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Electroweak Interactions - Standard and Beyond*

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ABSTRACT

Several important topics *within* the standard model raise questions which are likely to be answered only by further theoretical understanding which goes *beyond* the standard model. In these lectures we present a discussion of some of these problems, including the quark masses and angles, the Higgs sector, neutrino masses, W and Z properties and possible deviations from a pointlike structure.

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I. OVERTURE

I.1 Plan of Lectures

It is often claimed that the *standard* model is fully understood and that the physics *beyond* it is essentially unknown. However, somewhere between *"standard"* and *"beyond"* there is a border area – a gray fringe containing topics which are part of the standard model but which are far from being well understood. These lectures are devoted to several such topics, including the systematics of fermion masses and angles, the Higgs sector, neutrino physics, W and Z properties and possible deviations from a pointlike behavior.

We do not attempt to discuss in detail any specific theory which goes beyond the standard model. However, such theories often have important implications for the topics listed above. Consequently, we will comment on some of these implications whenever necessary, referring to general classes of theories including left-right symmetric models, grand-unified theories, Supersymmetry, horizontal symmetries, composite models and the so-called "String Inspired Phenomenology" (SIPH).

The present chapter deals with preliminaries, introducing the standard model, its parameters, its theoretical "loose ends" and the classes of theories which go beyond it. We also introduce the general ground rules of SIPH. We then move on to five specific topics.

The first topic deals with fermion masses and mixing angles. In the most minimal version of the standard model there are 18 arbitrary parameters. Of these, 13 arise from the quark and lepton mass matrices, corresponding to 9 masses, 3 angles and one phase. In chapter II we present a brief discussion of several issues related to these poorly understood parameters.

Chapter III deals with the Higgs particles, their properties, their accompanying Higgsinos in Supersymmetric theories and the possible existence of supermultiplets including quarks, leptons and Higgsinos.

Chapter IV is devoted to neutrinos, their masses, neutrino oscillations in vacuum and in matter, the recent proposal concerning the solar neutrino puzzle and the neutrino spectrum in SIPH.

Chapter V deals with future probes of the W and Z bosons, including the possibility of additional $Z's$ in SIPH and in Left-Right-Symmetric theories. We also discuss the possibility of a substructure for W and Z and deviations from the normal W and Z gauge couplings.

We conclude with a final chapter discussing the future of multi-TeV accelerator physics in view of the decreasing cross sections for all processes among pointlike particles. We consider the possibility of future deviations from a pointlike behavior.

I.2 Counting the Parameters of the Standard Model

The *minimal* version of the standard model is based on the gauge group $SU(3)_c \times SU(2) \times U(1)$ with three generations of quarks and leptons and *one* physical Higgs particle. If we assume that there are no right-handed neutrinos and that there is no strong CP violation, the minimal model contains 18 arbitrary parameters:

(*i*) Three gauge couplings for the three gauge groups. These can be chosen *e.g.* as g_1, g_2, g_3 or α, α_s, $\sin^2\theta_W$. At present energies g_1 and g_2 are of the same order of magnitude but g_3 is substantially larger (or, equivalently, $\alpha_s > \alpha$, $\sin^2\theta_W \sim O(1)$).

(*ii*) Two parameters representing the Higgs sector, even in the absence of fermions. These can be chosen, *e.g.* as M_W and M_ϕ or, alternatively, as $\langle\phi\rangle$ and M_ϕ. We have no detailed information about M_ϕ but expect it to be within one order of magnitude from M_W.

(*iii*) Nine masses for the six quarks and three charged leptons of the three generations. The nine mass values are spread over at least five orders of magnitude.

(*iv*) Three generalized Cabibbo angles for the quark sector.

(*v*) One Kobayashi-Maskawa (KM) phase for the quark sector.

The possible existence of "strong" CP-violation would add a 19th arbitrary parameter (whose value must be tiny).

Of these parameters, three are fundamental gauge couplings representing the strength of the three fundamental interactions at present energies. We may hope to relate them to each other only if we succeed in unifying two or more of these interactions. The other 15 parameters are related, in one way or another, to the Higgs sector of the theory. Their origin is obscure. Any attempt to reduce the number of these parameters must involve a deeper understanding of the symmetry breaking mechanism of the model.

The standard model may have several different extensions which will add no fundamental new physics but will increase the number of arbitrary parameters. The three most direct extensions are the following:

(*i*) *Neutrino masses.* If neutrinos are not exactly massless, we start by adding three additional neutrino mass parameters. However, the existence of these masses opens the door to generation mixing among the leptons, allowing for three leptonic Cabibbo angles and one leptonic KM phase. If the neutrinos have both Dirac and Majorana masses, the number of parameters is even larger, but, in that case, additional Higgs (and possibly Goldstone) particles must exist. The existence of non-vanishing neutrino masses therefore adds at least seven new parameters, possibly many more.

(*ii*) *Additional Higgs particles.* The standard model may include any number of Higgs doublets without changing its main features. However, the introduction of such additional doublets opens the way to a variety of additional terms in the Higgs potential. The couplings of the new Higgs fields as well as their masses and vacuum expectation values are additional free parameters.

(*iii*) *Additional generations.* It is entirely possible that additional generations of quarks and leptons, following the pattern of the first three generations, will be discovered. Since we have no reason to expect precisely three generations, we should not consider the possible existence of additional generations as a major extension of the model. However, a fourth generation will add nine additional arbitrary parameters (if all neutrinos are massless) and at least fourteen parameters if neutrinos have masses.

We therefore conclude that even the least controversial extensions of the standard model are likely to increase its number of arbitrary parameters to anywhere between 25 and 40.

It would have been bad enough if the standard model included 18 or 25 or 40 arbitrary parameters whose observed experimental values obeyed some simple patterns. For instance, a reasonable unbiased guess in the minimal standard model would suggest that M_W, M_ϕ and all quark and lepton masses are roughly of the same order of magnitude. If that were the case, we might still wonder about the origin of so many independent parameters but their general behavior would have posed no striking puzzles. Instead, we have $\frac{m_e}{M_W} = 6 \times 10^{-6}$, quark masses ranging from 4 MeV to at least 30 GeV, etc. Not only we cannot calculate the various parameters, we have no understanding of their general orders of magnitude and no explanation for the observed hierarchy of masses.

I.3 "Loose Ends" of the Standard Model

There are no experimental facts which force us to go beyond the standard model (with the possible exception of the non-vanishing baryon number of the universe) and there are no theoretical internal inconsistencies within the framework of the model. Consequently, all possible motivations for expecting physics beyond the standard model are, to a certain extent, a matter of taste. Nevertheless, there is a wide-ranging consensus among high energy physicists that there must be some new physics beyond the standard model. Every physicist may have his or her own list of motivations. I present here my own list:

(*i*) *The Fine Tuning Problem.* Why is M_ϕ of order M_W and not of order M_{Planck}?

(*ii*) *The Generation Puzzle.* Why do we have several generations? What distinguishes among them? Why do we have different orders of magnitudes for fermion masses in different generations?

(*iii*) *The Quark-Lepton Connection.* How are the quarks and the leptons related to each other? Why do they have simple charge ratios? What is the origin of the miraculous anomaly cancellation between quarks and leptons in one generation?

(*iv*) *The Origin of P and CP Violation.* Is parity conserved at short distances? Is it broken explicitly or spontaneously at present energies? Why are neutrinos massless or very light? What is the origin of "weak" and "strong" CP-violation?

(*v*) *The Unification Problem.* Can we unify the three basic fundamental interactions of the standard model, represented by the three commuting gauge groups?

(*vi*) *The Gravity Connection.* Can we construct a quantum field theory of gravitational interactions and relate it to the three interactions of the standard model? If we can do it, can we relate the physics of the Planck scale to the physics of present energies or to anything which may become accessible experimentally within the next few decades?

(*vii*) *"The 4-3-2-1 Puzzle".* Why 4 dimensions of space-time? Why $SU(3)$? Why $SU(2)$? Why $U(1)$?

None of these questions can be answered by the standard model. All of them require explanation. All such explanations can only come from some new physics beyond the standard model. The new physics is likely to emerge from a combination of new theoretical ideas and, most important, new experimental results which will indicate deviations from the predictions of the standard model. It is disappointing that no such experimental results exist at the moment. We must hope that they will appear soon. We now turn to a brief discussion of the general classes of experiments which may give us the required hints for the new physics.

I.4 Common Features of "Beyond Standard" Theories

Several theoretical approaches and numerous explicit models based on these approaches have been proposed for describing the physics beyond the standard model. All of them assume that the standard model will remain an excellent approximation at low energies (say, below $E \sim O(M_W)$). Among the various approaches we might mention Technicolor, Horizontal Symmetries, Left-Right Symmetry, Supersymmetry, Grand Unification, Compositeness of Quarks, Leptons and possibly W and Z, and – last but not least – Superstring Theory. Many models are based on combinations of the above ideas (*e.g.* Supersymmetric Grand Unified Theories with Horizontal Symmetries, etc.). What is common to all of these approaches is the existence of a new fundamental underlying theory, valid at energies well above present energies, leading to an effective low energy approximation which is consistent with the standard model.

In all "beyond standard" theories there is always a new high energy scale $\Lambda \gg M_W$ which characterizes the new Lagrangian. In some cases we may have several such scales, all larger than M_W. At energies around or above Λ many new phenomena are always expected. In particular, every theory predicts a large number of new particles with $M \sim O(\Lambda)$. We do not know the value (or values) of Λ. It can be anywhere between $O(TeV)$ and M_{Planck}. It is not likely to be below

1 TeV or else some indirect effects would have probably been already observed.

If and when we can do experiments at $E \sim O(\Lambda)$, there would be no great difficulty in discovering evidence for the new physics. We would be able to produce particles with $M \sim O(\Lambda)$ and will directly observe effects due to any possible new fundamental interactions. However, within the next 20 years, experiments at $E \sim O(\Lambda)$ will be possible only if $\Lambda \sim 1\ TeV$ *and* a multi-TeV collider is constructed. If Λ is larger and/or if such a collider is delayed, we will be reduced to experimentation at energies well below Λ.

At such energies we can still learn about the new "beyond standard" physics by using indirect experimental methods. The crucial ingredient here is the existence, in all "beyond standard" theories, of particles which are much lighter than Λ. Such particles are *approximately massless* on the Λ scale. We have mentioned earlier that the typical expected mass scale for all particles is actually $O(\Lambda)$. How do we then obtain approximately massless particles in all "beyond standard" models?

The answer is simple and well-known. A theory with a typical scale Λ allows massless particles if and only if there is some symmetry principle which protects these particles from acquiring a mass. We know at least four such principles:

(*i*) *Gauge Symmetry.* An unbroken gauge symmetry provides us with a mechanism for obtaining massless vector particles (*e.g.* photons, gluons and possible technigluons or hypergluons).

(*ii*) *Chiral Symmetry.* A chiral symmetry may prevent spin $\frac{1}{2}$ particles from acquiring a mass (*e.g.* left-handed neutrinos in a minimal standard model without right-handed neutrinos).

(*iii*) *Goldstone mechanism.* A spontaneously broken global symmetry will produce massless spin 0 Goldstone bosons (such as pions, axions, majorons etc.).

(*iv*) *Supersymmetry.* The above three mechanisms can produce massless particles with spins $1, \frac{1}{2}, 0$ respectively. When combined with supersymmetry, any one of them will lead to additional massless particles with "neighboring" spin values. Thus a massless spin-$\frac{1}{2}$ fermion can occur as a result of chiral symmetry or it can be the supersymmetric partner of a massless gauge boson or a massless Goldstone boson.

