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Resonant Enhancements In WIMP Capture By The Earth

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ABSTRACT

The exact formulae for the capture of WIMPs (weakly interacting massive particles) by a massive body are derived. Capture by the earth is found to be significantly enhanced whenever the WIMP mass is roughly equal to the nuclear mass of an element present in the earth in large quantities. For Dirac neutrino WIMPs of mass 10 to 90 GeV, the capture rate is 10 to 300 times that previously believed. Capture rates for the sun are also recalculated and found to be from 1.5 times higher to 3 times lower than previously believed, depending on the mass and type of WIMP. The earth alone, or the earth in combination with the sun is found to give a much stronger annihilation signal from Dirac neutrino WIMPs than the sun alone over a very large mass range. This is particularly important in the neighborhood of mass of iron where previous analyses could not set any significant limits.

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1. Introduction

As part of their argument that WIMPs (weakly interacting massive particles) could explain both the 'dark matter problem' and the 'solar neutrino problem' Press and Spergel (1985) gave an estimate of the capture rate by a massive body, of WIMPs in a Maxwell-Boltzmann distribution in the galactic halo or galactic disk. Their argument made admittedly crude assumptions about WIMP phase space which they hoped would introduce errors of no more than a factor of two. They were satisfied with this level of accuracy because of the 'order of magnitude' character of their argument. The Press and Spergel calculation was equally valid when the probability of a given WIMP interacting with the body was of order one and when it was much less than one. This was an important feature for them because, to solve the solar neutrino problem, it is best to have WIMPs with much larger than weak interaction cross-sections.

Subsequently, a number of workers have realized that if WIMPs and anti-WIMPs were both present in the galactic halo, they would tend to collect in the sun (Silk *et al.* 1985; Gaisser *et al.* 1986b; Srednicki *et al.* 1987; Griest and Seckel 1987) and earth (Freese 1986; Krauss *et al.* 1986; Gaisser *et al.* 1986b) and annihilate there, possibly giving rise to a neutrino signal which could be measured by proton-decay detectors. These authors have made use of the Press and Spergel formula for capture. The typical WIMP of interest in these analyses comes from some elementary particle theory such as supersymmetry and is expected to have roughly weak interaction cross-sections. Thus, for these WIMPs, it is only necessary to consider the limit where the probability for interaction with the massive body is much less than one. In this limit, the formulae for capture can be calculated exactly. Further, there is good reason to use the exact solution. Whereas Press and Spergel were only interested in demonstrating an order of magnitude plausibility, these more recent papers have had the aim of placing definite upper limits on the abundance of various types of particles. If the capture rate is 50% higher than the Press and Spergel formula, then the limits on these abundances are 33% lower. This alone would be reason for doing the exact calculation.

As it happens, the Press and Spergel formula is remarkably accurate when applied to the sun. In most cases, the errors introduced by the phase space approximation are negligibly small. The two main errors in their calculation come from other sources. First, they dropped a factor of two between their equations (2.6) and (2.7). Second, they neglected to take account of the fact that the sun is moving with respect to the frame in which the WIMP distribution is isotropic. This motion reduces the capture rate by a factor of $\sim .75$. (There are additional smaller errors which tend to cancel one another.) Thus, in many applications, one can correct calculations done using the Press and Spergel formula simply by multiplying by 1.5. However, in other cases, notably that of heavy WIMPs, the calculation cannot be simply corrected and must be done over.

On the other hand, when the Press and Spergel formula is applied to the earth, it produces results which are too low by one or several orders of magnitude. This has led to the belief (Gaisser *et al.* 1986b) that the annihilation signal from the center of the earth would be unimportant compared to that from the sun. Press and Spergel assumed that those WIMPs whose *average* energy loss in a single collision was at least as great as their energy-at-infinity would be captured, and the remainder would not. For the sun, this cut-off occurs well into the Boltzmann tail. The escape velocity even at the surface of the sun is 618 km/sec whereas the mean WIMP velocity (taken to be equal to the mean halo velocity) is only 300 km/sec. Thus, according to this assumption, nearly all the WIMPs which interact with the sun would be captured by it. For the earth however, the escape velocity is only 11.2 km/sec at the surface and 14.8 km/sec at the center. Therefore, only WIMPs from a very small and sparsely populated region of phase space would be captured. When the WIMP mass is not closely matched to the mass of the nucleus with which it interacts, this is a reasonable approximation. In this case, not only is the *average* energy loss less than what is needed for capture, but the entire *range* of energy losses is also less. However, if the masses are closely matched, and if the collision is 'head on,' or nearly so, then the WIMP will lose nearly all its energy. The higher the WIMP velocity, the more nearly head on must be the collision for the WIMP to be captured, and hence, the less likely it is that a given collision will result in capture; but this suppression is more than compensated for by the fact that these high-energy WIMPs come from a large and densely populated region of phase space.

The net result is that for WIMPs whose mass is near that of ^{16}O, ^{24}Mg, ^{28}Si, ^{32}S, ^{40}Ca, ^{56}Fe, or ^{58}Ni the capture rate is greatly enhanced. 'Near' must be more precisely defined, but it will turn out that virtually the entire mass range from 10 to 90 GeV is near enough to one or more of these resonances to make previous calculations irrelevant.

In section 2, I derive the general formula for capture of WIMPs in a homogenous (but not necessarily isotropic) distribution, by a spherically symmetric body. I apply this general formula to the case where the distribution is Maxwell-Boltzmann (possibly as seen by a moving observer), the WIMPs have an isotropic, velocity independent cross-section, and the body is at zero temperature.

In section 3, the resulting formulae are applied to calculate the capture rate of Dirac neutrinos by the earth. In section 4, the capture rates for Dirac neutrinos and photinos by the sun are calculated.

In section 5, the calculations for the sun and the earth are combined to place limits on the halo density of heavy Dirac neutrinos. These limits are twice as good as those previously reported over most of the mass range between 15 and 70 GeV, and are about 20 times better in the neighborhood of the mass of iron (52.4 GeV). The previous limits in this neighborhood had been particularly weak relative to the expected abundance of such particles (if they exist), because these particles would have been strongly depleted in the early universe by annihilation through a virtual Z_0. Limits are also placed on scalar neutrinos.

In section 6, I discuss the angular distribution of the flux coming from WIMP-anti-WIMP annihilations in the earth's core. The angular dispersion is not negligible but it poses no significant obstacle to distinguishing between the signal and the neutrino background.

In section 7, I derive the formula for capture of WIMPs in a Maxwell-Boltzmann distribution, by a body at *finite* temperature. This formula is very considerably more complicated than the corresponding zero temperature formula and it introduces corrections of order 1% in cases of interest. For this reason, it is of little 'practical value.' However, it has a beautiful symmetry with the analytic formula for WIMP evaporation from a body at finite temperature.

In section 8, I estimate the probability that a WIMP which is newly captured into a high orbit will 'escape' before it 'settles down' into a low energy orbit in the earth's core.

In the main body of this paper, I assume that the scattering cross-section is isotropic and velocity independent. For Dirac neutrino WIMPs this assumption is valid provided that the WIMP does not 'see' the structure of the nucleus; that is, that the WIMP scatters off the nucleus coherently. After the paper was originally written, Spergel (1987) pointed out that this coherence tends to break down for heavy WIMPs.

In the appendix I derive the formulae for capture which take this lack of coherence into account. When applied to the earth, these formulae produce no qualitatively new effects and do not significantly alter the quantitative results except over a narrow mass range. Moreover, they are much more complex and difficult to interpret than the formulae based on a constant cross-section. For these reasons, it is appropriate to present the main analysis using the simpler, and fairly realistic, assumption of constant cross-section. Note however, that all figures are based on the more precise formulae contained in the appendix.

On the other hand, lack of coherence has dramatic effects on the capture of WIMPs by the sun, especially in the mass range above 20 GeV. These effects are also analyzed in the appendix.

2. Analytic Theory Of WIMP Capture

Consider the problem of the capture of WIMPs by a thin spherical *shell* of material, which is in a spherically symmetric gravitational field. The shell has radius r and thickness dr. The escape velocity at the shell is v. The WIMPs have a velocity distribution (away from the gravitational field) $f(u)du$. I will initially assume the distribution is isotropic. I define $\Omega_v^-(w)$ to be the rate *per unit time* that a WIMP with velocity w will scatter to a velocity less than v when traveling through the medium which makes up the shell.

Consider now an imaginary surface bounding a region of radius R, which is so large that the gravitational field at R is negligible. Then the flux of WIMPs going *inward* across the surface is (Press and Spergel 1985)

$$\frac{1}{4}f(u)\ u\ du\ d\cos^2\theta \quad 0 \le \theta \le \frac{\pi}{2}, \tag{2.1}$$

where θ is the angle relative to the radial direction. Changing variables to the angular momentum per unit mass,

$$J = Ru\sin\theta \tag{2.2}$$

and summing over all the area elements on the surface, one obtains the total number of WIMPs entering the region per unit time,

$$4\pi R^2 \frac{1}{4} f(u)\ u\ du \frac{dJ^2}{R^2u^2}. \tag{2.3}$$

A WIMP whose velocity at infinity is u, will have a velocity at the shell w, where

$$w = \left(u^2 + v^2\right)^{\frac{1}{2}}. \tag{2.4}$$

To be captured, it must scatter to a velocity v or less. For any given WIMP entering the imaginary shell, the probability that this will happen is simply

$$\Omega_v^-(w)\frac{dl}{w} \tag{2.5}$$

where

$$\frac{dl}{w} = \frac{1}{w}\left[1 - \left(\frac{J}{rw}\right)^2\right]^{-\frac{1}{2}} dr\ 2\ \theta(rw - J), \tag{2.6}$$

is the total time the WIMP spends in the shell material. In equation (2.6), the 2 and the θ-function appear because the WIMP intersects the shell twice or not at all, depending on whether rw is greater or less than the angular momentum. The radical appears because a WIMP with finite angular momentum will not hit the shell orthogonally.

