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ϵ'/ϵ from the lattice*

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ABSTRACT

Results from a lattice calculation of the matrix elements of the weak Hamiltonian which determine ϵ' are presented. They suggest a smaller value of ϵ'/ϵ than previously estimated.

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1. Introduction. In this letter we take the first steps towards a complete evaluation of the ratio ϵ'/ϵ from first principles. To do this, one must combine a nonperturbative evaluation of certain hadronic matrix elements (ME) with standard renormalization group scaling. Here we calculate the matrix elements using the lattice approximation.

Our result is a qualitative conclusion: the effects we find tend to reduce ϵ'/ϵ from standard estimates. Combining this with a recent reevaluation of the importance of isospin breaking [1] puts ϵ'/ϵ below the sensitivity of present experiments, though possibly measurable in planned experiments. The present result for ϵ'/ϵ is $+.0035 \pm .003 \pm .002$ [2] , and the next generation of experiments hope to reach a sensitivity of .0005.

Our calculation uses the staggered discretization of fermions, rather than that proposed by Wilson. With the lattice technology available at present, this choice is preferred for an evaluation of ϵ'. The problem for Wilson fermions is the lack of GIM cancellation after the top quark has been integrated out [3] . This presents no difficulty for staggered fermions [4] [5] , and we work here in the effective theory just above the charm quark scale.

In the three generation standard model both ϵ and ϵ' are, to good approximation, proportional to $\sin\delta$, δ being the CP violating phase in the KM matrix. Thus the ratio ϵ'/ϵ can, in principle, be predicted, once one knows the various KM angles (θ_i) and the top quark mass (m_t). Necessary ingredients for this prediction are the ME of operators in the weak Hamiltonian ($\mathcal{H}_W$) between appropriate hadronic states. Those ME needed for ϵ can be estimated using a variety of continuum techniques, giving a result uncertain to about a factor of two. Future lattice calculations will reduce this uncertainty, though present calculations cannot. By comparison, the continuum estimates of the ME needed for ϵ' are less reliable. Thus even a semi-quantitative result from the lattice is of interest, and this is what we provide here.

To be precise, what we actually calculate are the ME needed for the imaginary part of the $K \to \pi\pi$ decay amplitudes. To convert these into a result for ϵ' we need also to know the real parts of these amplitudes. These we take from experiment. To further convert to a result for ϵ'/ϵ we must face the uncertainties in $s_i = \sin(\theta_i)$ and in m_t, as well as those in the ME needed to calculated ϵ. We do this by using the experimental value for ϵ (2.27×10^{-3}) and lumping all the uncertainties into the product $s_2 c_2 s_3 s_\delta$.

The dominant contribution to ϵ' comes from operators with a LR chiral structure. These operators also contribute to the real part of the $K \to \pi\pi$ amplitude. It has been suggested that they have large matrix elements, and that this can explain the $\Delta I = \frac{1}{2}$ rule [6] . The arguments for this are carried out at a scale $\mu << m_c$, where our calculations do not apply, and all perturbative calculations are suspect. The same arguments can also be used to predict a value for ϵ'. This allows us to test this line of reasoning indirectly, since we can roughly evaluate ϵ'. Our results suggest that LR operators are subdominant even for $\mu << m_c$. The only escape is the possibility that the Wilson coefficients of these operators have been underestimated, as has been suggested recently [7] .

2. Theory. At scales μ in the range $m_c < \mu < m_b$, the dominant CP violating part of $\mathcal{H}_W$ is

$$\mathrm{Im}\,\mathcal{H}_W = \frac{G_F}{\sqrt{8}}(s_1 s_2 c_2 s_3 s_\delta) \sum_{i=6,7,8} \widetilde{c}_i O_i. \tag{1}$$