It is therefore not too difficult to produce particles with masses well below Λ. In fact, it is fairly easy to account for *exactly* massless particles. It is much more difficult to allow for particles which have small, finite masses.

The existence of particles with masses which are much smaller than Λ enables us to describe physics phenomena at energies well below Λ in terms of a "low-energy" effective Lagrangian. Such a Lagrangian will, however, contain traces of the original theory at the Λ scale which led to it. All experiments at energies below Λ relate to this phenomenological effective Lagrangian.

I.5 Classes of Experiments which Probe "Beyond Standard" Theories

How can we probe a new theory which corresponds to a characteristic energy scale Λ by performing experiments at energies well below Λ?

We assume that some new theory, beyond the standard model, is described at $E \sim O(\Lambda)$ by a new fundamental Lagrangian $\mathcal{L}_{NEW}$. At energies $E \ll O(\Lambda)$, $\mathcal{L}_{NEW}$ leads to a low-energy effective Lagrangian $\mathcal{L}_{EFF}$ which describes low energy processes involving particles whose masses obey $M \ll O(\Lambda)$. The effective Lagrangian may be schematically written as:

$$\mathcal{L}_{EFF} = \mathcal{L}_{SM} + \mathcal{L}_{LP} + \mathcal{L}_{HD} + \mathcal{L}_{G}$$

Each of the components of $\mathcal{L}_{EFF}$ represents a large class of low energy phenomena which can be tested experimentally. We now consider these terms:

(*i*) $\mathcal{L}_{SM}$. This is the Standard Model Lagrangian. The experimental success of the standard model tells us that $\mathcal{L}_{EFF}$ cannot be very different from $\mathcal{L}_{SM}$ or else we would easily detect experimental phenomena which go beyond the standard model. We therefore conclude that the most obvious and severe constraint on $\mathcal{L}_{NEW}$ is that it must lead to a low-energy Lagrangian which approximately reproduces the standard model.

(*ii*) $\mathcal{L}_{LP}$. These are terms in the Lagrangian representing interactions of additional Light Particles *with couplings of "normal" strength* (i.e. coupling constants comparable to ordinary standard model couplings). Such particles have masses $M \ll O(\Lambda)$ but do not appear in the standard model. They may correspond to simple extensions of the standard model, such as an additional generation of quarks and leptons or additional Higgs doublets. They might also correspond to entirely new features which are required in some "beyond standard" models. The most important example of such particles are the supersymmetric partners of all standard model particles which appear in supersymmetric theories. If a given $\mathcal{L}_{NEW}$ leads to terms of the type $\mathcal{L}_{LP}$, there is no *a priori* reasons for $\mathcal{L}_{LP}$ to be less significant than $\mathcal{L}_{SM}$. However, since experimentally there is no evidence for $\mathcal{L}_{LP}$, we must conclude that its additional light particles must be somewhat heavier than the corresponding particles in $\mathcal{L}_{SM}$. The possible discovery of a $\mathcal{L}_{LP}$ term will not enable us to determine the value of Λ but will certainly provide us with hints concerning possible extensions of the standard model or some of the features of the new Lagrangian $\mathcal{L}_{NEW}$.

(*iii*) $\mathcal{L}_{HD}$. These are High-Dimension terms ($d > 4$), reflecting the new physics at $E \sim O(\Lambda)$. A typical term in $\mathcal{L}_{HD}$ would be an effective four-fermion term of dimension six, preceded by a coefficient of order $\frac{1}{\Lambda^2}$:

$$\mathcal{L}_{HD} = \frac{g^2}{\Lambda^2}\ \bar{f}_i f_j \bar{f}_k f_\ell .$$

There could also be $\mathcal{L}_{HD}$ terms of dimension 5, 7 or more, always accompanied by coefficients which are inversely proportional to positive powers of Λ. Such terms may lead to additional, nonstandard, contributions to existing processes (such as $e^+ + e^- \to e^+ + e^-$) or to new processes which cannot occur in the standard model (such as $p \to e^+ + \pi^0$ or $\mu \to e + \gamma$). In the first case we expect to observe experimental deviations from quantitative predictions of the standard model. In the second case we should begin to observe processes for which we presently have only experimental upper limits. In both cases the amplitudes are substantially smaller than typical standard model amplitudes because of the $\frac{1}{\Lambda^2}$ factor in $\mathcal{L}_{HD}$. Any observation of an experimental effect due to a $\mathcal{L}_{HD}$ term will provide us with some information on Λ. A precise determination of Λ requires knowledge of the effective coupling g.

(*iv*) $\mathcal{L}_G$. These are terms of dimension four, involving Goldstone or pseudo-Goldstone particles which are generated by symmetry breaking at a scale Λ. Their (Yukawa) couplings are inversely proportional to Λ. Typical examples involve particles such as axions, majorons, familons etc. Their couplings to fermions are typically of the form:

$$\mathcal{L}_G = \frac{m_f}{\Lambda}\chi \bar{f} f$$

where χ is the Goldstone (or pseudogoldstone) particle. Such couplings are clearly weaker than ordinary standard model couplings. For sufficiently large values of Λ they cannot be detected in terrestrial experiments and the only available information may come from astrophysical and cosmological arguments.

From the above analysis it is quite clear that we have several classes of experiments which may probe the new physics of the Λ scale at energies well below Λ. Some of these experiments can actually be performed at fairly low energies (*e.g.* measuring $(g-2)_{e,\mu}$ probes terms of the $\mathcal{L}_{HD}$ type). Others may not even require accelerators (*e.g.* searches for proton decay). All present experiments and most experiments in the next few decades will belong to the classes of experiments described here. We will somehow have to learn about $\mathcal{L}_{NEW}$ by probing $\mathcal{L}_{EFF}$.

I.6 String Inspired Phenomenology (SIPH)

String theory is certainly the first serious candidate for a quantum theory of gravity with the added attraction of an intimate connection between gravity and other gauge interactions. The theory itself is remarkable. However, it relates to a ten-dimensional world with 496 gauge-group generators and it makes predictions for physics at the Planck scale. The standard model describes physics at an energy scale which is 17 orders of magnitude lower in a world with four dimensions and with twelve gauge bosons. Somehow we must learn to make the transition between these two situations.

Eventually one would hope to derive the number of space-time dimensions at our present energies as well as all the parameters of the standard model (including the choice of its gauge group) from the fundamental ten-dimensional theory at the Planck scale. At the present time we do not know how to do this. We are therefore reduced to *assuming* that somehow six of the ten dimensions "compactify" and that, after the compactification, we remain with a gauged subgroup of the original large gauge group which was, presumably, $E_8 \times E_8$. The leading candidate for the subsidiary gauge group is E_6 whose 27-dimensional multiplets allegedly contain quarks, leptons, Higgsinos and their supersymmetric partners. The E_6 symmetry is broken to a subgroup containing $SU(3)_c \times SU(2) \times U(1)$. That subgroup may include at least one (possibly two) extra neutral Z', and perhaps an extra $SU(2)$.

There is no proof that the above scenario is a necessary consequence of the $E_8 \times E_8$ heterotic string theory. On the contrary, it is perfectly possible that the remaining gauge group after compactification is $SU(5)$ or $SO(10)$ or some other group. It is also possible that the extra Z boson (or bosons) will appear at intermediate energies between the Planck scale and M_W, leaving no observable effects at present energies. On the other hand, there is no clear experimental evidence *against* the E_6 scenario and it may be useful to pursue it both as an example of a String Inspired Phenomenology (SIPH) or merely as a candidate Grand Unified Theory with no Strings attached.

We will therefore discuss here and in the next chapters several implications of this possible phenomenology.

The ground rules are the following[1]: We assume that the *gauge couplings* obey the relations of an E_6 symmetry. There are 78 gauge bosons, corresponding to the adjoint representation of E_6. The corresponding gauginos are also in a 78 of E_6. All other fermions (quarks, leptons and Higgsinos) are in 27 or $\overline{27}$ representations of E_6. We will usually discuss only the three "normal" 27 representations and ignore the possible $\overline{27}'s$. There are also corresponding $27's$ containing the supersymmetric partners: squarks, sleptons and Higgses.

The masses of all gauge bosons, matter fermions and their corresponding supersymmetric partners may be anywhere between the present low-energy scale and the Planck scale. The 27 of E_6 has the following $SO(10)$ decomposition:

$$27 = 16 + 10 + 1$$

while the $SO(10)$ representations contain the following $SU(5)$ multiplets:

$$16 = 10 + \bar{5} + 1; \qquad\qquad 10 = 5 + \bar{5}$$

It is clear that only 15 of the 27 states correspond to the usual quarks and leptons. They, together with an additional neutrino N, form the 16 dimensional multiplet of $SO(10)$. The remaining 11 fermions are:

(*i*) A charge $-\frac{1}{3}$ quark, usually denoted by g, belonging to the 5 of $SU(5)$ and to the 10 of $SO(10)$.

(*ii*) The antiquark $\bar{g}$.

(*iii*) A doublet of Higgsinos H^+, H^0 in the 5 of $SU(5)$ and 10 of SO(10).

(*iv*) The antiparticles of the above Higgsinos.

(*v*) An extra neutrino-like particle S, which is an $SU(5)$ and $SO(10)$ singlet.

The g-quark is "dangerous" because it may lead to proton decay as well as to flavor-changing neutral currents. Most models would suggest that its mass is well above M_W, probably around the GUT scale. We will return to the neutrino spectrum in section IV.4. The remaining 19 states may be light and they may form three generations of quarks, leptons, Higgsinos and their associated supersymmetric partners.

Strictly speaking, in String models the Higgs particles need not belong to the same 27 multiplets as the matter fermions. They may be part of "incomplete" multiplets and their Yukawa couplings need not obey the usual E_6 relations.

However, the Higgs states *must* belong to 27 or $\overline{27}$ representations, in contrast with the usual situation in Grand Unified Theories ($SU(5), SO(10)$ or E_6) in which the Higgs particles belong to representations which are different from those of the matter fermions.

It is important to understand that what we call SIPH is not a well- defined framework and that any confirmation or failure of its predictions will neither confirm nor destroy the fundamental concepts of String Theory. Unfortunately, the predictive power of String Theory is still unsatisfactory. On the other hand, SIPH leads to a variety of interesting and useful phenomenological observations which may be helpful, independently of the validity of String Theory. We return to some of them in every one of the following chapters.

II. MASSES, ANGLES AND PHASES – STANDARD AND BEYOND

II.1 Experimental Values

In the minimal standard model we have 13 parameters representing the fermion masses (nine parameters), mixing angles (three) and KM-phase (one). If we *assume* that the top quark will be found somewhere in the 30-60 GeV range and that the observed value of ϵ in the $K^0 - \bar{K}^0$ system is fully accounted for by the KM-phase, we can quote either precise values or reasonable estimates for all of these parameters. The values are (all masses in GeV units):

First generation masses: $m_u = 0.004$; $m_d = 0.007$; $m_e = 0.0005$.
Second generation masses: $m_c = 1.3$; $m_s = 0.15$; $m_\mu = 0.1$.
Third generation masses: $m_t = 45 \pm 15$; $m_b = 5$; $m_\tau = 1.8$.
Mixing between adjacent generations: $\theta_{12} = 0.22$; $\theta_{23} = 0.05$.
Mixing between "distant" generations: $\theta_{13} \sim 0.01$.
KM-phase: $\delta \sim 90° \pm 30°$.

A few comments are in order:

(*i*) All masses of the five known quarks are approximate, but their order of magnitude is correct and we will not need here more than that.

