplitude in the $K^{*}(1410)$ region is
much flatter, as is expected from a heavier production mechanism. Moreover, since the $K^{-} \pi^{+}$decay of this state is so suppressed, it would have been essentially unobservable in earlier $K^{-} \pi^{+}$experiments [5-6].

### 5.3 Higher waves

The dominant structures observable in the higher spin waves arise from the leading orbitally excited $K^{*}$ series. The resonance parameters for these states are extracted by fitting the partial wave magnitudes to the Breit-Wigner form given in eq. 5.1. In performing these fits, the turn-on of the partial waves associated with each of these states is assumed to be resonance dominated.

The D-wave magnitude clearly shows the presence of the well established $K_{2}^{*}(1430)$. The fit to the D -wave magnitude using eq. 5.1 gives the fit curve shown in fig. 18 , with the parameters of table VI.

Results from the three-body channel $\bar{K}^{0} \pi^{-} \pi^{+}$measured in this experiment have shown evidence for a second resonance in the D-wave around $2.0 \mathrm{GeV} / \mathrm{c}^{2}[19]$. The Argand plot for solution type B shows a cusp-like structure around $1.7 \mathrm{GeV} / \mathrm{c}^{2}$ followed by the arc shaped path around $2.0 \mathrm{GeV} / \mathrm{c}^{2}$ region which may be interpreted as arising from a similar resonance. However, the relatively small phase motion, the large background, the slow drift of this solution toward nonunitarity at high mass, and the overall phase uncertainty prevent us from drawing any firm conclusions regarding such a structure, and in any case solution type A contains no evidence for a structure. The most that can be said is that these data are not inconsistent with the existence of a D -wave resonance around $2.0 \mathrm{GeV} / \mathrm{c}^{2}$.

The F-wave amplitude is required above a $K \pi$ mass of $1.58 \mathrm{GeV} / \mathrm{c}^{2}$. The leading

Table VI
Resonance parameters for the leading D, F, G, and H states as determined by Breit-Wigner fits to the high spin waves as described in the text.

| $K_{2}^{*}(1430)$ |  |
| :---: | :---: |
| $\begin{gathered} \text { Mass }\left(\mathrm{MeV} / \mathrm{c}^{2}\right) \\ \text { Width }\left(\mathrm{MeV} / \mathrm{c}^{2}\right) \\ \epsilon \\ \mathrm{r}\left(\mathrm{GeV} / \mathrm{c}^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} 1431.2 \pm 1.8 \pm 0.7 \\ 116.5 \pm 3.6 \pm 1.7 \\ 0.485 \pm 0.006 \pm 0.020 \\ 2.7 \pm 1.2 \pm 0.6 \\ \hline \end{gathered}$ |
| $\chi^{2} / N D F$ | 15.9/10 |
| $K_{3}^{*}(1780)$ |  |
| $\begin{gathered} \hline \text { Mass }\left(\mathrm{MeV} / \mathrm{c}^{2}\right) \\ \text { Width }\left(\mathrm{MeV} / \mathrm{c}^{2}\right) \\ \epsilon \\ \mathrm{r}\left(\mathrm{GeV} / \mathrm{c}^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} 1781 \pm 8 \pm 4 \\ 203 \pm 30 \pm 8 \\ 0.187 \pm 0.008 \pm 0.008 \\ 2.0 \text { (Fixed) } \\ \hline \end{gathered}$ |
| $\chi^{2} / N D F$ | 11.0/5 |
| $K_{4}^{*}(2060)$ |  |
| $\begin{gathered} \text { Mass }\left(\mathrm{MeV} / \mathrm{c}^{2}\right) \\ \text { Width }\left(\mathrm{MeV} / \mathrm{c}^{2}\right) \\ \epsilon \\ \mathrm{r}\left(\mathrm{GeV} / \mathrm{c}^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} 2055 \pm 51 \\ 245 \pm 124 \\ 0.099 \pm 0.012 \\ 2.0 \text { (Fixed) } \\ \hline \end{gathered}$ |
| $\chi^{2} / N D F$ | 0.04/1 |
| $K_{5}^{*}(2380)$ |  |
| $\begin{gathered} \text { Mass }\left(\mathrm{MeV} / \mathrm{c}^{2}\right) \\ \text { Width }\left(\mathrm{MeV} / \mathrm{c}^{2}\right) \\ \epsilon \\ \mathrm{r}\left(\mathrm{GeV} / \mathrm{c}^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} 2382 \pm 34 \\ 147 \pm 80 \\ 0.061 \pm 0.012 \\ 2.0 \text { (Fixed) } \\ \hline \end{gathered}$ |
| $\chi^{2} / N D F$ | 0.06/1 |