The $\widetilde{c}$ are Wilson coefficients, multiplying the operators

$$\begin{aligned} O_6 &= \bar{s}_a \gamma_\mu (1+\gamma_5) d_b \sum_{q=u,d,s,c} \bar{q}_b \gamma_\mu (1-\gamma_5) q_a \\ O_7 &= \bar{s}_a \gamma_\mu (1+\gamma_5) d_a \sum_{q=u,d,s,c} \frac{3Q_q}{2}\, \bar{q}_b \gamma_\mu (1-\gamma_5) q_b \\ O_8 &= \bar{s}_a \gamma_\mu (1+\gamma_5) d_b \sum_{q=u,d,s,c} \frac{3Q_q}{2}\, \bar{q}_b \gamma_\mu (1-\gamma_5) q_a . \end{aligned} \tag{2}$$

These operators appear when the top quark is integrated out. O_6 is induced by strong interaction penguin diagrams, O_7 by electromagnetic penguin graphs. O_7 in turn gives rise to O_8 upon renormalization group scaling. $\widetilde{c}_7$ and $\widetilde{c}_8$ are both proportional to α_{em}. Inserting $\mathrm{Im}\,\mathcal{H}_W$ in the standard expression for ϵ', and using lowest order current algebra to convert $K \to \pi\pi$ amplitudes into $K \to \pi$ ME, we find

$$\begin{aligned} \left|\frac{\epsilon'}{\epsilon}\right| &= 5.8 \times 10^3\,\mathrm{GeV}^{-4}\, |s_1 s_2 c_2 s_3 s_\delta|\, \left|\langle \pi^+ | \widetilde{c}_6 O_6^{subt} | K^+ \rangle\right|\, |1 - \Omega| \\ \Omega &\equiv \Omega_{emp} + \Omega_\eta + \Omega_{\eta'} \\ \Omega_{emp} &= \sqrt{2}\frac{1-\sqrt{2}\omega}{3\omega} \frac{m_\pi m_K}{m_K^2 - m_\pi^2} \frac{\langle \pi^+ | \widetilde{c}_7 O_7 + \widetilde{c}_8 O_8 | K^+ \rangle}{\langle \pi^+ | \widetilde{c}_6 O_6^{subt} | K^+ \rangle} \end{aligned} \tag{3}$$

Here $\omega \approx 1/22$ is the ratio of $I = 3/2$ to $I = 1/2$ kaon decay amplitudes. The factors of Ω_η ($\approx .3 - .4$), and $\Omega_{\eta'}$ ($\approx .04 - .06$) arise from isospin breaking [1]. The combination of angles that appears is bounded above by 4×10^{-4}. The superscript *subt* will be discussed later. We stress that (3) is only true to zeroth order in an expansion in m_K^2/Λ^2, where $\Lambda \approx 1\,\mathrm{GeV}$. For the present, though, this is sufficient.

In the $N_c \to \infty$ limit, one can show that, to $O(m_q)$

$$\langle \pi^+ | O_8 | K^+ \rangle = \langle \pi^+ | O_8 | K^+ \rangle_{VIA} \equiv f_\pi f_K \Big(6 \frac{m_K^2}{m_u + m_s} \frac{m_\pi^2}{m_u + m_d} - 4 m_\pi m_K \frac{m_K^2}{m_s^2}\big(\frac{f_K}{f_\pi} - 1\big)\Big). \tag{4}$$

Note that this tends to a constant as $m_q \to 0$. In fact, to work consistently to $O(m_q)$ we should drop the second term in (4), though this has little effect numerically. One can also show [7][8] that

$$\langle \pi^+ | O_6^{subt} | K^+ \rangle = \langle \pi^+ | O_6^{subt} | K^+ \rangle_{VIA} \equiv 8 f_\pi f_K m_\pi m_K \frac{m_K^2}{m_s^2}\big(\frac{f_K}{f_\pi} - 1\big). \tag{5}$$

This vanishes like $m_\pi m_K$ in the chiral limit. The conventional vacuum insertion approximation (VIA) also yields (4) and (5), but in addition has

$$\langle\pi^+|O_7|K^+\rangle_{VIA} = \frac{1}{3}\langle\pi^+|O_8|K^+\rangle_{VIA}. \tag{6}$$