(*ii*) The top-quark mass may still turn out to be above the range listed here. In fact, the only upper limit[2] we have is $m_t \leq 180\ GeV$, obtained from the maximum value of $m_t - m_b$ consistent with the present experimental value of the parameter $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}$.

(*iii*) We are using a choice of mixing angles which is the most convenient for most considerations. We will comment on this issue in detail in section II.3.

(*iv*) Direct searches for $b \to u$ transitions give only an upper limit for θ_{13}. No direct determination of δ is available. However, if we want to explain the observed value of ϵ *and* the upper limit on $\frac{\Gamma(b\to u)}{\Gamma(b\to c)}$ in terms of a minimal three-generation standard model without additional Higgs particles or "beyond standard" physics, we must assume θ_{13} and δ values in the general range quoted above.

Consequently, all the above numbers should be considered as an approximate representation of the emerging pattern of mass, angle and phase values. Some modifications are still possible, especially as a result of "beyond standard" ideas.

In searching for regularities among the above 13 parameters, it is instructive to consider the dimensionless ratios of fermion masses in different generations. In fact, we may wish to consider the following:

Quantities relating generations 1 and 2:

$$\sqrt{\frac{m_u}{m_c}} = 0.06; \ \sqrt{\frac{m_d}{m_s}} = 0.22; \ \sqrt{\frac{m_e}{m_\mu}} = 0.07; \ \theta_{12} = 0.22 \ .$$

Quantities relating generations 2 and 3:

$$\sqrt{\frac{m_c}{m_t}} \sim 0.18; \ \sqrt{\frac{m_s}{m_b}} = 0.18; \ \sqrt{\frac{m_\mu}{m_\tau}} = 0.24; \ \theta_{23} = 0.05 \ .$$

Quantities relating generations 1 and 3:

$$\sqrt{\frac{m_u}{m_t}} \sim 0.01; \ \sqrt{\frac{m_d}{m_b}} = 0.04; \ \sqrt{\frac{m_e}{m_\tau}} = 0.017; \ \theta_{13} \sim 0.01 \ .$$

II.2 Numerology

So far, no one has offered a satisfactory explanation for the observed pattern of masses and angles. Eventually, we might hope that some new physics will enable us to calculate the exact values of some or all of these parameters. But before we attempt to do that, we should have at least some qualitative understanding of the general orders of magnitude and the observed hierarchy of mass values. Here we are essentially reduced to naive "numerological" attempts and to possible relations between mass ratios and mixing angles.

In order to pursue some of these attempts, we should first inspect the observed pattern of the parameter values and try to identify simple regularities.

A brief inspection indicates that all mixing angles and square roots of mass ratios connecting *adjacent* generations are of order $\frac{1}{10}$. In fact, if we arbitrarily define a parameter $\alpha = 0.1$, the following empirical relations are not too wrong:

$$\theta_{ij} \sim O(\alpha^{|(j-i)|}); \qquad \sqrt{\frac{m_i}{m_j}} \sim O(\alpha^{i-j}).$$

We will refer to this empirical pattern as "Numerology I".

A somewhat more detailed numerical observation is the fact that all the above mass ratios and angles actually cluster around three values: 0.2; 0.05; 0.01. Consequently, one may introduce a parameter λ such that $\lambda \sim 0.22$ and:

$$\sqrt{\frac{m_d}{m_s}} \sim \sqrt{\frac{m_c}{m_t}} \sim \sqrt{\frac{m_s}{m_b}} \sim \sqrt{\frac{m_\mu}{m_\tau}} \sim \theta_{12} \sim \lambda;$$

$$\sqrt{\frac{m_u}{m_c}} \sim \sqrt{\frac{m_e}{m_\mu}} \sim \sqrt{\frac{m_d}{m_b}} \sim \theta_{23} \sim \lambda^2;$$

$$\sqrt{\frac{m_u}{m_t}} \sim \sqrt{\frac{m_e}{m_\tau}} \sim \theta_{13} \sim \lambda^3.$$

We refer to this empirical pattern as "Numerology II".

"Numerology I" is a simple, easy to remember pattern. However, it is correct only within factors of two. "Numerology II" is much more accurate, but seems to follow an irregular pattern.

At present, the above numerological observations are useful either as a simple method of remembering the orders of magnitude of the parameters or as an approximation procedure for certain calculations, keeping terms up to a certain order of α or λ. There is no convincing explanation or theoretical foundation for the observed pattern.

As we will see in the next sections, these naive numerological observations may provide us with some guidance in attempting to obtain relations between mass ratios and mixing angles.

II.3 A Recommended Choice of Mixing Angles and Phases

The mixing among the three generations of quarks is defined by a unitary 3×3 matrix V whose matrix elements can be parametrized in terms of the three generalized Cabibbo angles and a single KM-phase. In the general case of N generations, we have an $N \times N$ matrix, described in terms of $\frac{1}{2}N(N-1)$ angles and $\frac{1}{2}(N-1)(N-2)$ phases. Clearly, there are many ways of choosing the angles and phases. Among the well-known choices: the original KM-choice[3] (probably the least convenient for any purpose), the Maiani choice[4] (convenient for angles but less so for phases), the Wolfenstein choice[5] (convenient for phases but based on "Numerology II" for angles) and others. We strongly recommend that the standard choice of angles and phases become the choice first introduced by Chau and Keung[6] for three generations (incorporating the main ideas of both Maiani and Wolfenstein) and later generalized[7] to the case of N generations.

In this choice every angle has a clear and direct relation to one matrix element of the matrix V. All angles are denoted by θ_{ij} (for any $j-i>0$), representing the mixing among generations i and j. Each phase is denoted by δ_{ij} (for any $j-i>1$), and the related $e^{i\delta_{ij}}$ factor always multiplies the corresponding $\sin\theta_{ij}$. Assuming that the pattern of "Numerology I" persists in the general case of N generations, the above choice of parameters obeys, for any N and for all $j-i>0$:

$$V_{ij} = s_{ij}(1 + O(\alpha^4))$$

where $s_{ij} = \sin\theta_{ij}$ for $j-i=1$ and $s_{ij} = \sin\theta_{ij}e^{i\delta_{ij}}$ for $j-i>1$. In practice, this means that all V_{ij} values above the main diagonal are given, to an accuracy of three or more significant figures, by the corresponding values of s_{ij}.

The explicit form of the matrix V in the case of three generations is:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}^* & c_{12}c_{23} - s_{12}s_{23}s_{13}^* & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}^* & -c_{12}s_{23} - s_{12}c_{23}s_{13}^* & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij} e^{i\delta_{ij}}$.

A detailed discussion of our recommended choice of parameters can be found in Reference 8.

II.4 A Fourth Generation?

There is no known fundamental reason for the existence of three generations of quarks and leptons. There is no good argument for or against the existence of additional generations. Accepting the pattern of "Numerology I", we would guess that the mixing angles of the a possible fourth generation with the first two are probably very small: $\theta_{14} \sim 10^{-3}$, $\theta_{13} \sim 10^{-2}$. It is unlikely that the fourth generation will have a substantial influence on low energy quantities involving the first two generations, with the possible exception of CP-violating amplitudes in the $K^0 - \bar{K}^0$ system. Even in this latter case, we do not expect fourth generation effects to dominate, but they may lead to terms which are comparable to those induced by the third generation particles.

There are several interesting experimental and theoretical constraints concerning a possible fourth generation:

(*i*) The UA1 collaboration[8] obtained a lower limit of 41 GeV for the mass of a possible fourth generation charged lepton σ. Note that this limit already indicates that

$$\frac{m(\sigma)}{m(\tau)} > \frac{m(\tau)}{m(\mu)}$$

while the $\frac{\tau}{\mu}$ mass ratio is smaller than the $\frac{\mu}{e}$ mass ratio.

(*ii*) Cosmological and astrophysical considerations seem to limit the number of light neutrino generations to at most four.

(*iii*) Measurements of the Z width should soon provide us with strong limits on the number of light neutrinos. At present, however, they cannot compete with the astrophysical bound.

(iv) The present value of the ρ-parameter in the standard model is within 2% from its expected value of 1. This leads to an upper limit[9] on the *mass difference* between the two quarks of a hypothetical fourth generation. We obtain[2]

$$m(t') - m(b') < 180\ GeV.$$

If we assume (for no good reason) that the approximate relation $\frac{m(s)}{m(c)} \sim \frac{m(b)}{m(t)} \sim \frac{1}{10}$ carries over to a fourth generation, we conclude that $m(t')$ must be below 200 GeV. However, we cannot exclude heavier $t' - b'$ pairs which are almost degenerate.

(v) If the mass difference within a hypothetical fourth generation of quarks allows the decays $t' \to b' + W^+, t' \to b' + \phi^+$ where ϕ^+ is a charged Higgs particle, such decays should dominate over the usual weak decays $t' \to b' + e^+ + \nu_e,\ t' \to b' + u + \bar{d}$.

(vi) If quark masses in the fourth generation exceed a few hundred GeV's, the Yukawa couplings of these quarks may become strong, leading to a variety of unpleasant effects of the so-called "strong weak interactions". However, such a situation cannot be excluded and it may very well happen.

Our overall conclusion from the above assortment of comments is the following: There is no need for additional generations. If they exist, they are not likely to solve or to illuminate any presently existing problem in the standard model. Extrapolating present mass patterns and using various bounds it is reasonable to guess that *at most* one additional generation exists. If it does, the t'-mass should not be too far from 200 GeV and the b'-quark and the σ-lepton would be lighter than the Z.

II.5 Why Do We Expect Relations Between Masses and Angles?

Within the standard model, all masses and angles are free parameters. There are no relations among them. It appears that the standard model would remain self-consistent for any set of mass and angle values.

Within some new "beyond standard" theory which describes physics at a high energy scale Λ, we may be able to calculate *all* the masses, angles and phases, starting from some new set of (hopefully few) fundamental parameters.

Until such a time comes, it may be interesting to try to find some relations among the observed mixing angles and the pattern of masses. We may not be able to derive the masses and the angles from first principles, but we may be able to relate quantities which we do not yet know to compute.

Why do we believe that such relations must exist? Within the standard model, there are several "low-energy" quantities which we can calculate both in the tree approximation and in higher orders. We often discover that some low-energy quantity depends on the masses of intermediate particles which can be exchanged in a one-loop diagram. That, by itself, is no surprise. However, our physics intuition tells us that it is unlikely that a low-energy quantity will become indefinitely *larger* if the mass of such an intermediate particle *increases.* Such is the case at least in three simple examples which we now list:

(*i*) $\Delta M(K_S^0 - K_L^0)$. In this case the contribution of the top quark is such (because of the GIM mechanism) that for $m_t \to \infty$ we find $\Delta M \to \infty$.

(*ii*) $\mu \to e+\gamma$. Here, again, a GIM mechanism operates. The rate of the process depends on the masses of intermediate neutrinos in a way which does not disappear for $m_\nu \to \infty$.

(*iii*) The ρ-parameter of the standard model gets a contribution[9] from any pair of quarks with charges $\frac{2}{3}$,$-\frac{1}{3}$ which have a non-vanishing coupling to W^+ (in other words: when the relevant mixing angle does not vanish). Here, again, we may consider *e.g.* the contribution of a loop with a t-quark and a d-quark. If we hold everything else fixed and send the t-quark mass to infinity, we obtain a divergent contribution to M_W (and to ρ).