Multiplying the conditional probability (2.5) by equation (2.3), substituting in equation (2.6), and integrating over all angular momenta gives the number of WIMPs captured per unit time per unit velocity,

$$4\pi r^2 dr \frac{f(u)du}{u} w\Omega_v^-(w). \tag{2.7}$$

Thus the total WIMP capture rate per unit shell volume is just

$$\frac{dC}{dV} = \int_0^\infty du \frac{f(u)}{u} w\Omega_v^-(w). \tag{2.8}$$

In the above equation, w is regarded as a dependent variable which is given by equation (2.4).

In deriving equation (2.8), I assumed that $f(u)$ was isotropic. Had this assumption been dropped, then the flux going across a *single* area element could not have been given in the simple form of equation (2.1). In this case, however, the *summing* over all such area elements would have, in effect, averaged over the angular distribution in all directions. This average is, of course, isotropic, so the assumption of isotropy was not really necessary.

I now specialize to the case where the shell is at zero temperature and the interaction cross section, σ, is isotropic and velocity independent. Let the shell be composed of a single type of nucleus with mass m and number density n. Let the WIMPs have mass M. Then simple kinematics tells us that the fractional WIMP energy loss in a given collision, $\Delta E/E$, will be in the interval

$$0 \leq \frac{\Delta E}{E} \leq \frac{\mu}{\mu_+^2} \tag{2.9}$$

where

$$\mu \equiv \frac{M}{m} \quad \mu_\pm \equiv \frac{\mu \pm 1}{2}. \tag{2.10}$$

Moreover, the distribution of energy loss is uniform over this interval. On the other hand, scattering from velocity w to a velocity less than v, requires an energy loss of *at least*

$$\frac{\Delta E}{E} \geq \frac{w^2 - v^2}{w^2} = \frac{u^2}{w^2}. \tag{2.11}$$

Combining expressions (2.9) and (2.11) gives the probability that a given scattering will leave the WIMP with less than escape energy,

$$\frac{\mu_+^2}{\mu} \cdot \left(\frac{\mu}{\mu_+^2} - \frac{u^2}{w^2}\right) \theta\left(\frac{\mu}{\mu_+^2} - \frac{u^2}{w^2}\right). \tag{2.12}$$

The rate of scattering from w to less than v is just the product of the total rate of scattering, $\sigma n w$, with the conditional probability (2.12). This result may be written,

$$\Omega_v^-(w) = \frac{\sigma n}{w}\left(v^2 - \frac{\mu_-^2}{\mu}u^2\right) \; \theta\left(v^2 - \frac{\mu_-^2}{\mu}u^2\right). \tag{2.13}$$

Now let the WIMPs have a Maxwell-Boltzmann distribution,

$$f_0(u)du = n_W \frac{4}{\pi^{\frac{1}{2}}} x^2 \exp(-x^2) dx, \tag{2.14}$$

or a Maxwell-Boltzmann distribution as seen by an observer moving with velocity

$\tilde{v}$,

$$f_\eta(u) = f_0(u)\exp(-\eta^2)\frac{\sinh(2x\eta)}{2x\eta} \tag{2.15}$$

where x is the dimensionless velocity,

$$x^2 \equiv \frac{M}{2kT_W}u^2, \tag{2.16}$$

η is the dimensionless observer velocity,

$$\eta^2 \equiv \frac{M}{2kT_W}\tilde{v}^2, \tag{2.17}$$

and T_W is the temperature of the WIMP distribution. Equation (2.15) may be derived by writing equation (2.14) in component form and then making a Gallilean transformation. Following Press and Spergel (1985), I define a 'velocity dispersion', $\bar{v}$, in terms of the WIMP temperature,

$$\bar{v}^2 \equiv \frac{3kT_W}{M}. \tag{2.18}$$

I will assume a velocity dispersion

$$\bar{v} = 300 \text{ km/sec}. \tag{2.19}$$

(Later I will calculate the change in the capture rate for a small change in the velocity dispersion.) Since the earth and the sun are orbiting the galactic center at $\sim$ 250km/sec, η may be evaluated

$$\eta \sim 1.0 \tag{2.20}$$

In terms of the dimensionless velocity, equation (2.13) may be written

$$w\Omega_v^-(w) = \frac{\sigma n v^2}{A^2}(A^2 - x^2)\ \theta(A - x), \tag{2.21}$$

where

$$A^2 \equiv \frac{3}{2}\frac{v^2}{\bar{v}^2}\frac{\mu}{\mu_-^2}. \tag{2.22}$$

Using equations (2.14), (2.15), and (2.21), one may evaluate equation (2.8) to find the capture rate: For an observer at rest with respect to the thermal distribution, the rate is

$$\frac{dC_0}{dV} = \left(\frac{6}{\pi}\right)^{\frac{1}{2}} \sigma n n_W \bar{v} \frac{v^2}{\bar{v}^2}\left[1 - \frac{1-\exp(-A^2)}{A^2}\right]; \tag{2.23}$$

for an observer moving with (dimensionless) velocity η, it is

$$\begin{aligned}\frac{dC_\eta}{dV} =& \left(\frac{6}{\pi}\right)^{\frac{1}{2}} \sigma n n_W \bar{v} \frac{v^2}{\bar{v}^2} \frac{1}{2\eta A^2} \\ & \left[(A_+A_- - \frac{1}{2})(\chi(-\eta,\eta) - \chi(A_-,A_+)) + \frac{1}{2}A_+ e^{-A_-^2} - \frac{1}{2}A_- e^{-A_+^2} - \eta e^{-\eta^2}\right],\end{aligned} \tag{2.24}$$

where

$$\chi(a,b) \equiv \int_a^b \exp(-y^2)dy = \frac{\pi^{\frac{1}{2}}}{2}[\mathrm{erf}(b) - \mathrm{erf}(a)] \tag{2.25}$$

and

$$A_\pm \equiv A \pm \eta. \tag{2.26}$$

If the WIMP has only a small probability of scattering during its entire trajectory through the body, one may calculate the total probability of capture by dividing the body into a series of shells and summing the probabilities of capture due to each. Then the capture rate for the entire body may be written

$$C = \int_0^{R_B} 4\pi r^2 dr \frac{dC(r)}{dV} \tag{2.27}$$

where R_B is the radius of the body. For convenience, define v_{esc} as the escape velocity at the surface and introduce the dimensionless gravitational potential

$$\hat{\phi} \equiv \frac{v^2}{{v_{\mathrm{esc}}}^2}. \tag{2.28}$$

For the earth $\hat{\phi}$ ranges from 1 to 1.8; for the sun it ranges from 1 to 5.1.

In any actual calculation, one should use equation (2.24) rather than (2.23). However, the functional form of equation (2.23) lends itself much more readily to analysis. In order to take advantage of this simplicity while maintaining formally correct expressions, I introduce the correction factor, $\xi_\eta(A)$,

$$\xi_\eta(A) \equiv \frac{dC_\eta(A)/dV}{dC_0(A)/dV}. \tag{2.29}$$

One may obtain the limiting forms of $\xi_\eta(A)$ by respectively examining equations (2.23) and (2.24), and equation (2.15). The limits are

$$\xi_\eta(\infty) = \frac{\chi(0,\eta)}{\eta} \qquad \xi_\eta(0) = e^{-\eta^2}. \tag{2.30}$$

For the case $\eta = 1$, the limits are $\sim .75$ and $\sim .37$. Thus, moving with respect to the halo *decreases* capture. This is reasonable because the average kinetic energy of the thermal WIMPs increases in this frame, making capture more difficult.

Using equations (2.23) and (2.29), equation (2.27) may now be written in the suggestive form

$$C = \left[\left(\frac{8}{3\pi}\right)^{\frac{1}{2}} \sigma n_W \bar{v}\right] \left[\frac{M_B}{m}\right] \left[\frac{3{v_{\rm esc}}^2}{2\bar{v}^2} \left\langle \hat{\phi} \right\rangle\right] \left[\xi_\eta(\infty)\right] \left\langle \frac{\hat{\phi}}{\left\langle \hat{\phi} \right\rangle} \left(1 - \frac{1 - e^{-A^2}}{A^2}\right) \frac{\xi_1(A)}{\xi_\eta(\infty)} \right\rangle, \tag{2.31}$$

where M_B is the mass of the body and Dirac brackets indicate averaging over the mass of the body. Each of the five bracketed quantities can be identified as playing a distinct role. The first is the interaction rate of a flux of WIMPs with a single nucleus in free space. The second is the number of nuclei in the body. The third is a 'focusing factor' which determines the maximum capture rate of the body. The fourth is an overall suppression factor due to the motion of the body. The fifth is a (possible) suppression factor due to mismatching of the masses. The first four quantities are easy to calculate and (apart from factors hidden in σ) independent of the WIMP mass. In the limits $A \gg (\ll) 1$, the last quantity also has a simple dependence on the WIMP mass. For $A \gg 1$ it is one. For $A \ll 1$ it is

$$\frac{\mu}{\mu_-^2}\Big[\frac{3}{4}\frac{\xi_\eta(0)}{\xi_\eta(\infty)}\frac{{v_{\rm esc}}^2}{\bar{v}^2}\frac{\left\langle\hat{\phi}^2\right\rangle}{\left\langle\hat{\phi}\right\rangle}\Big]. \tag{2.32}$$