Using these approximations we can rewrite (3) as

$$\begin{aligned}\left|\frac{\epsilon'}{\epsilon}\right| &= 5\times10^{-3}\left|\frac{s_1s_2c_2s_3s_\delta}{4\times10^{-4}}\right|\left|\frac{\widetilde{c}_6}{.1}\right|\left(\frac{125\,\mathrm{MeV}}{m_s}\right)^2\left|\frac{\langle\pi^+|O_6^{subt}|K^+\rangle}{\langle\pi^+|O_6^{subt}|K^+\rangle_{VIA}}\right||1-\Omega| \\ \Omega_{emp} &= .23\ \frac{\frac{\widetilde{c}_7}{3}+\widetilde{c}_8}{\alpha_{em}\widetilde{c}_6}\ \frac{\langle\pi^+|\widetilde{c}_7O_7+\widetilde{c}_8O_8|K^+\rangle}{\left(\frac{\widetilde{c}_7}{3}+\widetilde{c}_8\right)\langle\pi^+|O_8|K^+\rangle_{VIA}}\ \frac{\langle\pi^+|O_6^{subt}|K^+\rangle_{VIA}}{\langle\pi^+|O_6^{subt}|K^+\rangle}.\end{aligned} \tag{7}$$

The estimates of the $\widetilde{c}$ discussed below suggest that the ratios of Wilson coefficients appearing in (7) are close to 1. In VIA all the ratios of ME are also unity, so that in this approximation we have $|\epsilon'/\epsilon| = 1-2\times10^{-3}$. What we can best evaluate on the lattice are the ratios of ME calculated with and without the VIA, as this cancels some of the systematic errors. Plugging the results into (7) allows us to extract a lattice estimate of ϵ'/ϵ.

Our method for transcribing O_6 onto the lattice has been explained in detail in ref. [5]. In the notation given there

$$\langle\pi^+|O_6^{subt}|K^+\rangle = -2(P_0^2+P_4^2)+(2I_1^1+I_1^1(s)-2I_3^1-I_3^1(s))+O(g^2) \tag{8}$$

The "eye" contractions, denoted by I_1^1 etc, are subdominant, and we drop them. They do not have the $1/m_s^2$ enhancement factors of (5), and can be ignored even if they are enhanced sufficiently to explain the $\Delta I=\frac{1}{2}$ rule. The $O(g^2)$ corrections have not been calculated, and so we work to $O(g^0)$. The symbol P – for "penguin" – indicates that "figure eight" and eye contractions have been combined. It is this that gives the suppression factor of f_K-f_π in (5)[9] .

In ref. [5] the $(8_L, 8_R)$ operators O_7 and O_8 were not considered. It is simple to extend the notation defined there to accommodate these operators. One must introduce $\Delta=0$ and $\Delta=4$ eight contractions. Those with one color loop we call E_0^1 and E_4^1, respectively, and those having two color loops E_0^2 and E_4^2. Then the transcription is

$$\begin{aligned}\langle\pi^+|O_7|K^+\rangle &= 3(E_0^1-E_4^1)+P_0^1+P_4^1+\tfrac{1}{2}(I_1^2-I_1^2(s)-I_3^2+I_3^2(s))+O(g^2)\\ \langle\pi^+|O_8|K^+\rangle &= 3(E_0^2-E_4^2)+P_0^2+P_4^2+\tfrac{1}{2}(I_1^1-I_1^1(s)-I_3^1+I_3^1(s))+O(g^2)\end{aligned} \tag{9}$$

Again we drop the $O(g^2)$ parts, and the subdominant eye contractions. As discussed below equation (4), to be consistent we should not include P_j^i either. The contractions leading to the E^i_j are, in fact, the figure eight part of the P^i_j, and so their numerical evaluation requires no extra labor. For example, E^2_0 is the average over configurations of the product of the kaon correlator and the pion correlator. E^2_0 is the only one of the E^i_j which does not vanish in lattice VIA, and its approximant is obtained by averaging the pion and kaon correlators separately, and then taking their product.

Since we drop the I^i_j from (8) and (9) the charm quark does not appear in any of the contractions we calculate. The charm quark influences the calculation only indirectly through its effect on the Wilson coefficients. Thus we need only calculate propagators for light quarks.