In all of these cases, there is a very simple way out of the paradox. The contribution of the intermediate quark or lepton is always multiplied by a mixing angle. If we assume that the mixing angle *must* decrease when the fermion mass increases to infinity, we will encounter no difficulty whatsoever. Thus, for instance, if the angle θ_{23} is proportional to $\sqrt{\frac{m_c}{m_t}}$, the contribution of the t-quark to $\Delta M(K^0_S - K^0_L)$ will not "explode" when $m_t \to \infty$. Similarly, if $\theta_{13} \to 0$ fast enough for $\frac{m_d}{m_t} \to 0$, the t-quark contribution to the ρ- parameter will not "explode".

We have therefore reached a remarkable conclusion: We have supplemented the standard model by a simple physical assumption stating that low-energy quantities must remain stable when masses of intermediate particles in higher order corrections increase indefinitely. We then find that this simple assumption forces us to have relations between masses and angles. More specifically: It tells us that mixing angles between a given pair of generations must decrease when the mass ratios of the fermions in the same generations decrease. We cannot derive a precise relation but the necessity of having some such relation is a significant result.

Since both the masses and the angles are obtained in the standard model from the mass matrices (which, in turn, are based on the Yukawa couplings of the Higgs fields), we must therefore conclude that *within the mass matrices*, some new symmetries or relations must exist. It is possible that some elements of the mass matrices vanish because of some new symmetry or that some otherwise unrelated matrix elements become related as a result of some new principle. Only such relations can yield the necessary connections between masses and angles.

We can now formulate two approaches to the problem of understanding the observed values of the masses, angles and phases:

(*i*) *The theoretical approach.* We search for the new theory, discover the new Lagrangian $\mathcal{L}_{NEW}$, derive the new symmetries which appear in the mass matrices and find the resulting relations among masses, angles and phases.

(*ii*) *The phenomenological approach.* We start from the observed pattern of masses and angles. Assuming what we earlier called "Numerology I" or "Numerology II" *and imposing mass-angle relations of the type suggested above* (i.e. $\theta_{ij} \sim \sqrt{\frac{m_i}{m_j}}$) we search for simple patterns in the mass matrices. On the basis of these, we guess the new symmetry or principle and then, hopefully, try to start building a convincing new model for the new physics at the high-energy scale.

Clearly, the first method is superior, if we can pursue it. No one has succeeded in doing so. The second method is less ambitious and much less profound. Several interesting attempts have been made along its lines but no great success can be reported. In the following section we briefly review some such attempts, mainly in order to show the type of work that can be done, at present.

II.6 Playing with Mass Matrices

Consider the quark mass matrix for the case of three generation. For simplicity, we assume that all mass matrices are Hermitian (in general they are not, but we are only illustrating the methods here). The simplest game one can play is to assume that certain matrix elements vanish (presumably as a result of a new symmetry of the Higgs Yukawa couplings). With a sufficient number of vanishing matrix elements , one can derive new relations between masses and angles.

The best known ansatz is the one proposed by Fritzsch[10] several years ago. According to his hypothesis, the 3×3 mass matrices for the up and down sectors have the form:

$$M_u = \begin{pmatrix} 0 & X_u & 0 \\ X_u^\star & 0 & Y_u \\ 0 & Y_u^\star & Z_u \end{pmatrix} \; ; \; M_d = \begin{pmatrix} 0 & X_d & 0 \\ X_d^\star & 0 & Y_d \\ 0 & Y_d^\star & Z_d \end{pmatrix} .$$

In this case we can express all masses, angles and phases in terms of eight real parameters. Since we have ten measurable quantities (six masses, three angles and one phase) we may obtain two relations. These relations are, at present, consistent[11] with the available experimental information.

Another ansatz, based on a different theoretical motivation has been proposed by Stech.[12] He postulates different forms for the mass matrices in the up and down sectors. According to Stech:

$$M_u = S;\ M_d = \beta S + A$$

where S and A are, respectively, a symmetric and an antisymmetric 3×3 matrix. Here, again, we are able to describe the ten measurable quantities in terms of a smaller number of parameters, obtaining relations which are, at the present time, consistent with the data.

Using the empirical fact that:

$$\frac{m_u}{m_c} \ll \frac{m_d}{m_s}$$

we obtain from both the Fritzsch ansatz and the Stech ansatz:

$$\theta_{12} \sim \sqrt{\frac{m_d}{m_s}}.$$

We also obtain, for the Fritzsch case:

$$\theta_{23} \sim \sqrt{\frac{m_s}{m_b}} - \sqrt{\frac{m_c}{m_t}}$$

and for the Stech case:

$$\theta_{23} \sim \sqrt{\frac{m_s}{m_b} - \frac{m_c}{m_t}}.$$

All of these results are consistent with the data. Moreover, both schemes provide us with an explanation to one interesting feature of the pattern of masses and angles. We have noticed in section II.2 that θ_{23} was significantly smaller than

θ_{12}. In fact, in what we called "Numerology II" we gave them the values $\theta_{23} \sim \lambda^2$, $\theta_{12} \sim \lambda$. Now we learn that the smallness of θ_{23} is related to the similar values of $\frac{m_s}{m_b}$ and $\frac{m_c}{m_t}$ while the difference between $\frac{m_u}{m_c}$ and $\frac{m_d}{m_s}$ is related to the fact that θ_{12} is larger. Thus, both the Fritzsch and the Stech guesses account for an important regularity in "Numerology II".

This is precisely the type of qualitative features which we may be able to understand by "playing" with mass matrices.

Another interesting exercise was recently proposed by Gronau et al.[13] They subscribe to *both* the Fritzsch and the Stech hypotheses and combine them to suggest the following mass matrices:

$$M_u = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} ; \; M_d = \beta \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} + \begin{pmatrix} 0 & ia & 0 \\ -ia & 0 & ib \\ 0 & -ib & 0 \end{pmatrix} .$$

Here all masses, angles and phases are expressed in terms of only six real parameters (A, B, C, β, a, b) and the predicted relations are still in reasonable agreement with the existing data.

The above "games" can teach us something about the physics beyond the standard model only if they can be based on some reasonable theoretical foundations. Typically, one would have to introduce some kind of a "horizontal symmetry" according to which different generations are labeled by different values of a new (spontaneously broken) quantum number. By applying such a symmetry to the fermion sector *and* to the Higgs sector, one immediately obtains selection rules preventing certain Higgs particles from coupling to certain fermions, depending on their generation. In this way we obtain vanishing matrix elements in the mass matrices, leading to one pattern or another.

Unfortunately, all the "horizontal symmetries" which were suggested so far, appear to be fairly artificial in the sense that they are designed to produce a specific ansatz for the mass matrices without explaining or solving other important issues of the standard model. Nevertheless, we believe that the problem of masses and angles is so important that we should continue to pursue it even at the simple-minded level described here with the hope of obtaining some clues to the real mystery behind the experimentally observed pattern. In section III.3 we will briefly return to this problem, suggesting yet another simple form for the mass matrices.

II.7 Masses, Angles and SIPH

String theories lead to a new interesting explanation for the existence of generations. The observed particles, which are approximately massless on the Planck scale, are assumed to be the so-called "zero-modes" of the string, obtained after the compactification of the ten-dimensional space-time into the usual four dimensional space-time. The most widely discussed version of String Theory is the one based on an $E_8 \times E_8$ gauge group, where the remaining four-dimensional symmetry is a broken E_6. In that case, all matter fermions (as well as their supersymmetric partners) belong to 27 and $\overline{27}$ representations of E_6. The number of 27-dimensional multiplets and the number of the $\overline{27}$ multiplets are determined by the topological properties of the relevant compactified manifold. We are then naturally led to the existence of several massless multiplets of fermions. This is considered a great triumph for string theories, since it provides us for the first time with a theoretical reason for the replication of fermions which have a small mass on the Planck scale.

Models can be constructed with almost any number of generations. In particular there are several variants which lead to exactly three generations of quarks and leptons. In principle, the theory should dictate the number of generations. However, at the present time we can only *choose* a solution with the correct number of generations and study it.

The theory does not provide us with a clear indication of the features that distinguish between particles belonging to different generations. There is no explicit quantum number which labels the generations in such models.

We also have no idea on how to proceed with an explicit calculation of the quark and lepton masses. In principle, the Yukawa couplings of all Higgs fields are computable from the fundamental string interactions. The mass values should then follow. In practice, not only we do not know how to derive the relevant numbers but we also do not yet have a qualitative argument either for the observed scale of the fermion masses ($10^{-18} - 10^{-22}$ in Planck units) or for the mass ratios among different generations (i.e. our "Numerology I and II").

One is reduced again, at least for the time being, to "playing games" with symmetries of the mass matrices. The first interesting attempt in that direction, within the framework of SIPH, was recently made by Greene et al.[14] They considered a three generation model with a Z_3 discrete symmetry, in which E_6 is broken into a Grand Unified $SU(3) \times SU(3) \times SU(3)$ gauge group, which is then spontaneously broken into a subgroup containing the standard model. They obtain quark mass matrices which are consistent with the observed masses, but the only explicit prediction is $m_u = 0$, which can be considered satisfactory or not, depending on one's point of view. We hope that future analysis of mass matrices along similar lines will enable us to derive additional relations among masses, angles and phases and eventually lead even to a complete calculation of the parameters of the standard model. This seems, however, a distant goal.

III. HIGGS – STANDARD AND BEYOND

III.1 Introduction

Twenty years ago, in 1966, some of the most interesting puzzles in physics were the following:

- The $\mu - e$ puzzle.
- Calculating the proton-neutron mass difference.
- Calculating the Cabibbo angle.

- The origin of CP violation.

- Why are the neutrinos massless?

Today, twenty years later, we still face the same five questions. The first three have now been generalized, respectively, to "the generation puzzle", understanding the quark masses, and calculating the generation mixing angles. The last two problems remain unchanged (except that we now have three neutrinos).

The common feature of all of these old unsolved problems is their direct dependence, in the standard model, on the Higgs parameters of the theory. The Higgs sector remains the most mysterious sector of the standard model.

In the minimal version of the standard model we have only one physical neutral Higgs particle. In simple extensions of the standard model we may have one or several Higgs particles; they may be neutral or charged; they may be light or heavy; their interactions may be weak or strong. There are no experimental hints concerning any of these properties. Some of these questions are extremely important and their answer may have a deep influence on the future of theoretical *and experimental* particle physics.

A trivial example can illustrate this: Imagine a situation in which we have at least two Higgs doublets. This is the case in many "beyond standard" models. It follows that we must have at least one charged positive Higgs particle ϕ^+. Assume further that the top quark is found below the W-mass and that the mass of the physical ϕ^+ obeys:

$$m_t - m_b > M_{\phi^+}.$$

For m_t anywhere between 30 GeV and M_W, there is a fair chance that the above inequality is obeyed. In such a case, the dominant decay of the t-quark is likely to be $t \to b + \phi^+$. Most produced t-quarks will yield charged Higgs particles. This would certainly change both our plans for studying t-decays and our hopes for performing Higgs-physics experiments.

More significant consequences occur if Higgs particles are very heavy and if they have strong interactions.

The present chapter deals with an assortment of topics related to the Higgs sector of the standard model, using hints which may be derived from "beyond standard" ideas.

III.2 One or Many? Charged or Neutral? Light or Heavy?

The first question we wish to address is : Do we have one physical Higgs particle or several?

Allowing ourselves complete freedom with the choice of Yukawa couplings, the standard model is perfectly self-consistent with having one physical Higgs particle whose Yukawa couplings differ from each other by at least five orders of magnitudes (if neutrinos are massless) and possibly by nine or more orders of magnitudes (if neutrinos have small Dirac masses). This is extremely unattractive but not necessarily incorrect.