If the velocity dispersion, $\bar{v}$, is slightly greater (less) than the value assumed in equation (2.19), the capture rate will be slightly less (greater). From equations (2.31) and (2.32) one may evaluate this effect in the limits $A \gg (\ll) 1$, assuming that the velocity of the sun in the disk does not change:

$$\frac{\Delta C}{C} = -\frac{\Delta \bar{v}}{\bar{v}}\left\{\begin{matrix} \eta e^{-\eta^2}/\chi(0,\eta) \\ 3 - 2\eta^2 \end{matrix}\right\} \sim -\frac{\Delta \bar{v}}{\bar{v}}\left\{\begin{matrix} .5 \\ 1 \end{matrix}\right\}. \tag{2.33}$$

3. Resonant WIMP Capture By The Earth

For purposes of this calculation, the earth may reasonably be divided into two zones, the mantle and the core. Each may be regarded as uniform in composition. The fraction of total earth mass which is due to mantle elements is (Stacy 1977) O(30%), Si(15%), Mg(14%), Fe(6%), Ca(1.5%), Al(1.1%), and Na(0.4%). There are two main competing models for the core. The first (Stacy 1977; Ringwood 1979) has Fe(24%), Ni(3%),and S(5%). The second (Ringwood 1979) has Fe(26%), Ni(3%), and O(3%). I will show results for both models, and use the minimum of the two when setting limits. In my numerical calculations, I evaluated $\hat{\phi}$ at each point in the earth, based on the earth model in Allen (1967). However, for purposes of qualitative discussion, it is sufficient to note that $\hat{\phi}$ ranges between 1 and 1.4 in the mantle and between 1.4 and 1.8 in the core, and to simply take the values of $\hat{\phi}$ to be 1.2 and 1.6 respectively.

With this approximation, A may be evaluated for the mantle and the core

$$A \sim \frac{\mu^{\frac{1}{2}}}{\mu_-} \left\{ \begin{matrix} 1/20 \\ 1/17 \end{matrix} \right\}. \tag{3.1}$$

From this and equation (2.32), it is clear that for most WIMP masses and for any given earth element, A will be much less than one and capture will be suppressed by a factor

$$\frac{\mu}{\mu_-^2} \left\{ \begin{matrix} 1/1600 \\ 1/1200 \end{matrix} \right\}. \tag{3.2}$$

However, in a narrow mass range

$$|\, \mu - 1 \,| < \left\{ \begin{matrix} 1/10 \\ 1/8.5 \end{matrix} \right\}, \tag{3.3}$$

there will be a 'resonance.'

It is therefore to be expected that the graph of capture as a function of WIMP mass will be dominated by a series of dramatic peaks at each of the elements which is well represented in the earth. In fact, a number of peaks tend to merge. Figure 1 shows the separate contributions of each of the most important elements in the earth to the capture of Dirac neutrino WIMPs. (The first core composition model is assumed.) The cross section for each element is assumed to be (Griest and Seckel 1987)

$$\sigma = \frac{\mu}{\mu_+^2} Q^2 \frac{mM}{(\mathrm{GeV})^2} \times 5.2 \times 10^{-40}\mathrm{cm}^2 \tag{3.4}$$

where

$$Q \equiv N - (1 - 4\sin^2\theta_w)Z \simeq N - (.124)Z \tag{3.5}$$

is slightly less than the neutron number. For each element in the earth, equation (2.31) can be written

$$C = 4.0 \times 10^{16}/\mathrm{sec}\ \bar{\rho}_{.4} \frac{\mu}{\mu_+^2} Q^2 f \left\langle \hat{\phi} \left(1 - \frac{1 - e^{-A^2}}{A^2} \right) \xi_1(A) \right\rangle, \tag{3.6}$$

where $\bar{\rho}_{.4}$ is the halo WIMP density normalized to .4 GeV/cm^3, f is the fraction of the earth's mass due to this element, and the brackets indicate averaging over the mass distribution of the element. From this formula, one may approximately calculate the capture rate by hand.

Figure 2 shows the total capture rate for both models of the earth's core. Also shown on this graph is the capture rate for the sun. The scales of the rates from the two bodies are adjusted so that equal heights on the graph represent equal annihilation fluxes at the surface of the earth.

4. Correction Factors For The Sun

WIMP capture by the sun is, of course, described by equations which are formally identical to those used above for the earth. In the case of the sun, however, A tends to be greater than 1 unless the mass of the WIMP is grossly mismatched relative to that of the nucleus:

$$A \sim 2.5\hat{\phi}^{\frac{1}{2}}\frac{\mu^{\frac{1}{2}}}{\mu_-}. \tag{4.1}$$

Thus, A will remain above 1 even at the surface of the sun and even if the masses are mismatched by a factor of 25. Using the standard solar model (Bahcall *et al.* 1982), I have calculated

$$\left\langle\hat{\phi}\right\rangle_{\mathrm{H}} = 3.16 \quad \left\langle\hat{\phi}\right\rangle_{\mathrm{He}} = 3.40 \quad \left\langle\hat{\phi}\right\rangle_{\mathrm{Tot}} = 3.23, \tag{4.2}$$

the average values of $\hat{\phi}$ for hydrogen, helium, and an element which traces the mass of the sun. Using the last, one finds that A does not drop below 1 unless the masses are mismatched by more than a factor of 80. However, since the sun is mainly hydrogen, and since one is, in principle, interested in the capture of heavy WIMPs, even these drastic mass mismatches could come into play.

For Dirac neutrinos this turns out not to be the case. The reason is that in every mass range, under the assumption of constant cross-section, capture is dominated by elements which are roughly equal in mass to the WIMP mass (Griest and Seckel 1987). As an example, consider the capture of a 50 GeV WIMP. It is true that, averaged over the sun as a whole, the suppression factor for hydrogen is about .3, but in this mass range, even unsuppressed hydrogen would account for only 8% of the capture rate. I have numerically calculated the capture rate using the standard solar model (Bahcall *et al.* 1982) for the solar structure and hydrogen and helium distribution, and Cameron (1983) for the abundances of other elements. I find (see Fig. 3) that the error made by assuming no suppression ($A = \infty$) is less than 10% over the entire mass range of 1 to 100 GeV. (When one takes account of lack of coherence, there is considerable additional suppression for high mass WIMPs.)

Figure 2 shows the capture rate by the sun for Dirac neutrinos assuming a halo WIMP density of 0.4 GeV/cm^3. For each element in the sun the capture rate of equation (2.31) can be written,

$$C = 4.1 \times 10^{25}/\text{sec}\ \bar{\rho}_{.4} \frac{\mu}{\mu_+^2} Q^2 f \left\langle \hat{\phi}\left(1 - \frac{1 - e^{-A^2}}{A^2}\right) \xi_1(A) \right\rangle, \tag{4.3}$$

where f is the fraction of the sun's mass due to this element,

$$Q_H^2 = 3g_A^2 = 3(1.25)^2, \tag{4.4}$$

and the Q of the remaining elements is as before.

If equal numbers of WIMPs and anti-WIMPs are captured, and if the halo density is sufficiently large, all the WIMPs which are captured by either the sun or the earth will be annihilated. To properly compare the expected fluxes from the two bodies, one should scale down equation (4.3) by the square of the ratio of the distances of the sources, before comparing it with equation (3.6). When this is done, the pre-factor in the solar equation is only about 1.8 times the pre-factor in the earth equation. This shows that the relative strength of the annihilation signals depends on the details of the 'resonance structure' of the earth, a conclusion which is borne out by Figure 2.

On the other hand, for WIMPs with overwhelmingly axial couplings, suppression due to mass-mismatching can play a very significant role. This is because hydrogen is virtually the only thing in the sun which couples axially. (In the earth, there is some ^{23}Na, ^{27}Al, and ^{29}Si which couple axially (Goodman and Witten 1981), but this proves insignificant). Figure 3 shows the 'correction factor' to the naive, unsuppressed calculation for photinos and Dirac neutrinos as a function of mass. For Dirac neutrinos, both the constant cross-section and the coherence-suppressed capture rates are shown. To correct previous calculations, such as those by Gaisser *et al.* (1986b), one must fold in the factor of 1.5 mentioned above. Note that in the range of 50 to 100 GeV, the combination of these two correction factors is of order 1/2 to 1/3 for photinos. Given the already extremely low expected event rates reported by Gaisser *et al.*, this suppression means that it will be very difficult to either detect or place limits on heavy photinos by using the sun.

5. Combined Earth-Sun Limits On Dirac Neutrinos

According to Figure 2, the earth and sun are roughly comparable sources of Dirac neutrino annihilation fluxes. However, before this comparison can be made precise, three differences in these sources must be taken into account. First, the flux from the sun is virtually a point source whereas that from the earth has a finite angular distribution. Assuming one had detectors of sufficiently good angular resolution, this means that one could more easily distinguish the solar flux from background. In the next section I will show that this is not a significant factor.

Second, the signal from the earth will be 'filtered' (by the earth itself) 100% of the time whereas the signal from the sun will only be filtered about 50% of the time. This is important if one is counting events which happen in the rock outside detectors. Events which occur above the detector must be thrown out because they are mostly due to cosmic rays. This means that in experiments of this type and for equal fluxes from the sun and the earth, statistics are twice as good for the earth. In this section, however, I will be considering event rates occurring inside the detector.