To complete the definitions we must explain the superscript *subt* on $\mathcal{O}_6$ in (3). As first noted in [10] and elucidated in [3], $\mathcal{O}_6$ can mix with the dimension 4 operator

$$\mathcal{S} = i\bar{s}\gamma_\mu(1+\gamma_5)(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)d.$$

One can undo this mixing in a variety of ways [5], all equivalent at lowest order in m_q, though differing in higher orders. The most straightforward method is to form $\mathcal{O}_6^{subt} \equiv \mathcal{O}_6 - \rho\mathcal{S}$, and adjust ρ so that $\langle 0|\mathcal{O}_6^{subt}|K^0\rangle = 0$. This we use for the results of Table 2. We will comment below on the variations between methods. No subtraction is needed for the E^i_j, which cannot mix with $\mathcal{S}$.

3. Lattices and Renormalization Group Scaling. Our results come from 25 configurations on a $12^3 \times 30$ lattice. We use an improved action, which in the notation of ref. [11] is

$$S = 10.5\big(\mathrm{Re\,Tr}(U_F) - .12\,\mathrm{Re\,Tr}(U_6) - .12\,\mathrm{Re\,Tr}(U_8) - .04\,\mathrm{Re\,Tr}(U_{1\times 2})\big). \qquad (10)$$

We calculate quark propagators with masses .005 and .040 in lattice units, applying antiperiodic boundary conditions in all directions. We use five base points within the 2^4 hypercube, which allows us to measure the ME of all the required operators. We also have propagators from one base point at other masses with which we can calculate masses and decay constants.

The improved action is designed to be as near as possible to the renormalized trajectory for pure gauge theory. Using the tree level relation to determine the bare charge, we find it much larger than at the corresponding point on the Wilson axis with the same lattice spacing (based on the string tension), i.e. $g^2/4\pi = .41$ compared to $g^2/4\pi = .08$. It is also larger than the continuum α_s evaluated at a scale $\mu = 1/a$. However, a calculation of the non-perturbative β function with this action [12] gives a result for $\Delta\beta$ much smaller than the asymptotic value. This shows that the tree level relation is not valid and that we should use a smaller effective bare charge. The relationship between lattice and continuum charge needs to be calculated. For the present we simply assume that the continuum and lattice coupling constants are equal.

To complement the lattice calculation of matrix elements, we need the coefficients $\tilde{c}_6$, $\tilde{c}_7$ and $\tilde{c}_8$. To the accuracy we are working, it is appropriate to use the leading logarithm results embodied in the Renormalization Group (RG) equations of ref. [13] . The only unknown is the value of α_s to put in these equations, or, equivalently, the value of the one-loop Λ to use. Numerically, the difference can be significant because the ratios of continuum to lattice Λ values are typically large. However, as discussed above, for our action we use the continuum formulae of ref. [13] with the lattice values of Λ set equal to typical continuum values. Thresholds are treated in the usual way, matching occurring when the quark mass satisfies $ma = 1$. The RG equations are run down to $\mu = 1/a = 1.7\,\mathrm{GeV}$. We use $m_b = 4.5\,\mathrm{GeV}$, and vary m_t from 30 to 70 GeV. For $\Lambda = .1(.3)$ GeV we find $-\tilde{c}_6 = .08 - .09\ (.12 - .15)$, $-\tilde{c}_7/\alpha_{em} = .14 - .21\ (.10 - .17)$, and $-\tilde{c}_8/\alpha_{em} = .03 - .06\ (.04 - .09)$, where the ranges come from varying m_t. Thus we have $\tilde{c}_6 \approx -.1$ and $(\tilde{c}_7/3 + \tilde{c}_8)/\alpha_{em} \approx -.1$, which explains the values used in (3). Note, these coefficients are smaller by a factor of roughly 2 than those obtained by running the RG equations down to μ such that $\alpha_s(\mu) = 1$.

4. Results. We first clarify the expected behavior of the ME as m_π and m_K vanish. We should find that $\langle\pi^+|\mathcal{O}_6{}^{subt}|K^+\rangle \propto m_\pi m_K + O(m_q^2)$ while $\langle\pi^+|\mathcal{O}_{7,8}|K^+\rangle \propto 1 + O(m_q)$, as exemplified by (4) and (5). These behaviors are guaranteed on the lattice, provided the lattice correlators are dominated by the leading pion and kaon poles, and that m_π and m_K are small enough. One can also show under these conditions, that the lattice VIA gives both (4) and (5). The crucial question is then whether these assumptions are valid for our lattice.