The success of the Weinberg mass relation $M_W = M_Z \cos\theta_W$ requires that the only Higgs bosons contributing to the W and Z masses are SU(2) doublets. We may have as many doublets as we wish without influencing this relation. Higgs triplets or higher $SU(2)$ representations would have to have extremely small vacuum expectation values or else their contributions to the W and Z mass will destroy the mass relation. Singlet Higgs fields are harmless but also useless, with one exception: We may have an SU(2)-singlet which transforms nontrivially under some other symmetry (gauged, global or discrete), serving as a symmetry breaking mechanism for that other symmetry without influencing the Weinberg mass relation or other features of the standard model. Such is the case in Left-Right Symmetric theories, in some Grand Unified Theories, in majoron schemes and in Horizontal symmetry schemes.

If we have more than one doublet we immediately face several consequences:

(*i*) We must have charged Higgs bosons. Those are easier to detect experimentally.

(*ii*) We may have several scales for the vacuum expectation values of the different Higgs fields. This may allow us to have Yukawa couplings which cover a much smaller range of values, the hierarchy of quark and lepton masses being attributed to the different vev's rather than to different Yukawa couplings.

(*iii*) The GIM mechanism is not guaranteed in a model with many Higgs multiplets. The additional Higgs doublets must obey certain constraints[15] or be sufficiently heavy to avoid flavor changing neutral transitions.

(*iv*) We have an additional source of CP violation, beyond the KM-phase. It is logically possible that CP-violation is due *only* to the Higgs sector.[16] However, since we know that we have at least three generations and there is no reason to assume that the KM-phase is small, we assume that part of the CP-violating effects is definitely due to the KM-phase while another part may be due to multi-Higgs effects. The interplay between these two sources of CP-violation has not been sufficiently studied[17] and we suspect that a complete understanding of CP-violation in light hadron processes may not be possible without it.

(*v*) There are several quantities related to "beyond standard" physics which depend on the number of Higgs fields. A well-known example is the SU(5) prediction for the proton lifetime which decreases by almost a full order of magnitude with every additional Higgs doublet,[18] as a result of the Higgs influence on the rate in which the coupling constants "run". There are several other examples of such a dependence.

We should note at this point that practically *all* "beyond standard" models predict the existence of several Higgs multiplets. Since we are fairly confident that there is physics beyond the standard model and that some of the present ideas are likely to be among the correct ingredient of a future "beyond standard" theory, we argue that the existence of several Higgs particles (including charged ones) is almost certain. We believe that it does not make too much sense to rely on predictions which are based on the single Higgs hypothesis.

The unknown mass of the Higgs particle(s) leads to another well-known ambiguity which we will not discuss in these notes. It is well known that as the Higgs mass approaches the TeV range, Higgs couplings must become strong,[19] leading to copious production of Higgs particles and longitudinal W's,[20] to possible bound states of Higgs particles and to an entirely new range of hadron-like physics at the TeV scale. This possibility may become even more complicated if additional generations of quarks and leptons exist with masses larger than 0.5 TeV or so. In that case, the Yukawa couplings may become large, preventing us from using perturbative methods, and adding the heavy quarks and leptons[9] to the list of particles "enjoying" strong weak interactions. Note, however, that such heavy quarks and leptons must come in approximately degenerate pairs, implying the existence of heavy left-handed neutrinos.

III.3 Generations of Higgs Particles?

We can think of at least two motivations for considering the possibility that Higgs particles, like quarks and leptons, come in generations. The first motivation is based on supersymmetry. In supersymmetric models we must have Higgses and Higgsinos, leptons and sleptons. All of them may be SU(2) doublets. There is no fundamental difference between the properties of Higgsinos and leptons, or between Higgses and sleptons. While we do not know the reason for the existence of repetitive generations, these reasons may apply equally to leptons and to Higgsinos, leading to generations of quarks, leptons and Higgsinos, accompanied by the corresponding squarks, sleptons and Higgses.

In such a case, each generation would have to include at least the usual quarks and leptons and four Higgsinos arranged in two doublets $(\tilde{H}^+, \tilde{H}^0), (\tilde{\bar{H}}^0 \tilde{H}^-)$.

Counting left-handed states only (particles and antiparticles) we obtain a minimum of 19 states per generation (instead of the usual 15).

A second, independent motivation for suggesting generations of Higgs fields follows from the observed mass hierarchy of the fermions in the three known generations. The different energy scales for the fermion masses in different generations may be due to:

(*i*) Yukawa couplings of different orders of magnitude (as in the minimal standard model).

(*ii*) Several Higgs fields possessing vev's of different orders of magnitude.

(*iii*) Masses in different generations being due to different powers of the same vev.

The last two possibilities seem to be more natural than the first one. The second possibility requires that the masses in a given generation are actually dominated by the vev of a corresponding Higgs field, with all Yukawa couplings being of the same order of magnitude.

It is interesting to ask whether it is possible to construct a phenomenological model in which Higgs fields appear in generations, possessing the same generation labels as ordinary quarks and leptons.

We have studied this possibility and found some interesting consequences. We assume that all quarks and leptons possess some generation label X such that the three generations are labeled by $X = X_1, X_2, X_3$. We then assume that we also have three Higgs doublets with the same X-values, respectively, belonging to the same generations. The label X may correspond to a gauged quantum number, a global symmetry or a discrete symmetry. The X-symmetry must, of course, be spontaneously broken by the three Higgs doublets, leading to non-diagonal mass matrices, generation mixing, etc. However, the Lagrangian, including all Yukawa couplings, is assumed to conserve X-symmetry.

We then find that there is only one set of X-values (up to a multiplicative factor) which does not lead to contradictions or to trivial solutions. It is: $X_1 = 2$; $X_2 = 1$; $X_3 = 0$. The resulting quark or lepton mass matrix has the form:

$$M_{u,d} = \begin{pmatrix} 0 & 0 & \lambda_1 \langle\phi_1\rangle \\ 0 & \lambda_1' \langle\phi_1\rangle & \lambda_2 \langle\phi_2\rangle \\ \lambda_1 \langle\phi_1\rangle & \lambda_2 \langle\phi_2\rangle & \lambda_3 \langle\phi_3\rangle \end{pmatrix}.$$

If we now assume:

$$\frac{\langle\phi_1\rangle}{\langle\phi_2\rangle} \sim \frac{\langle\phi_2\rangle}{\langle\phi_3\rangle} \sim \alpha \sim 0.1,$$

we obtain mass values and mixing angles which obey the general pattern suggested by "Numerology I" in section II.2, i.e.

$$\theta_{ij} \sim O(\alpha^{|(j-i)|}); \qquad \sqrt{\frac{m_i}{m_j}} \sim O(\alpha^{i-j}).$$

Furthermore, we find that the predictions of such a scheme are consistent with the known values of the masses and angles. It should be interesting to pursue this form of the mass matrices and to see whether it leads to useful constraints among masses and angles.

The above pattern of Higgs fields and quarks and leptons in each generation should appear in some Grand Unified Supersymmetric models. We have already indicated that the smallest number of left-handed fermions in each generation in such a model must be 19. We do not know of any reasonable gauge group which has a 19 or a 20-dimensional representation, consistent with the above set of quantum numbers. The smallest schemes which can accommodate two Higgs doublets and a full generation of quarks and leptons in one large multiplet are E_6 and its subgroup $SU(3) \times SU(3) \times SU(3) \times Z_3$. Both of these groups appear in SIPH models in which we encounter 27-dimensional multiplets containing the above 19 or 20 states. The remaining states in these multiplets are g, $\bar{g}$ and S (see section I.6)

IV. NEUTRINOS – STANDARD AND BEYOND

IV.1 Massless or Light?

Neutrinos are either exactly massless or extremely light. If they are exactly massless, there must be a symmetry principle which prevents them from acquiring a mass to all orders in the standard model and in any possible "beyond standard" physics. We do not know any such symmetry and no one has made a proposal for it. If neutrinos are very light, compared with all other quarks and leptons, there still must be a convincing reason which explains why it is the neutrino and no other particle, which happens to be so light. Fortunately, there is such a class of mechanisms and they apply only to neutrinos. Only the neutrinos can have both a Dirac and a Majorana mass, leading to a 2×2 mass matrix even in the case of one generation. If, for some reason, the right handed neutrino has a large Majorana mass (corresponding to the scale Λ of some new physics) and the left-handed neutrino have no Majorana mass (or a tiny Majorana mass), we obtain a mass matrix of the form:

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

with eigenvalues:

$$m(\nu_1) \sim \frac{m^2}{M}; \qquad m(\nu_2) \sim M.$$

The eigenstates ν_1, ν_2 are approximately equivalent to ν_L, ν_R, respectively. The Majorana mass M originates from the new physics at the scale Λ, and the Dirac mass m is, presumably, of the same order of magnitude as the mass of the corresponding charged lepton.

The above mechanism[21] (often referred to as the "see-saw" mechanism) appears in Left-right Symmetric models, in some Grand Unified Theories (especially various versions of $SO(10)$) and in some Horizontal Symmetry schemes. In all of these cases the standard model provides the Dirac masses for the neutrinos and no Majorana mass. The Majorana mass is due to some "beyond standard" effects at the Λ scale and it applies to the right-handed neutrino and not to the left-handed one because of their different transformation properties under the gauge group in question. The scale Λ must be at least $O(2\ TeV)$ in LRS models, approximately $O(10^{15}\ GeV)$ in GUTs and could be somewhere in between in Horizontal Symmetry schemes. The present experimental upper limits on the masses of the three neutrinos imply only $M \geq 50\ GeV$. Note that M need not be equal to Λ. In fact, if M is due to some Higgs field with a vev of order Λ, we expect $M = h\Lambda$ where h is a Yukawa coupling of that Higgs field. Since Yukawa couplings are likely to be smaller than one, we expect $M \leq \Lambda$.

The above argument, which is quite general, leads us to suspect that neutrinos are actually light but not massless and that they enjoy both Majorana and Dirac masses, leading to one extremely light and one heavy particle for each neutrino generation. The existence of neutrino masses immediately implies a large number of additional effects including neutrino oscillations, leptonic Cabibbo-like angles, leptonic KM phase, possible neutrino decays, etc.

IV.2 Neutrino Oscillations in Vacuum and in Matter

Assuming that neutrinos have small masses, we may express the mass eigenstates ν_i for $i = 1, 2, 3$ in terms of the "weak" eigenstates ν_e, ν_μ, ν_τ. If we neglect the small mixing of antineutrinos into the ν_i eigenstates (as a result of the "see-saw" mechanism), we obtain a unitary 3×3 matrix similar to the matrix V discussed for the quark sector in section II.3. There is no reason to assume that the leptonic generation mixing angles vanish. If they do not, we expect neutrino oscillations. A beam of, say, ν_e may eventually contain ν_μ's or ν_τ's as a result of such oscillations. The effect may be amplified when the neutrinos go through dense matter,[22,23] in analogy with the familiar situation in the $K^0 - \bar{K}^0$ system.

In order to understand the main physics features of this effect, let us consider the case of two generations of neutrinos. All the qualitative results remain valid in the more realistic case of three generations.