The indicated errors are statistical and systematic respectively. Systematic errors are not estimated for the G- and H-wave states.
$K_{3}^{*}(1780)$ resonance can be clearly observed in the magnitude which rises steadily to a peak around $1.8 \mathrm{GeV} / \mathrm{c}^{2}$. There is a slight drop just above $1.8 \mathrm{GeV} / \mathrm{c}^{2}$ but the magnitude then remains rather large indicating a significant background to $K_{3}^{*}(1780)$ production above the resonance mass. The resonance fit for the leading state shown by the curve in fig. 18 with parameters as given in table VI is performed in the mass region below $1.88 \mathrm{GeV} / \mathrm{c}^{2}$ to avoid this background. In the region above $1.88 \mathrm{GeV} / \mathrm{c}^{2}$ the two solutions differ significantly. Solution B shows structure in amplitude and phase above $2.0 \mathrm{GeV} / \mathrm{c}^{2}$ which is suggestive of resonance behavior. However, this amplitude has most of the difficulties already discussed above for the high mass $D$ wave and in any case Solution A displays no significant effect.

A G-wave amplitude is required above a $K \pi$ mass of $1.92 \mathrm{GeV} / \mathrm{c}^{2}$. The leading $K_{4}^{*}(2060)$ can be observed in the magnitude which rises to a peak around 2.1 $\mathrm{GeV} / \mathrm{c}^{2}$. The fit to the G -wave magnitude shown in fig. 18 is independent of solution type and uses the independent mass bins as shown up to $2.2 \mathrm{GeV} / \mathrm{c}^{2}$. The resulting resonance parameters given in table VI agree with the results from the overall mass dependent fit to the moments presented in an earlier publication [4].

Evidence for a leading $K_{5}^{*}(2380)$ can be seen in the H-wave magnitude that peaks at about $2.4 \mathrm{GeV} / \mathrm{c}^{2}$ and is independent of solution type. The fit to this magnitude uses the four independent mass bins as shown, and the resulting parameters are given in table VI. These also agree with the results from the mass dependent fit to the moments presented earlier [4].

## 6. Discussion and Summary

Results from an energy independent partial wave analysis of a data sample more than four times larger than those used by earlier analyses provide new information on the $K \pi$ resonance structure. Clear resonance behavior can be seen in the partial wave amplitudes, which can be reproduced by simple Breit-Wigner modeling. Fits of the amplitudes with these models provide estimates for the masses, widths, and elasticities of the observed resonances.

The parameters of these states are summarized in table VII and in fig. 19.