We can test this by studying simpler quantities such as $m_\pi^2/m_1 + m_2$ and various definitions of f_π [4]. These should all behave as $a + b(m_1 + m_2) + O(m_q^2)$, with relationships between the coefficients a and b. Table 1 shows our results for $m_\rho, m_\pi, f_\pi^a, f_\pi^b, f_\pi^c$. Results for mesons with both equal mass quarks and unequal mass quarks are given. The chiral behavior agrees quite well with expectations: the various f_π have small linear variations with m. In addition the intercepts and slopes are consistent with the expected relationships, though within quite large errors. We can also extract the physical $f_K/f_\pi - 1$, and we find .22, consistent with the actual value. Thus the absolute values of the slopes b are reasonable.

We can find the lattice spacing using the values of f_π or m_ρ extrapolated to zero quark mass. Both yield $1/a \approx 1.7\,\mathrm{GeV}$. Thus our lightest pion has a mass of about 300 MeV. Using the ϕ mass, or the ratio $m_\phi/m_{\eta(ss)}$, we find $m_s a \approx .03$. Thus $m_q = 0.04$ is slightly heavier than the physical strange quark. Further, note that in physical units $m_s \approx 50\,\mathrm{MeV}$, which is much smaller than the known value of $\approx 125\,\mathrm{MeV}$. Similar low values have been found in other quenched simulations [14] . They must represent an artifact of the quenched approximation, for the following reason. We have constrained the

RHS of $m\langle\overline{\chi}\chi\rangle \approx 4m_\pi^2 f_\pi^2$ to match with the continuum values. Since $\langle\overline{\chi}\chi\rangle$ is overestimated in the quenched approximation, m_s must come out too small.

Our results for the ME are given in table 2. We quote physical ME, the $\mathcal{M}_{K\pi}$ of ref. [5]. The table includes results for P_0^1, which, though unimportant for ϵ'/ϵ, allow a test of the relation $P_0^1 = P_0^2/3$, which is true in VIA. The lattice VIA to P_0^2 and E_0^2 are also shown. Errors come from binning into subsamples. For comparison we show also $m_\pi m_K$, the expected chiral behavior for P_0^1 and P_0^2.

The best way to use the table is to form ratios to the VIA values, and insert the results in (3). This approach is actually essential because equations (4) and (5) suggest that the ME are inversely proportional to the lattice value of m_s. Since the lattice m_s is too small, we would obtain considerable overestimates of the ME if we converted to physical units directly.

VIA works very well at the heaviest quark masses, but breaks down as m_q decreases. This is particularly true of P_0^1 and P_0^2. P_0^2 drops to one half of VIA, while P_0^1 possibly changes sign. Despite the large difference between P_0^2 and its approximant, both are consistent with the required chiral behavior. In fact, it is the VIA result which deviates more from proportionality to $m_\pi m_K$. These deviations are presumably $O(m_q)$ corrections, their magnitudes being consistent with the variations in f_π. The same is not true for P_0^1. We stress that it is crucial to find the correct chiral behavior if the current algebra argument relating our ME to ϵ' is to hold. Clearly, we need more mass points and smaller errors to provide a significant test.

The results for E_0^1 and E_0^2 are much cleaner but are less dramatic. E_0^2 exceeds its approximant by 20% at small quark masses, while, within the errors, the relation $E_0^1 = E_0^2/3$ holds. All the results are consistent with m_q independence as $m_q \to 0$.

The data for P_4^1 and P_4^2 are too poor to allow stable fits in nearly all cases. In VIA these ME vanish, and so we expect them to be small at large quark masses. We can place bounds on the ME from our data, and we find that these are an order of magnitude below the P_0^1 and P_0^2 results for the heaviest two quark masses. For the lightest quark masses, however, the bounds are an appreciable fraction of the P_0^1 and P_0^2 results. Thus it is possible that the ratio of the total ME of $\mathcal{O}_6$ to its VIA value will be increased from the value coming from table 2.