Third, the rate at which Dirac neutrinos are annihilated is not necessarily equal to the rate at which they are captured, even if the halo is composed of equal numbers of particles and anti-particles. If the rates are equal, I will refer to this as 'full signal.' As I will show below, for any given WIMP mass, there is a definite halo density above which both the earth and sun will be at full signal. Below this level, the sun will still be at full signal, but the earth signal will be suppressed relative to this by a factor proportional to the halo density. In this halo density range, the earth signal would quickly dwindle relative to that of the sun. However, over much of the WIMP mass range of interest, the halo density at which this effect sets in is far below what could detected with today's detectors using either the earth or the sun. The stage will then be set to place specific limits on Dirac neutrinos in the halo by using the earth and sun signals in combination.

I will assume that the WIMPs which are captured collect near the center of the earth in an isothermal distribution at some temperature T, which is somewhat less than the central temperature of the earth. This latter quantity is a matter of some dispute among geophysicists (Sleep 1987), with estimates ranging from 4.5 to 5.5 $\times 10^3$ $^\circ k$. I will express my results in terms of T_5, the temperature normalized to the midpoint of this range. The density of the earth will be assumed to be $\rho = 12\ \mathrm{gm/cm}^2$ in the region of interest. Under these assumptions one may use formulae I derived previously (Gould 1987) to show that evaporation is not a significant factor for WIMPs above 12 GeV. Thus, in this mass range, the number density of WIMPs in the earth is entirely determined by competition between capture and annihilation. Griest and Seckel (1987) have shown that under these conditions, the rate at which Dirac neutrino WIMPs are annihilated is

$$C \tanh^2[(aC)^{\frac{1}{2}}\tau], \tag{5.1}$$

where

$$a = 32^{-\frac{1}{2}} \frac{\langle \sigma v \rangle}{V_1} \tag{5.2}$$

and $\tau = 4.6$ billion years is the lifetime of the earth. In equation (5.2), V_1 is the effective volume of the earth,

$$V_1 = \left(\frac{3kT}{2GM\rho} \right)^{\frac{3}{2}} = 6.9 \times 10^{24} \mathrm{cm}^3 T_5^{\frac{3}{2}} M_{20}^{-\frac{3}{2}} \tag{5.3}$$

and $\langle \sigma v \rangle$ is the non-relativistic annihilation cross section times velocity. This latter quantity may be evaluated (Griest and Seckel 1987), assuming 7.4 annihilation channels,

$$\langle \sigma v \rangle = 7.4 \times 10^{-25} \mathrm{cm}^3/\mathrm{sec}\ P_a M_{20}^2 \tag{5.4}$$

where M_{20} is the mass of the WIMP normalized to 20 GeV and

$$P_a = \frac{m_Z^4}{(4M^2 - m_Z^2)^2 + \Gamma_Z^2 m_Z^2} \tag{5.5}$$

accounts for the Z_0 propagator and resonance. I take the mass of the Z_0, m_Z, to be 93 Gev and its width, Γ_Z, to be 2.5 GeV. Combining these formulae one finds that the argument of the hyperbolic tangent is given by

$$(Ca)^{\frac{1}{2}} \tau = 20 P_a^{\frac{1}{2}} (C_{18} \bar{\rho}_{.4})^{\frac{1}{2}} M_{20}^{\frac{7}{4}} T_5^{-\frac{3}{4}} \tag{5.6}$$

where $\bar{\rho}_{.4}$ is the halo WIMP density normalized to $0.4 \mathrm{GeV/cm}^3$ and C_{18} is the capture rate in units of 10^{18}/sec when the density is at this normalized value. From this expression, one can see that the earth will be at full signal down to very low WIMP densities. For definiteness I will take full signal to mean that this argument is at least 2.5, since $\tanh^2 2.5 \sim .97$. (A similar analysis (Griest and Seckel 1987) shows that sun is at full signal down to much lower WIMP densities.)

I now turn to the problem of detection. I (arbitrarily) assume that event rates of at least 1 per kiloton-year are detectable, and lower event rates are not. If experimenters believe that their detectors are better or worse than this value, they have only to proportionately adjust the limits which I derive below. I assume (Gaisser *et al.* 1986b) that a fraction 0.21 of the annihilations will go directly to massless neutrinos and that the neutrino-nucleon cross-section is

$$\sigma_\nu + \sigma_{\bar{\nu}} = 1 \times 10^{-38} \frac{E_\nu}{\mathrm{GeV}} \mathrm{cm}^2. \tag{5.7}$$

(I have ignored all neutrino production arising from the decay of other annihilation products like b and c quarks. In terms of absolute numbers this production is of the same order as that which I am including. In terms of signal, however, it is much less significant for two reasons. First, these neutrinos are of much lower energy and so, by equation (5.7), are much less likely to give rise to a signal. Second, unlike the directly produced neutrinos, they are not monochromatic, and hence are more difficult to distinguish from the background). Keeping in mind that there are 1.9×10^{40} nucleon-sec in a kiloton-year and that it takes 2 WIMPs to annihilate, the event rate per kiloton-year at full signal is given by

$$\frac{C}{4\pi R_\oplus^2} \mathrm{cm}^2 \mathrm{sec} \frac{.21}{2} 10^{-38} \frac{M}{\mathrm{Gev}} 1.9 \times 10^{40} \sim 78 C_{18} \bar{\rho}_{.4} M_{20}. \tag{5.8}$$

Thus, under these assumptions, the earth can be used to put very good limits on the density of Dirac neutrinos in the halo.

Figure 4 shows the halo WIMP density which will produce a signal of 1 event per kiloton-year from the sun alone and from the combined earth-sun sources. To combine these sources, I used the square root of the sum of the squares of the fluxes due to each. (This method of combining signals is conservative. With sufficient angular and/or energy resolution, the background can be made negligibly small, in which case the simple sum of the two signals is more appropriate. See section 6.) Also shown on the graph is the WIMP density at which the earth ceases to be at full signal. Better detection means better limits on WIMP density until this line is reached. For halo WIMP densities well below this line, the earth signal is suppressed and one must use only the sun. If, for example, detectors were capable of seeing 0.1 events per kiloton-year, then the sun (dashes) and the earth-sun (solid) limit curves in Figure 4 could both be moved down by a factor of 10 (1 unit). However, wherever the earth-sun curve fell below the full-signal limit curve (dots), one would have to take account of the fact that the earth was not at full signal before setting limits on the halo WIMP density. In principle, at sufficiently low halo WIMP densities, use of the earth signal would also be compromised because it could not be distinguished from background. This is discussed in the next section. It turns out that this restriction comes into play only in a narrow mass range near 45 GeV. It is shown in dot-dash.

Over most of the mass range, these limits are about twice as good as those previously believed (Gaisser *et al.* 1986b). However, in the neighborhood of the iron resonance, centered at 52.4 GeV, the limits are 20 times better than before. This range is particularly important because the abundance of Dirac neutrino WIMPs of this mass range, if they existed, would be highly suppressed. They would have been depleted by annihilation in the early universe due to their proximity to the Z_0 resonance. The number of WIMPs that one 'expects' to be in the halo is proportional to the number which were created in and then survived the big bang. The proportionality constant depends on many model-dependent factors. In order to abstract from these factors as far as possible, I examine the *relative* detectability of WIMPs of various masses. This I take to be the product of the detection rate per unit halo WIMP density (eqn. (5.8)) with the inverse annihilation rate (eqn. (5.4)). In figure 5, I have plotted this quantity normalized to its value at a WIMP mass of 34 GeV. Detectabilities based on the sun alone, and on the earth together with the sun are shown. The role of the iron resonance in overcoming some of the effects of the Z_0 resonance is evident.

What has been said about Dirac neutrinos can easily be extended to scalar neutrinos (particles which are predicted by supersymmetry). The elastic cross-section of scalar neutrinos with nuclei (except hydrogen) is simply four times the cross-section for Dirac neutrinos (Gaisser *et al.* 1986b). They would probably annihilate almost 100% of the time into ordinary massless neutrinos. Thus, their signal would be $4/.21 \sim 20$ times greater than that of Dirac neutrinos, and so the limits on their halo abundance are 20 times lower. The annihilation cross-section for scalar neutrinos is highly model dependent. Typically it is larger than for Dirac neutrinos. However, before the earth is used to place limits on a specific model, it must be checked that the annihilation cross-section in this model is sufficient for the earth to be at full signal.

6. Angular Distribution of Annihilation Signal

As in the last section, I assume that the WIMPs collect in an isothermal distribution in the center of the earth and that the earth's density is uniform in the region of interest. In contrast to the previous section, however, no assumption need be made about the form of the annihilation cross-section, nor about the relative density of WIMPs and anti-WIMPs.

The number densities of WIMPs and anti-WIMPs are separately proportional to

$$\exp\left(\frac{-M\Phi(r)}{kT}\right) = \exp\left(\frac{-r^2}{2r_0^2}\right), \tag{6.1}$$

where

$$r_0^2 \equiv \frac{3kT}{4\pi G M \rho}. \tag{6.2}$$

The annihilation rate is proportional to the products of the number densities of the WIMPs and anti-WIMPs, that is, to the square of expression (6.1). Consider a cone whose apex is at the surface of the earth and whose axis goes through the earth's center. Let the cone have half-angle θ and angular thickness $d\theta$. Parameterize position along the cone by l, the distance to the apex. The distance from the center of the earth to the cone at its closest point is then

$$r_{\min}(\theta) = R_\oplus \sin\theta. \tag{6.3}$$

Let the distance from the apex to this closest point be $l_0(\theta)$. The volume element is

$$dV = 2\pi l^2 \; dl \; d\cos\theta. \tag{6.4}$$

Thus, the flux at the apex originating from this volume element is proportional to

$$\frac{\cos\theta}{l^2} \exp\left(-\left[\frac{r_{\min}^2(\theta) + (l - l_0(\theta))^2}{r_0^2}\right]\right) 2\pi l^2 \; dl \; d\cos\theta. \tag{6.5}$$

The integral of this expression over all l is proportional to

$$\exp\big(-\frac{r_{\min}^2(\theta)}{r_0^2}\big) d\cos^2\theta. \tag{6.6}$$

Finally, the definite integral of this expression from 0 to θ is proportional to

$$1 - \exp\big(-\frac{R_\oplus^2}{r_0^2}\sin^2\theta\big). \tag{6.7}$$

Since

$$\frac{r_0}{R_\oplus} = .12 T_5^{\frac{1}{2}} M_{20}^{-\frac{1}{2}}, \tag{6.8}$$

98% of the signal will originate within a 14° cone for 20 GeV WIMPs.