Table VII
The preferred quark state assignments and the measured mass values for the states observed in this experiment; the predicted values from the model of ref. 8 are given for comparison.

| Spin <br> Parity | Probable <br> $\bar{q} q$ state | Measured Mass <br> $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ | Predicted Mass <br> $\left(\mathrm{MeV} / \mathrm{c}^{2}\right)$ |
| :---: | :---: | :---: | :---: |
|  | $1^{3} P_{0}$ | (I) 1340 <br> (II) $1429 \pm 6$ | 1240 |
|  | $2^{3} P_{0}$ | (A) $1934 \pm 22$ <br> (B) $1955 \pm 13$ | 1890 |
|  |  | $1^{3} S_{1}$ | $895.9 \pm 0.6$ |
| $1^{-}$ | $2^{3} S_{1}$ | $1380 \pm 28$ | 900 |
|  | $1^{3} D_{1}$ | $1677 \pm 34$ | 1580 |
| $2^{+}$ | $1^{3} P_{2}$ | $1431 \pm 2$ | 1780 |
| $3^{-}$ | $1^{3} D_{3}$ | $1781 \pm 9$ | 1420 |
| $4^{+}$ | $1^{3} F_{4}$ | $2055 \pm 51$ | 1790 |
| $5^{-}$ | $1^{3} G_{5}$ | $2382 \pm 34$ | 2110 |

The quoted errors include the statistical and systematic errors added in quadrature.

For the most part, the observed states can be naturally assigned to levels expected in a quark model. The preferred quark model assignments are also summarized in table VII and fig. 19, along with predictions for their mass values taken from the $q \bar{q}$ potential model of Godfrey and Isgur [8].

The amplitudes confirm the first four leading resonances, the $K^{*}(892)$, the $K_{2}^{*}(1430)$, the $K_{3}^{*}(1780)$, and the $K_{4}^{*}(2060)$, and fits to these amplitudes provide new measurements for the parameters of these states. In the highest mass region analyzed, the behavior of the H-wave amplitude supports the earlier observation of a spin- $5 K_{5}^{*}(2380)$ state based on the moments fit [4]. In the quark model, these states can naturally be understood as the leading orbitally excited triplet series, the $1^{3} S_{1}, 1^{3} P_{2}, 1^{3} D_{3}, 1^{3} F_{4}$, and $1^{3} G_{5}$. These five states lie close to a linear Regge trajectory with a slope $0.84\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{-1}$, and the model predicts their masses well.

The lowest mass S-wave state, the $K_{0}^{*}(1350)$, is naturally assigned as the lowest $0^{+}$quark model state, the $1^{3} P_{0}$. The predicted and measured masses show a substantial difference ( $\sim 100-200 \mathrm{MeV} / \mathrm{c}^{2}$ ). However, the measured mass of this state depends critically on the model used for the estimation, so that this difference is difficult to interpret.

A second $0^{+}$state is observed around $1.95 \mathrm{GeV} / \mathrm{c}^{2}$, confirming an observation in a previous study of the $K^{-} \pi^{+}$channel [6], but with a much better determination of the parameters. Within the quark model, this state can only be assigned as a radial excitation of the $0^{+}$member of the $L=1$ triplet, most naturally the $2^{3} P_{0}$ state. This state is one of the clearest candidates known for a radial excitation of a light quark system. The measured mass value agrees well with the predictions of the model.