In the case of two generations there is only one mixing angle θ defined by:

$$\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta$$

$$\nu_\mu = -\nu_1 \sin\theta + \nu_2 \cos\theta.$$

The squared mass matrix which is relevant for the description of oscillations can be written as:

$$\begin{pmatrix} m_1^2 \cos^2\theta + m_2^2 \sin^2\theta & \frac{1}{2}\Delta \sin 2\theta \\ \frac{1}{2}\Delta \sin 2\theta & m_1^2 \sin^2\theta + m_2^2 \cos^2\theta \end{pmatrix}.$$

This can be rewritten as:

$$\frac{1}{2}\Sigma \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -\Delta \cos 2\theta & \Delta \sin 2\theta \\ \Delta \sin 2\theta & \Delta \cos 2\theta \end{pmatrix}$$

where:

$$\Sigma = m_2^2 + m_1^2; \qquad \Delta = m_2^2 - m_1^2.$$

It is clear that if $\Delta = 0$ or $\theta = 0$ there are no oscillations. If θ is small, the oscillations must be small. The mixing and the oscillations are controlled, as always, by the ratio between the off-diagonal matrix element $(\Delta \sin 2\theta)$ and the difference between the two diagonal matrix elements $(2\Delta \cos 2\theta)$. The unit matrix $(\frac{1}{2}\Sigma I)$ contributes to neither of these quantities and remains outside the "game". Experiments can search for oscillations (i.e. starting with neutrinos of one type and looking for neutrinos of the other type in the beam) or for depletion (i.e. starting with a known flux of neutrinos of a given type and measuring that same flux as a function of distance). A large number of neutrino oscillation experiments of both types have been performed and no convincing effects have been observed, leading to upper bounds on θ for a given Δ. All the experiments searched for oscillations (or depletion) in vacuum.

The formalism for oscillations in matter is similar but differs in one extremely important feature. The mean free path of ν_e in matter is different from that of ν_μ because of the reaction:

$$\nu_e + e \rightarrow e + \nu_e$$

which can proceed via W exchange, while ν_μ has no analogous reaction at low energies. The evolution of a neutrino beam, when it goes through matter, will be influenced by this reaction which will add an effective squared mass term to the above matrix. The revised squared mass matrix will now be:

$$\begin{pmatrix} m_1^2 \cos^2\theta + m_2^2 \sin^2\theta + A & \frac{1}{2}\Delta \sin 2\theta \\ \frac{1}{2}\Delta \sin 2\theta & m_1^2 \sin^2\theta + m_2^2 \cos^2\theta \end{pmatrix}$$

where:

$$A = 2\sqrt{2} G N_e E$$

and G is the Fermi constant, N_e is the density of electrons in the matter traversed by the neutrinos and E is the neutrino energy. This can, again, be rewritten as:

$$\frac{1}{2}(\Sigma + A)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} A - \Delta\cos 2\theta & \Delta \sin 2\theta \\ \Delta \sin 2\theta & -A + \Delta \cos 2\theta \end{pmatrix}.$$

Here, again, the unit matrix has no effect on the mixing. However, the important ratio is now: $\frac{\Delta \sin 2\theta}{2(\Delta \cos 2\theta - A)}$. it is clear that we now have a new situation in which the parameter A plays a crucial role. We can easily see that the effective mixing angle in matter θ_m obeys:

$$\sin^2 2\theta_m = \frac{\Delta^2 \sin^2 2\theta}{(A - \Delta\cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}.$$

For $A = 0$ we clearly obtain $\theta_m = \theta$. However, in the very special situation in which

$$A = \Delta \cos 2\theta$$

we find $\sin^2 2\theta_m = 1$, yielding maximal mixing, regardless of the value of θ, as long as $\theta \neq 0$. These are the "resonant" neutrino oscillations in matter[23] which can actually convert a pure neutrino beam of one type into a beam dominated by the other type of neutrinos even for an extremely small vacuum mixing angle, as long as the process is adiabatic.

The immediate potential practical application of this idea is the case of the missing solar neutrinos. It is well known that only a small fraction of the expected flux of solar neutrinos is detected on earth. There are several possible explanations for this, including modifications of the model of the solar interior, neutrino decays and several other exotic proposals. The experiment itself must still be verified by an independent apparatus. However, all of the above explanations are not very likely to solve the puzzle. The resonant neutrino oscillations in matter provide us with yet another potential explanation of the solar neutrino puzzle. It turns out[24] that, for a small vacuum mixing angle θ, the solar mass density and the neutrino energies are such that neutrinos with energies above several $MeV's$, originating in the solar interior, may well undergo resonant neutrino oscillations, converting from ν_e to ν_μ or ν_τ. In that case, the number of $\nu_e's$ observed on earth will be smaller than expected, in detectors which are sensitive to these higher energy neutrinos. On the other hand, other planned detectors such as the Gallium detector, are sensitive to lower energy neutrinos, enabling them to observe the full neutrino flux without suffering resonant oscillations.

In the $\Delta - \theta$ plane, three general solutions were found, all leading to observable effects in the flux of solar neutrinos. The first solution[24] assumes a small θ in vacuum, yielding $\Delta \sim 10^{-4}\ eV^2$ and, for, $m_2 \gg m_1$:

$$m(\nu_2) \sim 0.01\ eV.$$

The second solution[25] allows arbitrary values of θ and leads to Δ values ranging from $10^{-4}\ eV^2$ (for small θ) to $10^{-7}\ eV^2$ (for large θ). The resulting allowed range of $m(\nu_2)$ is between 10^{-2} and $10^{-4}\ eV$. The third solution[26] corresponds to large vacuum mixing angles and is physically much less surprising and less appealing than the other two solutions.

In the above results, ν_2 could be either ν_μ or ν_τ, depending on the neutrino type whose mass obeys the condition for resonant oscillations at the relevant matter densities and neutrino energies. Future solar neutrino experiments will be able to test this hypothesis and to distinguish among the various solutions.

IV.3 Neutrino Masses

We have no experimental evidence for non-vanishing neutrino masses. As outlined above, we do have several theoretical arguments for expecting such masses. We now proceed to review the various sources of information concerning the question of the masses of the three known generations of neutrinos.

Experimentally, we know that:

$$m(\nu_e) < 40\ eV; \qquad m(\nu_\mu) < 250\ keV; \qquad m(\nu_\tau) < 70\ MeV.$$

In the case of the ν_e mass there is one claim[27] of an observed nonvanishing mass of the order of 30 eV. Several recent experiments[28] are beginning to contradict this claim and the issue should be resolved within the next couple of years.

If all three neutrinos are stable, we have a cosmological limit[29] on their masses. Assuming that the neutrinos cannot contribute to the universe *more* than its *total* energy density, one can show that all masses of stable neutrinos should be below $O(100\ eV)$. This is not far from the present direct experimental limit on ν_e but it is approximately six orders of magnitude below the present limit for ν_τ.

The above limit does not apply to unstable neutrinos. In such a case, cosmological considerations give only a constraint relating the lifetime and the mass of an unstable neutrino.[30] By combining this constraint with theoretical estimates of the decay amplitudes of the relevant neutrino, it is possible, under fairly general assumptions,[31] to exclude neutrino masses above 100 eV.

On the other hand, if the "see-saw" mechanism[21] is in effect and if its origin is a Grand Unified Theory with a typical energy scale of $\Lambda \sim 10^{15}\ GeV$, we expect the neutrino masses to be much smaller. For instance, assuming the above value of Λ and the relations:

$$m(\nu_i) = \frac{[m(\ell_i)]^2}{\Lambda}$$

we expect:

$$m(\nu_e) \sim 10^{-13}\ eV;\ m(\nu_\mu) \sim 10^{-8}\ eV;\ m(\nu_\tau) \sim 10^{-6}\ eV.$$

In this case it is unlikely that we can ever see effects of nonvanishing neutrino masses. It is also unlikely that the solar neutrino puzzle has any relation to the resonant matter oscillations. We can reverse the argument and ask what should be the value of Λ which, through the see-saw mechanism, could yield the necessary mass range for resonant neutrino oscillations which solve the solar neutrino puzzle. The answer depends, of course, on whether the ν_e converts into ν_μ or into ν_τ as well as on the vacuum value of θ. The obtained range of Λ-values for all of these cases is between 10^9 and $10^{13}\ GeV$. This is a "no-man's-land" for most "beyond standard" theories with the possible exception of some String models which would like an intermediate energy scale half way (on a logarithmic scale) between the Planck scale and the weak interaction scale.

Another interesting question is how the ratios among the neutrino masses relate to the mass ratios of charged leptons. The simplest see-saw models seem to predict:

$$\frac{m(\nu_i)}{m(\nu_j)} = \left[\frac{m(\ell_i)}{m(\ell_j)}\right]^p$$

where $p = 2$ for a model in which all generations have the same Majorana masses and $p = 1$ for models in which Majorana masses of different generations are proportional to the Dirac masses of the same generations.

The summary of this somewhat confused discussion is that neutrino masses are likely to be well below their present experimental limits and they probably obey a generation hierarchy which is not very different from that of the corresponding charged leptons. We suspect that the mass of ν_τ will be found somewhere between 100 eV and 10^{-6} eV with the ν_μ and ν_e masses scaled down by the corresponding ratios of the squared masses of the charged leptons. Our "guess" is admittedly extremely poor, as it leaves no less than eight available orders of magnitude for each neutrino mass. However, it excludes six orders of magnitude between the present experimental limit and the top of our predicted range. Unfortunately, we cannot say more, at present, and some of our colleagues would not subscribe even to the above guess.

IV.4 Neutrino Proliferation in SIPH

This is neither the place nor the time to present a detailed discussion of neutrinos and neutrino-like states in SIPH. All we want to state are a few simple observations which may give us a preliminary impression of the complexity of the neutrino spectrum in such a theory. Each generation (or each 27-dimensional representation of E_6) contains five neutrinos. We may characterize each of these five states by their $SO(10)$ and $SU(5)$ representations. We will also mention the $SU(2)_L$ classification, although it is completely determined by the two other parameters. For each neutrino we then list three numbers: (d_{10}, d_5, d_2) for the dimensionality of the representations of the three groups. The five neutrino states in each generation are:

$$\nu_L(16,\bar{5},2),\ N(16,1,1),\ \tilde{H}^0(10,5,2),\ \tilde{\bar{H}}^0(10,\bar{5},2),\ S(1,1,1).$$

In a somewhat more familiar terminology these are a left-handed neutrino, a right-handed neutrino, a Higgsino and its anti-particle and a singlet neutrino. Since we must have at least three generations, we must have at least 15 neutrino states, five per generation. To these we must add several neutral colorless

Gauginos $(\tilde{\gamma}, \tilde{Z}, \tilde{Z}', \tilde{Z}'')$ which may mix with the above states. A complete understanding of the neutrino spectrum would then involve the diagonalization of a mass matrix involving *at least* 19 states, four of which belong to the adjoint representation of E_6 and 15 describing the three generations of matter particles. To these one may wish to add E_6 singlets and possible members of incomplete $27 + \overline{27}$ multiplets. The complete neutrino spectrum is incredibly complicated.

It would appear that such a proliferation would allow unlimited freedom in choosing the mass parameters and in obtaining an appropriate "generalized seesaw" mechanism for the masses of the three observed light neutrinos. However, it appears that this is not the case. Neither the tree approximation nor the one-loop approximation seem to lead to the existence of three very light $SU(2)$-doublet neutrinos. A detailed discussion of this problem[32] is beyond the scope of these lectures.