We have no data for E_4^1 and E_4^2 because of an oversight. Both of these vanish in VIA, and, based on the success of VIA for E_0^1 and E_0^2, we expect them to be small.

There are systematic errors in the subtraction procedure which are not included in the results of table 2. First, there is an error in the calculation of the subtraction coefficient ρ. We have estimated this by hand, and find it to be comparable to the quoted statistical errors. Second, there are other ways to do the subtraction, using the ME of $\mathcal{O}_6$ between a kaon and the vacuum, or using the induced value of $\langle\overline{s}d\rangle$ [5]. All methods should agree as

$m_q \to 0$. Although, they diverge for the higher masses, we find that they do agree for the lightest mass point, within the quite large errors. This gives us some confidence that our lowest mass result is reasonable.

A further source of concern is the "wrap around" contributions allowed by our boundary conditions. These introduce an oscillating component in our data for P_0^1 and P_0^2 at the lightest two masses. This is shown in the figures of ref. [15] . The uncertainties this introduces are supposedly included in our error estimates, but this needs to be checked by repeating the calculation using fixed BC.

5. Conclusions. Given all the caveats, what we extract from our results are two trends. The first and clearest is that the ME of $\mathcal{O}_6$ are smaller then expected in VIA. The second is that the ME of $\mathcal{O}_7$ and $\mathcal{O}_8$ are somewhat enhanced compared to the VIA, though this is less convincing. Returning to (7) the implications for ϵ'/ϵ are clear. Both differences from VIA reduce ϵ'/ϵ. Note that the factors Ω_η and $\Omega_{\eta'}$ are unaffected. Assuming the ME of ${\mathcal{O}_6}^{subt}$ is reduced by 2, while those of $\mathcal{O}_8$ are increased by 1.2, the factor $1 - \Omega$ nearly vanishes. The electromagnetic penguin contribution itself cancels more than half of that of the strong interaction penguin. Thus ϵ'/ϵ may be reduced significantly from its VIA value, and could even change sign. Since the VIA value is at the present experimental sensitivity, it may well be that increased sensitivity is needed to reach the standard model prediction for ϵ'/ϵ.

In conclusion, although the actual numbers we have extracted are not to be taken too seriously, it is more likely that the general trends we find are correct. Of course, the list of approximations, and consequent undetermined systematic errors, is the usual long one. In particular we need more data at small quark masses, using propagators calculated with fixed BC. Nevertheless we consider it significant that the methods we have adopted, in particular the use of staggered fermions, and the subtraction technique, appear to work well with our relatively small sample of lattices.

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Table 1						
$m_1(m_2)$	.0	.005	.01	.02	.005(.04)	.04
m_π		.180(15)	.245(10)	.335(10)	.358(7)	.469(5)
m_ρ	.45		.50(10)	.55(5)		.66(2)
f_π^a	.055	.056(6)	.064(7)	.073(5)	.072(3)	.088(2)
f_π^b	.053	.052(6)	.055(4)	.056(4)	.056(4)	.058(4)
f_π^c	.064	.067(5)				.087(5)

Masses and decay constants. Values for $m_1 = m_2 = 0$ are obtained by extrapolation. The various definitions of f_π are given in ref. [4]. f_π^a corresponds to the physically measured f_π, and should differ from the other definitions only at $O(m_q)$.

Table 2			
$m_1(m_2)$	.005	.005(.04)	.04
P_0^1	.001(1)	-.004(1)	-.018(2)
P_0^2	-.007(3)	-.014(3)	-.054(4)
$VIA\ to\ P_0^2$	-.016(5)	-.022(5)	-.053(6)
$m_\pi m_K/3$	.011	.021	.073
E_0^1	-.031(6)	-.031(4)	-.041(2)
E_0^2	-.091(15)	-.091(12)	-.124(5)
$VIA\ to\ E_0^2$	-.072(9)	-.082(8)	-.126(4)

Results for physical ME, in lattice units. The ME are taken between a pion with composition $\overline{q}_1 q_1$, and a kaon with composition $\overline{q}_2 q_1$.