The two P-wave resonances observed in earlier $K^{-} \pi^{+}$analyses [5-6], the $K^{*}(892)$, and the $K^{*}(1790)$, are clearly seen in the amplitudes. In addition, new evidence for a structure with a small elasticity around $1.4 \mathrm{GeV} / \mathrm{c}^{2}$ is provided which is most easily interpreted as a confirmation of the $K^{*}(1410)$ resonance seen in the results of the three-body $\bar{K}^{0} \pi^{-} \pi^{+}$PWA, decaying into the $K^{-} \pi^{+}$mode. The most recent of the three-body analyses, using data from this experiment, observed the two higher mass states at masses of $1420 \mathrm{MeV} / \mathrm{c}^{2}$ and $1735 \mathrm{MeV} / \mathrm{c}^{2}$ respectively [19], in good agreement with this analysis. In the potential model of ref. 8 , two non-leading $1^{-}$states, the $2^{3} S_{1}$ and the $1^{3} D_{1}$, are expected below a mass of $2.0 \mathrm{GeV} / \mathrm{c}^{2}$. The preferred assignment of these observed states to the levels predicted by the model has been discussed previously [19], and the strength of these arguments is enhanced by the observations made in this analysis. Even though mixing is not excluded, it is simplest to associate the higher mass state with the $1^{3} D_{1}$, based primarily on the small mass splitting between this state and the $K_{3}^{*}(1780)$. The lower state is then interpreted as the first radial excitation of the $K^{*}(892)$. The supression of the $K^{-} \pi^{+}$decay mode of this lower state can be understood in some models as being a dynamical effect resulting from the presence of a node in the radial wave function [20-22]. The agreement of the $K^{*}(1410)$ with the mass predictions of the model of ref. 8 for the $2^{3} S_{1}$ is rather poor. However, other models give mass predictions for the $2^{3} S_{1}$ that are consistent with the $K^{*}(1410)$ mass and also suggest that the $3^{3} S_{1}$ state should lie well below $2.0 \mathrm{GeV} / \mathrm{c}^{2}[20]$. The assignment discussed here implies that the nonet containing the $2^{3} S_{1}$ states should lie in the mass region between about 1250 and $1600 \mathrm{MeV} / \mathrm{c}^{2}$, and that these states will not be easily observable in the simple two body modes. Though many previous experiments have found evidence for a candidate isovector state, the $\rho(1250)$ [2], its present status is equivocal
[23,24], and there are no firm candidates for the other states in this nonet. On the other hand, a recent analysis has obtained a consistent picture of several different channels by postulating two $\rho^{\prime}$ resonances with masses 1.465 and $1.700 \mathrm{GeV} / \mathrm{c}^{2}[25]$.