V. PROBING W AND Z – STANDARD AND BEYOND

V.1 Introduction

The discovery of the W and Z bosons is clearly a great triumph of the standard model. Given the experimental value of $\sin^2\theta_W$ obtained from neutral current experiments, one can predict the masses of W and Z, *including one-loop radiative corrections*[2], to be:

$$M_W = \frac{38.7\ GeV}{\sin\theta_W}; \qquad M_Z = \frac{77.3\ GeV}{\sin 2\theta_W}.$$

This is consistent with present experiments. We expect a much better measurement of M_Z as soon as SLC is ready. The two $\bar{p}p$ colliders at CERN and Fermilab should soon be able to improve the accuracy of the W-mass determination.

There is no direct evidence yet for the WWW and $WWWW$ couplings required by the nonabelian nature of the $SU(2)$ gauge group (except for the obvious existence of an electric charge for W^+ and W^-). In particular, we have no measurement of the magnetic moment of the W. Such measurements are likely to come only from difficult processes such as $\bar{p} + p \to W^+ + \gamma + anything$ at $\bar{p}p$ colliders or from $e^+ + e^- \to W^+ + W^-$ at the second stage of LEP in the 1990's.

"Beyond standard" models may require additional W and Z bosons. Interest in such a possibility has recently increased as a result of the possibility of a single additional Z' in SIPH. It is amusing that such a Z' can still be as light as 130 GeV and not be detected by present experiments. We discuss this in section V.2.

On the other hand, the limits on *right-handed* $W's$ are much stronger. We have a lower bound[33] of approximately 2 TeV for $M(W_R)$. We discuss this issue in section V.3.

Another interesting possibility which was already briefly mentioned above in section III.2 is that of a strongly interacting Higgs particle. In such a case the longitudinal component of the W will also participate in such strong interactions. At energies of many TeV's the longitudinal W may then become the "pion of the weak interaction", being copiously produced[20] in any collision of quarks or leptons.

The present experimental limit on possible substructure of W and Z are still only around 1 TeV, possibly even less. This together with the $O(TeV)$ limits on quark and lepton substructure, still allow for the possibility of detecting such effects within the next decade. We discuss this in section V.4.

V.2 Low Lying Z-Bosons and SIPH

The possibility of adding an additional $U(1)$ gauge group to the standard model can, of course, be studied on its own, without reference to any specific theory at higher energy scales. All we have to do is assume that the complete gauge group is $SU(3)_c \times SU(2) \times U(1) \times U(1)$ and that the extra Z-boson is sufficiently heavy so as not to modify the low-energy predictions of the standard model for all neutral current phenomena which have already been experimentally tested.

In the above scenario, we would have no information on the detailed couplings of the extra Z to quarks and leptons. In other words, the new $U(1)$ charges of all standard model particles cannot be determined without additional information.

Only if we have a "beyond standard" theory at higher energies, which at low-energies yields the new extended gauge group, can we hope to determine the properties of the extra Z-boson. An example of such a theory is the simple version of the heterotic string theory based on $E_8 \times E_8$, with a leftover broken E_6 operating in the four-dimensional world below the Planck scale.

In such a theory, the E_6 gauge symmetry is broken by the so-called "Wilson loop" operators which transform like the adjoint 78-dimensional representation of E_6. In order to see how this leads to additional $Z's$, we may consider the decomposition of the 78 multiplet under $SU(3)_c \times SU(3)_L \times SU(3)_R$. We obtain:

$$78 \rightarrow (3,3,3) + (\bar{3},\bar{3},\bar{3}) + (8,1,1) + (1,8,1) + (1,1,8).$$

Among these, the only ones which can break E_6 and leave $SU(3)_c \times SU(2) \times U(1)$ intact are the (1,8,1) and (1,1,8) operators. However, the particular combination of these operators which breaks the symmetry must conserve the usual $U(1)$ of the standard model. It is easy to see that it must therefore conserve *at least* one additional $U(1)$ within $SU(3) \times SU(3) \times SU(3)$. If it conserves *exactly* one such $U(1)$, it can be chosen as $Y_L + Y_R$ (where $Y_{L,R}$ is the $U(1)$ charge which commutes with $SU(2)$ within $SU(3)_{L,R}$).

The simplest way to achieve this is to suggest that E_6 is actually broken at the Planck scale into $SU(3) \times SU(2) \times U(1) \times U(1)$ and that the latter symmetry persists down to energies well below 1 TeV. In this case we would have an extra low mass Z' boson with well defined couplings to all particles.

Before we briefly address the relevant phenomenological issues , we hasten to add that the above scenario is *definitely not* a necessary consequence of the heterotic string theory. It may be the simplest scenario, but other variations are possible. We have already mentioned that, to begin with, E_6 is not the only possible leftover gauge group. Even if it were, it might be broken to a larger subgroup. In fact, the following possibilities exist for the surviving gauge group *after* E_6 has been broken by the Wilson loop operators;

$$SU(3) \times SU(3) \times SU(3)$$

$$SU(3) \times SU(2) \times SU(2) \times U(1) \times U(1)$$

$$SU(3) \times SU(2) \times U(1) \times U(1) \times U(1)$$

$$SU(3) \times SU(2) \times SU(2) \times U(1)$$

$$SU(3) \times SU(2) \times U(1) \times U(1)$$

All of these groups are subgroups of E_6, and they all contain the standard model group as well as at least *one* additional Z'. The first case corresponds to a GUT and must be further broken by a "normal" Higgs mechanism at a relatively high energy scale. However, in the last four cases the remaining gauge groups may survive down to energies well below the typical GUT scale. In each of these cases we have either one or two extra Z-bosons. Both of them, one of them or none of them may be below 1 TeV. If both are, they may mix and we cannot determine the precise couplings of the lowest lying extra Z. If both of them are at masses well above 1 TeV (say, around the so-called "intermediate mass scale" of 10^{11} GeV) we expect no detectable effects at low energies. Only if the remaining gauge group contains *exactly one* extra Z *and* the symmetry breaking pattern is such that the extra Z is below 1 TeV, we can expect clear experimental signatures of a new neutral Z-boson with well-known couplings to all other particles.

Several authors[34] have analysed the experimental constraints on additional Z-bosons which are presently available or which are expected at the $\bar{p}p$ colliders, SLC, LEP, HERA and the SSC. One can distinguish among three kinds of experimental effects:

(*i*) Direct observation of a Z'. At sufficiently high energies such a particle can be produced both at hadron colliders and at lepton colliders. The present mass limits allow the observation of a Z' even at the Fermilab collider, LEP II and HERA.

(*ii*) Direct contributions of heavier Z'-bosons to specific amplitudes. This will happen in almost any process to which the ordinary Z contributes and the resulting amplitudes and cross sections will be modified.

(*iii*) Indirect effects due to the mixing of a new Z' with the ordinary Z. These could modify the properties of the lowest Z-boson, including its mass, width and other features. These last effects may be the first ones to be observed, once we have an e^+e^- collider at the Z mass.

The search for a possible extra Z is interesting on its own merit, regardless of its possible relation to SIPH.

V.3 Right Handed Weak Bosons

One of the most straightforward extensions of the standard model is the Left-Right symmetric theory (LRS) in which the gauge group is $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In this type of theory all left handed quarks and leptons are in $(\frac{1}{2}, 0)$ representations of the LRS group while their right-handed counterparts are in $(0, \frac{1}{2})$ representations. The usual Higgs field ϕ is in a $(\frac{1}{2}, \frac{1}{2})_0$ multiplet. In the minimal version of the theory[35] we also have an extra $SU(2)$-triplet Higgs field Δ. Because of the left-right symmetry we must then have a Δ_L in a $(1,0)_2$ and a Δ_R in a $(0,1)_2$. Parity is spontaneously broken by giving Δ_L^0 and Δ_R^0

different vev's. We denote:

$$\langle \Delta_L^0 \rangle = v_L; \qquad \langle \Delta_R^0 \rangle = v_R; \qquad \langle \phi \rangle = k.$$

We learn that in order to preserve the standard model predictions at low energies we must have:

$$v_R \gg k \gg v_L.$$

We immediately obtain the following results:

(i) The masses of the additional W and Z are related to v_R.

(ii) The right handed neutrino gets a Majorana mass related to v_R, leading to a "see-saw" mechanism and to a light left-handed neutrino.

(iii) All Higgs fields other than the usual standard model Higgs have masses of order v_R. Some of the additional neutral Higgs particles induce flavor-changing neutral currents.

(iv) The absolute values of right- and left-handed generalized Cabibbo angles are identical, but extra phases appear, leading to CP-violating effects which are smaller than ordinary weak interactions amplitudes by a factor $\left[\frac{M(W_L)}{M(W_R)}\right]^2$.

All of the above features are quite attractive, but the main question remains unanswered: What is the value of v_R and the mass of W_R?

Direct searches for right-handed currents give lower limits of a few hundred $GeV's$ for $M(W_R)$. However, a much stronger limit can be obtained from the $K_S^0 - K_L^0$ mass difference. That limit has an amusing history which is worth reviewing:

(i) The standard model contribution to $\Delta M(K_S^0 - K_L^0)$ is dominated by the "box diagram" involving the exchange of two ordinary $W's$. That diagram gives approximately the correct value for ΔM. The leading additional contribution in the LRS model is an identical diagram, with one of the ordinary "left-handed" $W's$ replaced by a W_R. The success of the standard model calculation means that the new diagram should contribute less than the usual diagram. When the new diagram is inspected superficially, it appears to be suppressed (relative to the usual diagram) only by a factor of $\left[\frac{M(W_L)}{M(W_R)}\right]^2$. There must also be a numerical factor corresponding to the different chiralities involved, but it appears to be of order one. If that were the case, all we would be able to deduce would be:

$$\left[\frac{M(W_L)}{M(W_R)}\right]^2 < 1$$

yielding a useless bound $M(W_R) > M(W_L)$.

(ii) Enter Beall, Bander and Soni.[33] They were the first to calculate the numerical factor preceding $\left[\frac{M(W_L)}{M(W_R)}\right]^2$ in the relative strength of the LRS box diagram and the standard model box diagram. They found that the numerical factor (which was allegedly of order one) was actually 430. The new result, *assuming equal left-handed and right-handed Cabibbo angles*, was:

$$430\left[\frac{M(W_L)}{M(W_R)}\right]^2 < 1.$$

This leads to a much stronger bound for $M(W_R)$:

$$M(W_R) \geq \ 1.7\ TeV.$$

(iii) However, while there are good reasons to expect an equal *magnitude* for the left- and right-handed Cabibbo angles, there is no reason to assume that they are equal in *phase*. Actually, the contribution of the LRS diagram should contain an additional arbitrary relative phase ϕ and the correct result becomes:

$$430 \left[\frac{M(W_L)}{M(W_R)} \right]^2 \cos\phi < 1.$$

Since there is no apriori reason for ϕ to have any specific value, no bound can be derived and the 1.7 TeV result appears to be lost.

(*iv*) The 1.7 TeV bound can be rescued by observing that the CP-violating parameter ϵ gets a contribution from the *same* box diagram involving one W_L and one W_R. That contribution depends on the *same* arbitrary phase ϕ. The LRS contribution to ϵ is:

$$\epsilon_{LRS} = \frac{1}{2\sqrt{2}} 430 \left[\frac{M(W_L)}{M(W_R)} \right]^2 \sin\phi.$$

We can now combine the contributions to ΔM and to ϵ and derive a new bound[36]:

$$430 \left[\frac{M(W_L)}{M(W_R)} \right]^2 < \sqrt{1 + 8\epsilon_{LRS}^2}$$

Assuming that ϵ_{LRS} is not much larger than $\epsilon \sim O(10^{-3})$, we may safely neglect ϵ_{LRS}^2 and recover:

$$M(W_R) \geq \; 1.7 \; TeV$$

(*v*) The above limit is a strict limit valid for any value of ϕ. However, in order to get $M(W_R) = 1.7 \; TeV$ we must actually have $\phi \leq 0.5°$. Such a small value of ϕ is allowed but appears to be unnatural. There is no reason for ϕ to be small. If we arbitrarily assume that ϕ is not very small (say, $\phi \geq 5°$), we immediately obtain[36] a much stronger bound on the mass of W_R:

$$M(W_R) > \; 5 \; TeV$$

This bound is based on reasonable "hand-waving" but is not as solid as the 1.7 TeV bound.