If the halo WIMP density were sufficiently low, then the signal from this 98% cone would be of the same order as the background. In this case, further improvement in the angular resolution of the detector could isolate the point-like solar signal from the background, but would yield no enhancement for the earth signal. Then it would be inappropriate to combine the earth and sun signals.

However, the background signal (from neutrinos produced by cosmic rays in the atmosphere) is (Gaisser *et al.* 1986a)

$$.033 \frac{M_{20}}{(M_{20} + .015)^3} \left[1 + \frac{.45}{1 + M_{20}} \right] \tag{6.9}$$

per kiloton-year per steradian, assuming an energy resolution of 1 GeV. This means that the background signal in the 98% cone is

$$.006 T_5 (M_{20} + .015)^{-3} \left[1 + \frac{.45}{1 + M_{20}} \right] \tag{6.10}$$

per kiloton-year.

Over most of the mass range of interest, this background signal would compete with the earth signal only when the earth signal was already well below full signal, and thus was of no use anyway; but in a narrow range near the Z_0 resonance, the background could become a problem before the earth fell below full signal. This is shown in Figure 4.

I must emphasize that the background limits discussed in this section are the *theoretical* limits assuming that detector angular resolution can be narrowed to the intrinsic angular width of the earth signal. The actual problem with background in any given experiment will, of course, depend on the actual angular resolution.

7. Capture By Bodies At Finite Temperatures

In this section I will relax the assumption, made half-way through section 2, that the capturing shell is at zero temperature. It will now be assumed to be at temperature T. The WIMPs will still be assumed to be at temperature T_W. I will consider only the case when the shell is not moving with respect to the thermal distribution. Equation (2.8) is still valid, but now $\Omega_v^-(w)$ must be re-evaluated,

$$\Omega_v^-(w) = \int_0^v dv' R(w \to v'), \tag{7.1}$$

where $R(w \to v')$ is the rate per unit time that a WIMP with velocity w scatters to velocity v'. I have previously evaluated this quantity (Gould 1987) and found

$$R(w \to v)dv = \frac{2}{\pi^{\frac{1}{2}}} \frac{\mu_+^2}{\mu} \sigma n \frac{v dv}{w} [\chi(\pm\beta_-, \beta_+) e^{-\frac{M}{2kT}(v^2 - w^2)} + \chi(\pm\alpha_-, \alpha_+)], \tag{7.2}$$

where

$$\alpha_\pm \equiv (m/2kT)^{\frac{1}{2}} (\mu_+ v \pm \mu_- w), \tag{7.3}$$

$$\beta_\pm \equiv (m/2kT)^{\frac{1}{2}} (\mu_- v \pm \mu_+ w), \tag{7.4}$$

and the upper (lower) sign in equation (7.2) refers to the case when $w < (>) v$.

Equation (7.1) may now be evaluated,

$$\Omega_v^\pm(w) = \pm \frac{1}{2\pi^{\frac{1}{2}}} \frac{2kT}{m} \frac{1}{\mu^2} \frac{\sigma n}{w} [\mu(\pm\alpha_+ e^{-\alpha_-^2} - \alpha_- e^{-\alpha_+^2}) + (\mu - 2\mu\alpha_+\alpha_- - 2\mu_+\mu_-)\chi(\pm\alpha_-, \alpha_+) + 2\mu_+^2 \chi(\pm\beta_-, \beta_+) e^{-\frac{M}{2kT}(v^2 - w^2)}], \tag{7.5}$$

Here Ω^+ is the rate per unit time that a WIMP with velocity w scatters to a velocity *greater than* v. It was derived in the aforementioned paper. The steps and identities necessary to evaluate Ω^- are identical to those used for Ω^+. Using equation (7.5), one may now integrate equation (2.8), giving

$$\left\{ \begin{array}{c} dE/dV \\ dC/dV \end{array} \right\} = \left\{ \begin{array}{c} \exp(-Mv^2/2kT_W) \\ 1 \end{array} \right\} \frac{2}{\pi}(\frac{2kT}{M})^{\frac{1}{2}}(\frac{T}{T_W})^{\frac{3}{2}}\sigma n n_W$$
$$\Big\{ \exp\big[-(\frac{\mu-\nu}{\xi^2}\frac{Mv^2}{2kT_W})\big]\big[\frac{\mu\mu_-}{\nu\xi}(\frac{\xi^2}{\nu} - \frac{\mu_+\mu_-}{\mu}) + \frac{\mu_+^3}{\xi(\nu-\mu)}\big]\chi(\pm\gamma_-,\gamma_+)$$
$$+ \frac{\mu}{\nu}[\alpha_+\alpha_- - \frac{1}{2\mu} + \mu_-^2(\frac{1}{\mu} - \frac{1}{\nu})]\chi(\pm\alpha_-,\alpha_+)$$
$$- \frac{\mu_+^2}{\nu-\mu}\chi(\pm\beta_-,\beta_+) \mp e^{-\alpha_-^2}\frac{\mu}{2\nu}\alpha_+ + e^{-\alpha_+^2}\frac{\mu}{2\nu}\alpha_- \Big\}. \tag{7.6}$$

In these equations,

$$\gamma_\pm \equiv (m/2kT)^{\frac{1}{2}}(\rho v \pm \xi w), \tag{7.7}$$

$$\xi^2 \equiv \mu_-^2 + \nu \qquad \rho \equiv \frac{\mu_+\mu_-}{\xi}, \tag{7.8}$$

$$\nu \equiv \frac{T}{T_W}\mu, \tag{7.9}$$

and all quantities are to be evaluated at $w = v =$ the escape velocity. Here dE/dV is the evaporation rate for WIMPs in a thermal distribution at temperature T_W which is cut-off at the escape energy, when they are driven by a truly thermal distribution of nuclei at temperature T. This was also derived in the aforementioned paper and again, all the steps and identities which are necessary to evaluate the capture integral are the same as those used in doing the evaporation integral. The reader will note that, apart from an overall Boltzmann factor and some sign changes, the equations for capture and evaporation are formally identical.

For the earth, most of the capture occurs when $\mu \sim 1$. In all cases of interest, the ratio of the earth temperature to the WIMP escape energy, $\epsilon = 2kT/Mv^2$, is much less than one. In this limit, equation (7.6) becomes

$$\frac{dC}{dV} = \left(\frac{6}{\pi}\right)^{\frac{1}{2}} \sigma n n_W \bar{v} \frac{v^2}{\bar{v}^2} \left[1 - \frac{\epsilon}{2}\right]. \tag{7.10}$$

This is a correction of $\epsilon/2$ relative to the zero temperature formula, equation (2.23). For WIMPs of order 25 GeV the escape energy and earth temperature should be evaluated in the region where there are elements of this mass range, namely in the mantle. For WIMPs of order 50 GeV, they should be evaluated in the core. In either region, ϵ is approximately 1%. Thus the finite temperature correction to the previous calculations is a reduction in capture rate of order 0.5%.

8. Securing Capture

In the above analysis, a WIMP was considered 'captured' when its energy was reduced below escape energy. It was implicitly assumed that once the WIMP was in a bound orbit, it would follow a trajectory which would take it through the earth until it again collided with a nucleus, lost more energy, and so on. The process would continue until the WIMP reached thermal energies at the center of the earth. But mightn't something happen to the WIMPs, especially those in high-energy orbits, that would prevent them from being 'securely captured'?

This question was posed by Krauss *et al.* (1986). They suggested that since it was very probable that the WIMP's first bound orbit would be a high-energy one, and since most of the evaporation of the 'securely captured' thermal WIMPs came from the high-energy part of their distribution, that it was possible that a significant fraction of the newly captured WIMPs would evaporate before they could 'settle down.' I will show below that this is an effect of order ϵ^2. However, there is another source of capture disruption, namely the moon, which has order ϵ effects.

As in the previous section, I restrict my attention to the most important (and simplest) case, $\mu = 1$. In this case, it is clear from equation (7.5), that the probability of evaporation is exponentially suppressed for WIMPs whose energy is more than kT from escape energy. But from the analysis of section 3, we know that of the WIMPS which are captured, only a fraction $\epsilon = 2kT/Mv^2$ initially scatter to energies above this threshold. Again using equation (7.5), one now finds that the probability that a WIMP in this energy range will scatter to a higher (as opposed to lower) energy is also ϵ. Thus the probability that a given newly captured WIMP will evaporate before it settles down is bounded above by ϵ^2.

The moon poses a much more serious threat to securing capture, and one which is much more difficult to analyze. The radius of the moon's orbit is about 60 times the radius of the earth. Thus, about 1% of the WIMPs which are captured by the core, and about 1.4% of those captured by the mantle will have essentially radial orbits taking them out to the 'lunar sphere.' These orbits will have a period on the order of a month. During each such period, the WIMP will travel through the earth. The 'optical depth' of the earth will vary widely depending on the mass and orbit of the WIMP, but may be generally estimated to be of order 10^{-2}. After roughly a hundred such orbits, the WIMP will scatter, losing about half its energy, and thus more effectively securing its capture. However, during these hundred orbits it is quite possible that the moon will disrupt the WIMP's orbit enough that it no longer intersects the earth. In this case, its further trajectory will be entirely governed by celestial mechanics until such time as it either escapes or is redirected into an earth-intersecting orbit.