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## Figure Captions

1. Plan view of the LASS spectrometer.
2. The $M M^{2}(K \pi)$ distribution for all candidates with $\left|t^{\prime}\right|<0.2(\mathrm{GeV} / \mathrm{c})^{2}$. The solid line indicates the result of the fit described in the text. The dashed line indicates the $\Delta^{0}$ background contribution and the dotted line indicates the $K^{-} \pi^{0} p$ background contribution determined by the fit. Neutron candidates are selected from this distribution using the cuts described in the text.
3. The $K^{-} \pi^{+}$invariant mass distribution for events with $\left|t^{\prime}\right| \leq 1.0(\mathrm{GeV} / \mathrm{c})^{2}$. The unshaded curve contains all events while the cross-hatched curve contains events with $n \pi^{+}$mass greater than $1.7 \mathrm{GeV} / \mathrm{c}^{2}$.
4. The $n \pi^{+}$invariant mass distribution for events with $\left|t^{\prime}\right| \leq 1.0(\mathrm{GeV} / \mathrm{c})^{2}$.
5. The $\cos \theta_{G J}$ vs $M_{K \pi}$ plot for; (a) events with $\left|t^{\prime}\right| \leq 0.2(\mathrm{GeV} / \mathrm{c})^{2}$; and (b) events with $0.2<\left|t^{\prime}\right| \leq 1.0(\mathrm{GeV} / \mathrm{c})^{2}$.
6. The acceptance corrected $\mathrm{M}=0$ unnormalized $K^{-} \pi^{+}$moments as a function of mass for the small $\left|t^{\prime}\right|$ region, $\left|t^{\prime}\right| \leq 0.2(\mathrm{GeV} / \mathrm{c})^{2}$. The notation is defined in eq. 3.1.
7. The acceptance corrected $\mathrm{M}=1$ unnormalized $K^{-} \pi^{+}$moments as a function of mass for the small $\left|t^{\prime}\right|$ region, $\left|t^{\prime}\right| \leq 0.2(\mathrm{GeV} / \mathrm{c})^{2}$.
8. The acceptance corrected $\mathrm{M}=2$ unnormalized $K^{-} \pi^{+}$moments as a function of mass for the small $\left|t^{\prime}\right|$ region, $\left|t^{\prime}\right| \leq 0.2(\mathrm{GeV} / \mathrm{c})^{2}$.
9. The acceptance corrected unnormalized $K^{-} \pi^{+}$moments as a function of $\left|t^{\prime}\right|$. Three different mass regions are shown; $0.87 \leq M_{K \pi} \leq 0.89 \mathrm{GeV} / \mathrm{c}^{2} ; 1.42 \leq$
$M_{K \pi} \leq 1.48 \mathrm{GeV} / \mathrm{c}^{2}$; and $1.80 \leq M_{K \pi} \leq 1.90 \mathrm{GeV} / \mathrm{c}^{2}$. The curves are the result of a fit to the production model described in the text.
10. The $t^{\prime}$ dependent parameters as a function of $K^{-} \pi^{+}$mass. The smooth curves drawn through these data points represent simple smoothing functions as described in the text.
11. The $K^{-} \pi^{+}$amplitude (Barrelet) zeros for solution A as a function of mass.
12. The magnitudes and phases of the amplitudes for solution A. All solutions are identical below the mass indicated by the broken line. The symbol $\circ$ indicates values fixed using the procedure described in the text. Where error bars are not shown, the plotted point is larger than the statistical error associated with the point.
13. The magnitudes and phases of the amplitudes for solution $B$. The notation is as indicated for fig. 12.
14. The Argand diagrams for solutions A and B. The overall phase is fixed as described in the text. The dashed line represents the $I=1 / 2$ S-wave amplitude determined by subtracting the $I=3 / 2$ component using the parametrization of ref. 5. The ticks mark $0.2 \mathrm{GeV} / \mathrm{c}^{2}$ intervals.
15. The magnitude (a) and phase (b) of the $I=1 / 2 \mathrm{~S}$-wave amplitude in the mass region below $1.6 \mathrm{GeV} / \mathrm{c}^{2}$. Where error bars are not shown, the points are larger than the statistical errors. The curve shows the results of the Model II fit described in the text.
16. The magnitudes and phases of the S-wave amplitude for solutions A (a) and $B(b)$ in the mass region between 1.76 and $2.14 \mathrm{GeV} / \mathrm{c}^{2}$. Where error bars are
not shown, the points are larger than the statistical errors. The curves show the results of the Breit-Wigner fits described in the text. The amplitudes from the two solutions are fitted separately.
17. The magnitude (a) and phase (b) of the $P$-wave amplitude in the mass region below $1.84 \mathrm{GeV} / \mathrm{c}^{2}$. Where error bars are not shown, the points are larger than the statistical errors. The curves show the results of the Breit-Wigner fits described in the text. The dashed line indicates the fit including the $K^{*}(892)$ and $K^{*}(1790)$ resonances only, while the solid line indicates the fit that includes another resonance around $1.4 \mathrm{GeV} / \mathrm{c}^{2}$.
18. The magnitudes of the leading $D, F, G$, and $H$-wave amplitudes in the leading resonance mass regions. The curves show the results of the Breit-Wigner fits described in the text.
19. The preferred quark model assignments for the states observed in this analysis compared with the predictions from ref. 8. The shaded regions indicate the errors in the measured mass as quoted in table VII.


Fig 1


Fig. 2


Fig 3


Fig. 4


Fig. 5


Fig. 6


Fig. 7


Fig. 8


Fig. 9


Fig. 10


Fig. 11


Fig. 12


Fig. 13


Fig. 14


Fig. 15


Fig. 16


Fig. 17


Fig. 18


Fig. 19


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[^1]:    * The naturality amplitudes are defined by taking the following combination of helicity amplitudes. $L_{0}=H_{0}^{L}, L_{\lambda}^{ \pm}=\frac{1}{\sqrt{2}}\left(H_{\lambda}^{L} \mp(-1)^{\lambda} H_{-\lambda}^{L}\right)$.