For WIMPs initially in orbits which go out to the lunar sphere, or even half or a third that far, it seems quite likely that their orbits will be initially so disrupted. It appears to be a fairly difficult problem in statistical celestial mechanics to determine the eventual fate of these WIMPs. However, one may estimate that some, perhaps a few per cent, of those initially captured, will either escape or go into orbits which have an extremely low probability of again intersecting the earth. This means that the above calculations should be modified by a small, but difficult to calculate correction.

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APPENDIX Lack of Coherence

In the main body of this paper, I assumed that the Dirac neutrino cross-section is velocity independent and isotropic. This assumption is strictly valid only if the WIMP does not 'see' the structure of the nucleus; that is, if the momentum transfer, q, is small compared to the inverse of (the root means square of) the nuclear radius, R:

$$qR \ll \hbar. \tag{A1}$$

Spergel (1987) has recently pointed out that this condition is not necessarily satisfied for heavy WIMPs hitting the earth with typical halo velocities.

In this appendix, I will assume that the cross-section is *anisotropic* and velocity *dependent*. I will take this dependence to be described by a 'form-factor' suppression

$$\mid F(q^2) \mid^2 = \exp\left(-\frac{q^2 R^2}{3\hbar^2}\right). \tag{A2}$$

In terms of the WIMP's energy loss, ΔE, this may be expressed

$$\mid F(q^2) \mid^2 = \exp(-\Delta E / E_0) \tag{A3}$$

where

$$E_0 \equiv \frac{3\hbar^2}{2mR^2} \tag{A4}$$

is the characteristic coherence energy. By generalizing the argument given in section 2, one may determine the scattering function, $\Omega_v^-(w)$:

$$\Omega_v^-(w) = (\sigma n w) \int_{\frac{u^2}{w^2}}^{\frac{\mu}{\mu_+^2}} \frac{\mu_+^2}{\mu} \exp(-\Delta E/E_0) d\left(\frac{\Delta E}{Mw^2/2}\right) \tag{A5}$$

where σ is now the low-energy total cross-section. This may be evaluated;

$$w\Omega_v^-(w) = \frac{\sigma n}{b}\left[\exp\left(-ax^2\right) - \exp\left(-bx^2\right)\exp\left(-A^2(a-b)\right)\right] \tag{A6}$$

where

$$a = \frac{M\bar{v}^2}{3E_0} \quad b = \frac{\mu}{\mu_+^2} a. \tag{A7}$$

The parameter a is the characteristic halo WIMP energy in units of the coherence energy. Using an estimate (Eder 1968)

$$R \sim \left[.91\left(\frac{m}{\mathrm{GeV}}\right)^{\frac{1}{3}} + .3\right] \times 10^{-13}\mathrm{cm} \tag{A8}$$

for the nuclear radius, a may be evaluated:

$$a \sim .014 \times \mu m_{20}^2\left(m_{20}^{\frac{1}{3}} + .12\right)^2. \tag{A9}$$

At the resonances of oxygen, silicon, and iron, the values of a are respectively .01, .04, and .22.

Using equations (2.15) and (A6), equation (2.4) may be evaluated

$$\begin{aligned}\frac{dC_\eta}{dV} = \left(\frac{8\pi}{3}\right)^{\frac{1}{2}} \frac{\sigma n n_W \bar{v}}{b} \times \frac{1}{2\eta}\Big\{&\left[\chi(-\hat{\eta},\hat{\eta}) - \chi(\hat{A}_-,\hat{A}_+)\right]\frac{\exp(-a\hat{\eta}^2)}{(1+a)^{\frac{1}{2}}} \\ &-\left[\chi(-\check{\eta},\check{\eta}) - \chi(\check{A}_-,\check{A}_+)\right]\frac{\exp(-b\check{\eta}^2)}{(1+b)^{\frac{1}{2}}}\exp\left[-(a-b)A^2\right]\Big\},\end{aligned} \tag{A10}$$

where

$$\hat{\eta} \equiv \frac{\eta}{(1+a)^{\frac{1}{2}}} \quad \check{\eta} \equiv \frac{\eta}{(1+b)^{\frac{1}{2}}} \tag{A11}$$

$$\hat{A} \equiv A(1+a)^{\frac{1}{2}} \quad \check{A} \equiv A(1+b)^{\frac{1}{2}} \tag{A12}$$

$$\hat{A}_{\pm} \equiv \hat{A} \pm \hat{\eta} \quad \check{A}_{\pm} \equiv \check{A} \pm \check{\eta}. \tag{A13}$$

While equation (A10) is itself fairly inscrutable, it is easy to compare its impact on WIMP capture by the earth relative to its simpler counterpart, equation (2.24). First note that if either of the conditions

$$1)\ b \ll 1 \quad \text{or} \quad 2)\ aA^2 \ll 1 \tag{A14}$$

are satisfied, then the two equations are essentially identical. The first condition says that the kinetic energy loss of virtually any WIMP hitting the earth is small compared to the coherence energy. The second condition says that for those WIMPs which may possibly be captured, the highest possible energy loss is small compared to the coherence energy. (Note that these interpretations make use of the fact that the kinetic energy which the WIMP acquires by falling into the earth's gravitational field is small compared to the coherence energy.) From the second condition one may immediately conclude that equation (A10) introduces no effect except possibly at resonances. Away from resonances, where by definition $A \ll 1$, the second condition is always satisfied. Moreover, for the oxygen and silicon resonances, the effect is very small by virtue of the first condition. Thus the principal effect of replacing equation (2.24) by equation (A10) is to somewhat suppress the iron resonance. One may explicitly evaluate the ratio of the two formulae at a resonance:

$$\frac{\exp(-a\hat{\eta}^2)}{(1+a)^{\frac{1}{2}}} \frac{\chi(0,\hat{\eta})}{\chi(0,\eta)} \sim (1+a)^{-2}. \tag{A15}$$

For oxygen, silicon, and iron this is respectively .99, .94, and .72.

The effect on the sun is more significant. Because the solar gravitational potential energy is much greater than the mean kinetic energy of WIMPs in the halo, the kinetic energy of the WIMP at impact is more or less independent of its energy-at-infinity. The relevant dimensionless parameter is thus

$$b\frac{3{v_{\rm esc}}^2}{2\bar{v}^2}\hat{\phi} \sim .3\frac{\mu^2}{\mu_+^2}m_{20}^2\left(m_{20}^{\frac{1}{3}}+.12\right)^2\frac{\hat{\phi}}{3.23} \tag{A16}$$

instead of b. This parameter represents the maximum possible energy loss in units of the coherence energy. Note that when the WIMP mass is much greater than the nuclei mass, expression (A16) has the limiting form

$$\sim 1.2m_{20}^2\left(m_{20}^{\frac{1}{3}}+.12\right)^2\frac{\hat{\phi}}{3.23}. \tag{A17}$$

Thus, lack of coherence will never have an effect on capture by helium no matter how heavy the WIMPs are. Similarly, there will only be a modest effect on capture by oxygen. However, there will be considerable suppression of capture by silicon and iron, even for WIMPs as light as 10 GeV. Indeed, explicit numerical calculation shows that, whereas iron had been thought to dominate capture of high mass WIMPs, in fact, it places third behind oxygen and helium. I mentioned in section 4 that, under the assumption of constant cross-section, capture is dominated by nuclei whose mass is of the same order as that of the WIMP. It is not surprising then, that when account is taken of lack of coherence, solar WIMP capture tends to be highly suppressed above the mass of oxygen. This is shown in Figure 3.

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FIGURE CAPTIONS

1. RESONANCES: Log_{10} of capture rate in inverse seconds of Dirac neutrino WIMPs of various masses (in Atomic Mass Units) for various elements in the earth. Shown are ^{16}O, ^{28}Si, ^{56}Fe-^{58}Ni (all solid), ^{24}Mg (dots), and ^{32}S (dashes). The envelope is total for all elements including those not shown. The first core model has been assumed. The assumed halo WIMP density is 0.4 GeV/cm^3.

2. EARTH VS SUN: Total capture rate in inverse seconds of Dirac neutrino WIMPs of various masses (in Atomic Mass Units), assuming a halo WIMP mass density of 0.4 GeV/cm^3. Rates are shown for first (solid) and second (dots) core models, and for the sun (dashes). The solar rate is scaled down by a factor of 5.5×10^8. Note change in scale at 46 A.M.U.

3. SUPPRESSION: True capture rates by the sun as a fraction of what the rate would be if mass-mismatching could be ignored. Ratios for photinos (solid) and Dirac neutrinos (dots and dashes) of various masses (in GeV) are shown. The lower Dirac neutrino curve takes account of lack of coherence (see appendix).

4. LIMITS: Maximum halo WIMP density (in GeV/cm^3) which would be undetectable at detection rates of 1 per kiloton-year. Limits are shown based on the sun alone (dashes) and the earth in combination with the sun (solid) for Dirac neutrino WIMPs of various masses (in GeV). Also shown is the minimum WIMP density at which the earth will be at 'full signal' (dots) and the density at which the earth signal would drop below background (dot dash).

5. RELATIVE DETECTABILITY (defined in the text) for Dirac neutrino WIMPs of various masses (in GeV). The logarithmic scale is arbitrarily normalized to the value at 34 GeV.

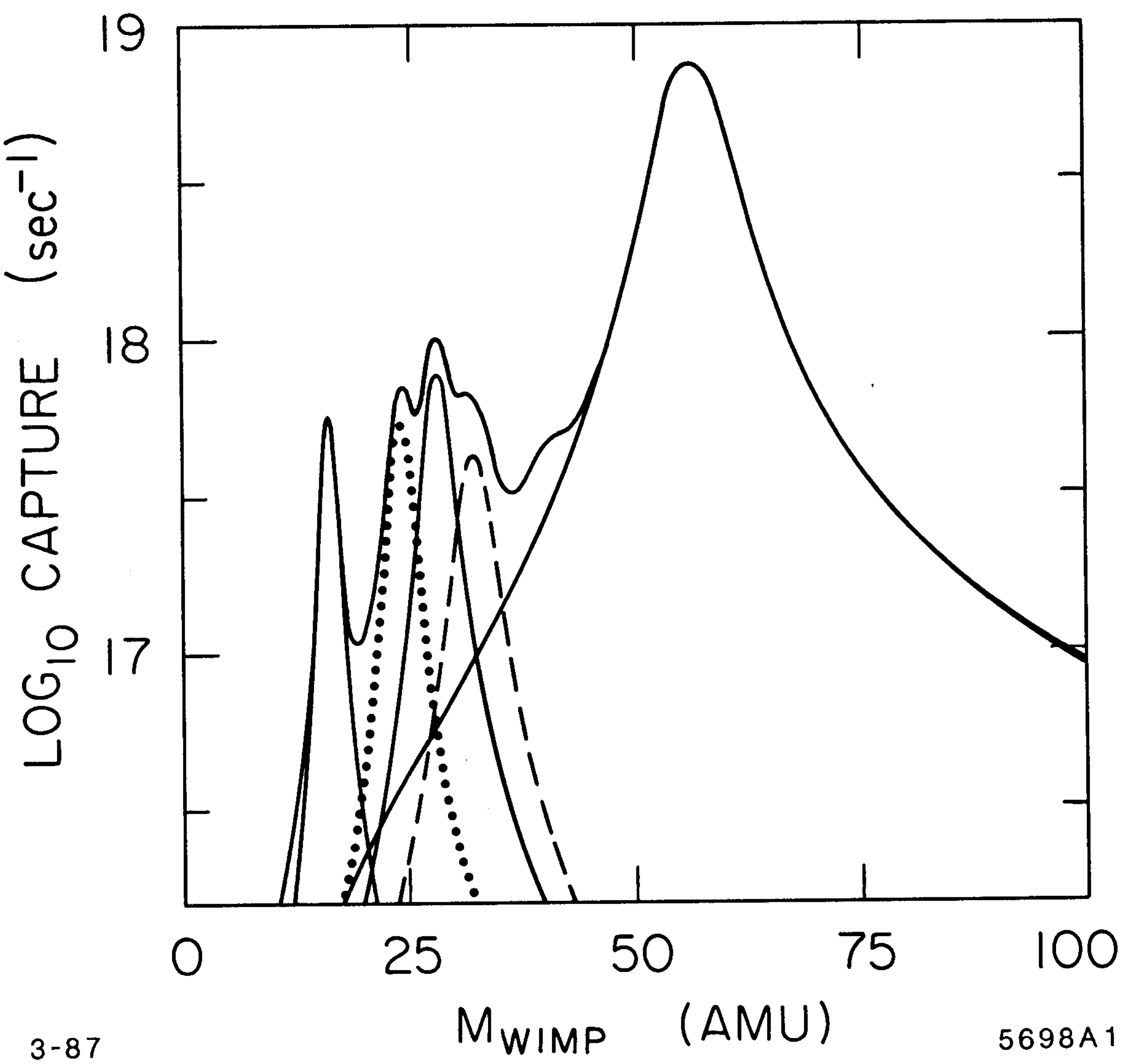


Fig. 1

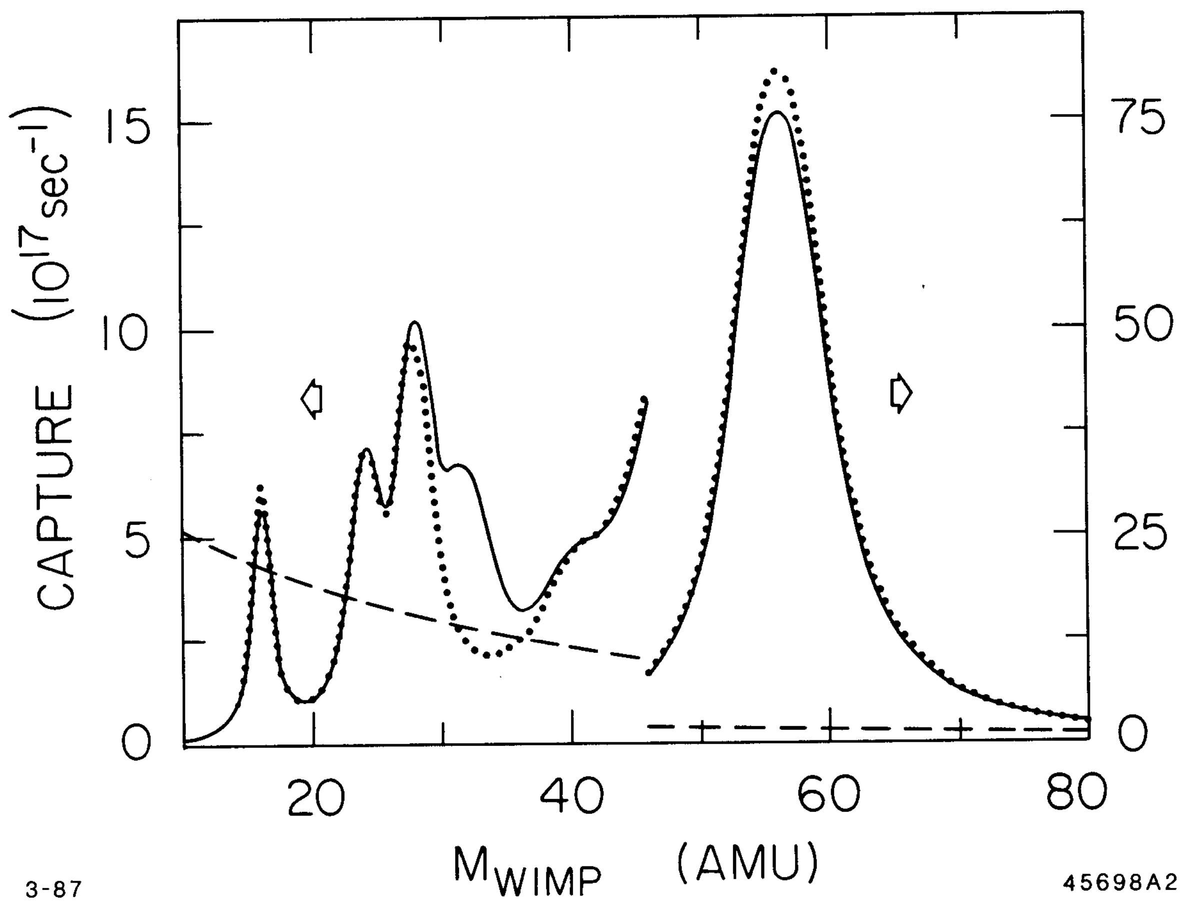

CAPTURE (10^{17}sec^{-1})
15
10
5
0
75
50
25
0
20
40
60
80
M_{WIMP} (AMU)
3-87
45698A2

Fig. 2

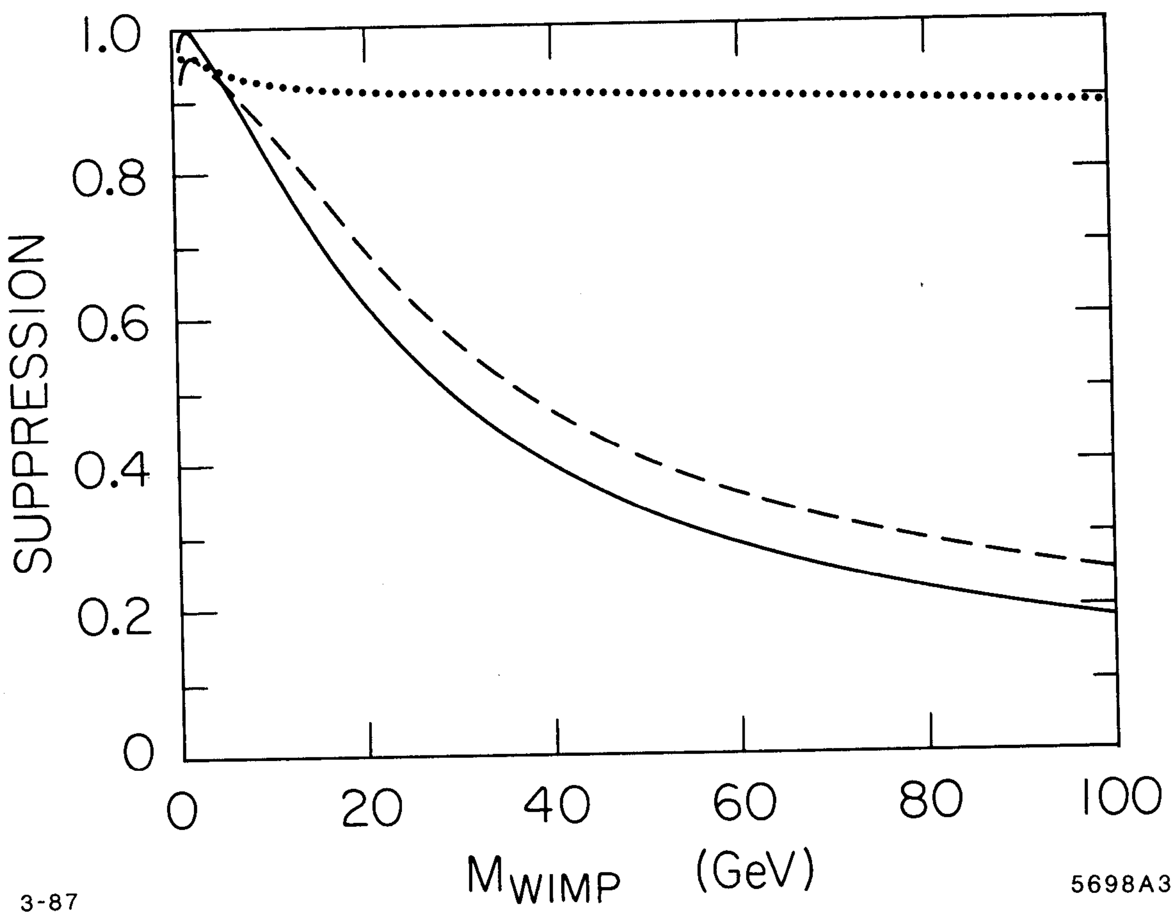
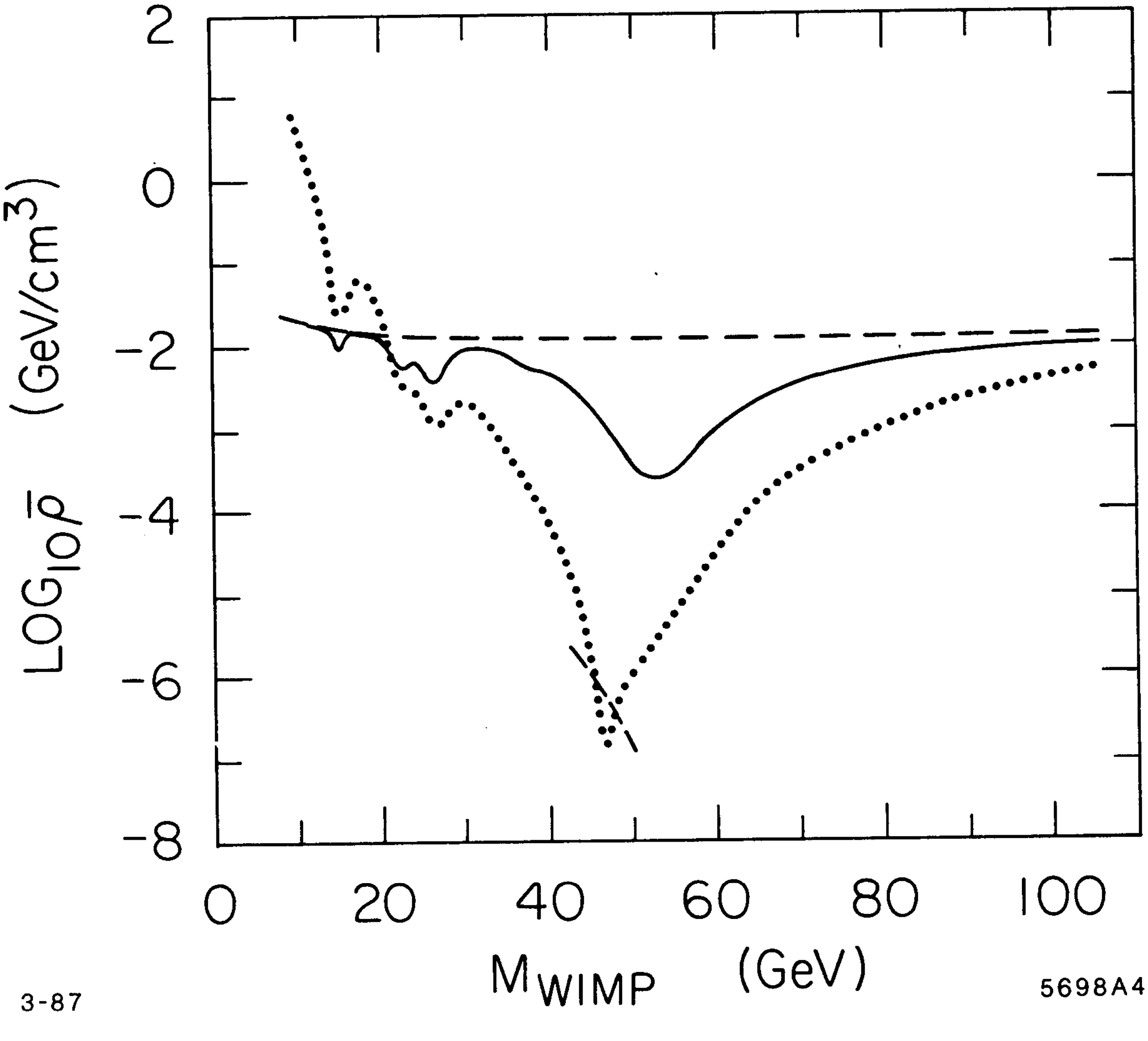
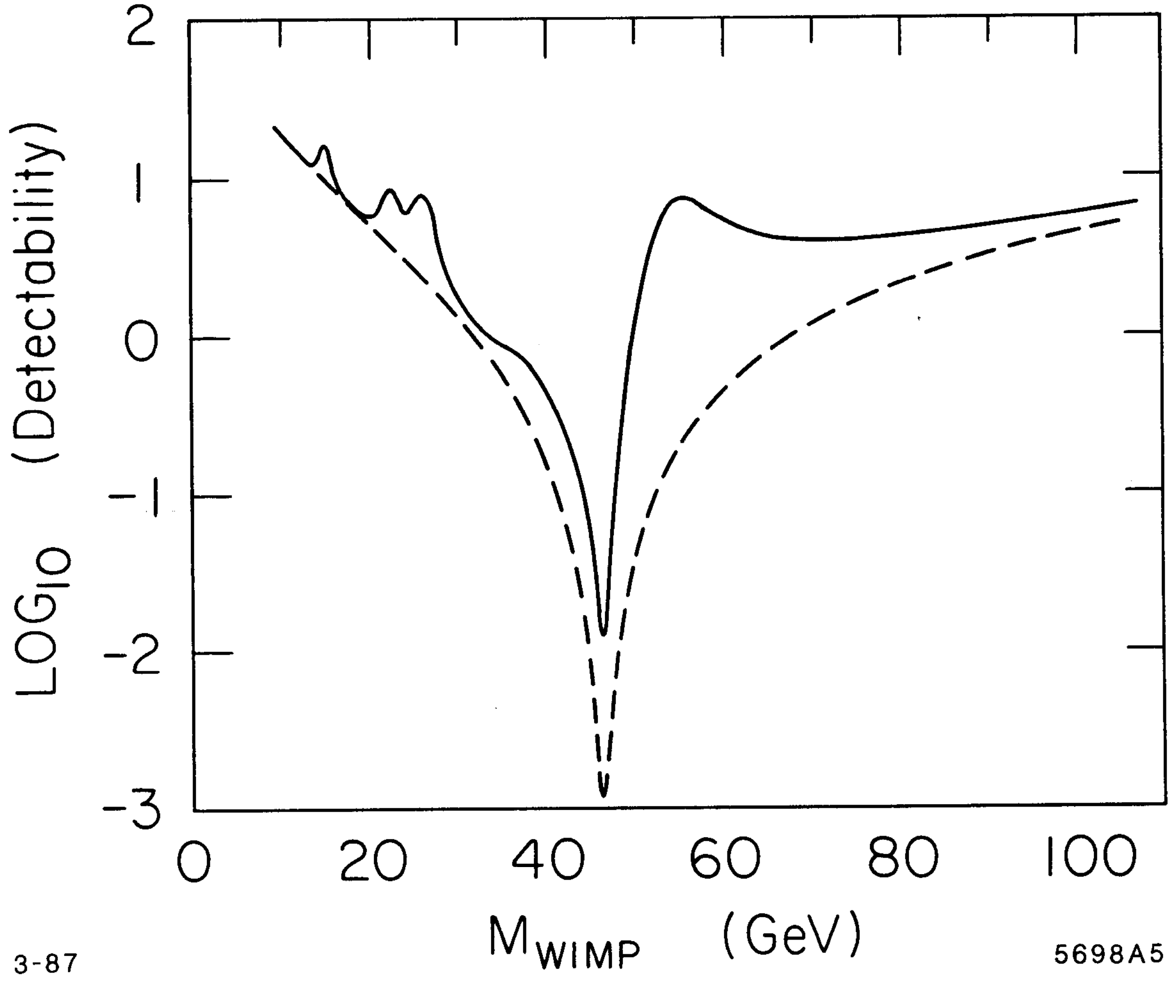

1.0
0.8
0.6
0.4
0.2
0
SUPPRESSION
0
20
40
60
80
100
MWIMP (GeV)
3-87
5698A3

Fig. 3



2
0
-2
-4
-6
-8
0
20
40
60
80
100
$LOG_{10}\bar{\rho}$ (GeV/cm^3)
M_{WIMP} (GeV)
3-87
5698A4

Fig. 4



2
1
0
-1
-2
-3
LOG10 (Detectability)
0
20
40
60
80
100
M WIMP (GeV)
3-87
5698A5

Fig. 5