(vi) The contribution of the neutral Higgs particle to ΔM can also be computed. Assuming that it is not larger than the standard model box diagram, we obtain a lower bound on the Higgs mass. It should be[37] somewhere above $5-10\ TeV$. We expect the Higgs mass to be of the same general order of magnitude as $M(W_R)$. Consequently, we conclude again that $M(W_R)$ is likely to be at least around $5-10\ TeV$. The Higgs diagram also contributes to ϵ.

The summary of all of these steps is the following: We definitely know that $M(W_R)$ is above 1.7 TeV. We strongly suspect that it is actually *at least* around 10 TeV. If the LRS diagrams (both the Higgs contribution and the $W_L - W_R$ box) contribute a *substantial* part of ϵ we also know that $M(W_R)$ cannot be much larger than,[38] say, 100 TeV. However, if it provides a negligible contribution to ϵ, W_R could be substantially heavier.

The LRS theory by itself does not provide answers to any of the problems of the standard model, except for the parity problem and possibly the CP and the neutrino mass problems. It certainly does not shed any light on Unification, Substructure, the Fine Tuning problem or the Generation Puzzle. However, LRS is built into some of the more detailed models such as $SO(10)$, E_6 and into some composite models. In some of these models the scale of $M(W_R)$ is determined uniquely.

V.4 Possible Substructure of W and Z

One of the two candidate solutions to the "fine tuning" problem is the suggestion that Higgs particles are composite objects (the other solution being supersymmetry). If Higgses are composite, so are the longitudinal W and Z that are "born" from the Higgs field. It is entirely consistent to assume that these are the *only* composite objects among the particles of the standard model. However, one may also entertain the hypothesis that other particles possess substructure. The prime candidates for such a substructure are the quarks and leptons, and the next candidates may be the W and Z bosons.

The scenario for composite W and Z goes as follows: The $SU(2)$ gauge symmetry is replaced by a global $SU(2)$, guaranteeing that all W and Z couplings obey the usual ratios and that the Weinberg mass relation is preserved. The W and Z consist of some subparticles bound together by a new fundamental interaction (possibly, but not necessarily, a color-like interaction). The usual weak interactions are then induced[39] as residual interactions among the composite quarks, leptons, $W's$ and $Z's$, in analogy to the usual hadronic forces being residual color interactions. Properties such as universality of W and Z couplings can be recovered by assuming Z-dominance of the electromagnetic currents. We may, in fact, use the present known accuracy of $e - \mu$ universality and quark-lepton universality in order to estimate the lowest bound on the mass of a possible excited W or Z. We obtain values around 600 GeV.[40]

An important possible experimental test of a W substructure (or, for that matter, of any deviation from a minimal nonabelian gauge coupling for the W) is the measurement of the anomalous magnetic moment of the W. Experimentally, we have no direct information on κ_W. The strongest *indirect* limit is obtained[41] from the contribution of a $W - \gamma$ loop to the W-mass. Such a contribution would lead to a deviation from the observed accuracy of the Weinberg mass relation (the ρ-parameter) unless we have $\kappa < 0.01$. Direct measurements of κ are unlikely to achieve such a level of accuracy within the next decade.

We have no clear idea about the compositeness scale of the W (if it is indeed composite). However, if that scale is too high above $M(W)$, we will face a new difficult problem. We will not be able to explain why $M(W)$ should be very small compared to its own compositeness scale. The only reasonable solution seems to be that, if W and Z are composite, the relevant energy scale should not be above 1 TeV or so. This is perfectly consistent with the compositeness scale of the Higgs particle according to Technicolor models. It is also consistent with all available experimental information on W and Z and with the experimental limits on quark and lepton compositeness.

If there is a substructure at the 1 TeV scale, a machine like the SSC cannot fail to uncover it.

VI. ON THE FUTURE OF ACCELERATOR HIGH-ENERGY PHYSICS

VI.1 Limits on Pointlike Behavior

All present experimental data are consistent with a "pointlike" behavior of all standard model particles. The phrase "pointlike" refers to the minimal couplings of the standard model Lagrangian. The only corrections to these couplings, allowed by the standard model, are the usual radiative corrections which are calculable and small.

At the same time, one cannot exclude the possibility that some or all of the standard model particles do have some internal structure at distance scales below the ones presently probed by experiments. Technicolor models and composite models for quarks, leptons and W and Z bosons necessarily lead to such a substructure. On the other hand, Grand Unified theories do not allow for any substructure at least up to an energy scale of the order of 10^{15} GeV. String theory insists on a substructure at the Planck scale, but (at least in its present version) does not allow for a deviation from pointlike behavior at lower energies.

In discussing models which suggest substructure,[42] we should distinguish among four possible versions:

(i) The most conservative departure from an overall pointlike behavior is the suggestion that Higgs particles are the *only* composite objects among the standard model particles. This is the approach advocated by the original Technicolor models and it is designed to solve the fine-tuning problem by avoiding fundamental scalars. Since we have no experimental evidence for Higgs particles, we clearly have no direct tests of their possible substructure.

However, theoretical arguments suggest that Higgs compositeness will solve the fine-tuning problem only if the compositeness scale is not too far from 1 TeV. Here one may wish to distinguish between Technicolor schemes in which the new scale must be of the order of 1 TeV and composite Higgs models in which one has more flexibility. In all cases we end up with a scale between 1 and 10 TeV.

(ii) The next logical step may be to assume that not only Higgs particles, but also quarks and leptons are composite. Here the main motivation is to explain the proliferation of parameters and the replication of generations in terms of some common substructure and a smaller number of building blocks. Experimentally, we have several *direct* tests of quark and lepton compositeness. All *model-independent* tests lead to lower bounds of the order of 1 TeV for the compositeness scale. These tests include the measurements of $(g-2)_{e,\mu}$, observed cross sections for $e^+ + e^- \rightarrow e^+ + e^-$ and $e^+ + e^- \rightarrow \mu^+ + \mu^-$ and upper limits on deviations from quark point-like behavior in deep inelastic scattering experiments and in e^+e^- collisions. The summary of all of these limits is that quarks and leptons could still have a typical "radius" of the order of an inverse TeV, without contradicting any known experimental data. There are additional important bounds which are *model-dependent*. The absence of processes such as $\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e, K^0 \rightarrow e^+\mu^-$, $K^+ \rightarrow \pi^+e^-\mu^+$, yields bounds of order 100 TeV. Stronger bounds above 1 PeV can be obtained from $\Delta M(K_S^0 - K_L^0)$. However, these bounds are valid only if the relevant processes are not suppressed or forbidden by the quantum numbers of the model. Since we do not have an accepted convincing description of the generation pattern, we cannot use these limits as a valid estimate on the possible "size" of quarks and leptons. We can only state that if quarks and leptons do have a "size" of order 1 TeV^{-1}, the resolution of the generation puzzle will occur on a different energy scale which is unlikely to be below 1 PeV. A similar conclusion follows from the present bounds on proton decay. If quarks and leptons are composite, either proton decay is exactly forbidden, or it is strongly suppressed by some dynamical mechanism (for instance – it may occur only in third order in the new fundamental interaction) or the compositeness scale is of the order of the usual GUT scale. We may not like this as an attractive theoretical option, but it is perfectly possible[43] that the "size" of quarks and leptons is given by one energy scale, the generation puzzle is solved at a different scale and baryon number violation occurs at yet another scale.

(*iii*) A more radical possibility is to suggest[39] that, not only Higgs, quarks and leptons are composite, but also W and Z. We have briefly discussed this possibility in section V.4 and indicated that, again, the compositeness scale is unlikely to be too different from 1 TeV. Experimentally, this is perfectly possible.

(*iv*) Finally, it is conceivable that *all* particles of the standard model (including the gluon and the photon) have some substructure. In that case it is likely that the relevant scale is not too far from the Planck scale. This is the case in String Theory.

The overall situation is therefore the following: The actual "size" of all Higgs particles, quarks, leptons and W and Z can still be as large as 1 TeV^{-1}. The scale for Higgs, W and Z substructure *must* be around that number while the "size" of quarks and leptons could assume any value above 1 TeV. It is entirely possible that there are several compositeness scales.

VI.2 Cross Sections and Pointlike Behavior

Any two-body reaction $a + b \to c + d$ among four pointlike particles, which is dominated by a direct-channel exchange and takes place at an energy scale well above the masses of the four particles, will follow a high-energy behavior of the form $\frac{1}{E^2}$. Examples include e^+e^- and $\bar{q}q$ collisions leading to lepton pairs, quark pairs, W-pairs etc. The exceptions to this rule are resonances (such as ψ, Υ or Z in e^+e^- scattering), possible new thresholds and other effects signalling the onset of some new physics.

If all standard model particles are pointlike up to the GUT scale or the Planck scale (as is the case *e.g.* in SIPH), we expect that most of the interesting amplitudes above 1 TeV or so will follow the $\frac{1}{E^2}$ behavior. On the other hand, if some particles have a substructure, we expect departures from the $\frac{1}{E^2}$ behavior at energies around the compositeness scale or somewhat below it. Thus, if quarks and leptons have a size of the order of TeV^{-1}, we expect the amplitudes for $e^+ + e^-$ and $q + \bar{q}$ scattering to follow a pattern which is different from the $\frac{1}{E^2}$ pointlike behavior. At energies well above the compositeness scale, we actually expect a more or less constant cross section, in accordance with a simple diffractive picture (assuming that the binding force of the constituents inside quarks and leptons is a sufficiently strong force). For instance, at $E \sim 10\ TeV$ we expect:

$$\sigma(e^+ + e^- \to \mu^+ + \mu^-) = \ 10^{-39} cm^2 \ (for\ pointlike\ behavior);$$

$$\sigma(e^+ + e^- \to \mu^+ + \mu^-) = \ 10^{-33} cm^2 \ (based\ on\ \sigma \sim 2\pi R^2\ with\ R \sim\ TeV^{-1}).$$

The difference between these two possibilities is remarkable, especially if you are planning an experiment at a high energy collider.

It is perhaps too optimistic to expect substructure at energies which are as low as 1 TeV. However, in the absence of such a substructure, we may run out of interesting measurable cross sections in the next few generations of high-energy accelerators.

VI.3 A Wild Extrapolation: Accelerator Experiments in 2030

Historically, the center-of-mass energy of the highest energy accelerators have advanced by one order of magnitude approximately every twelve years. We are presently reaching energies of order 1 TeV (more precisely: We should soon have almost 2 TeV at the Fermilab collider but the "useful" energy of quark and gluon subprocesses is only of the order of, say, 0.5 TeV). The graduate students attending this summerschool are now in their mid-twenties. They will therefore retire approximately around the year 2030. Can we try to predict how accelerator experiments will look at that time?

Extrapolating the progress of accelerators over the last fifty years, we conclude that the highest energy available in the year 2030 should be above 1 PeV (= 10^3 TeV). This would presumably require a linear collider with an entirely new technology which will allow for a much higher acceleration per unit length than anything imaginable now. We all know that to do physics at such energies will require innovations in accelerator physics, new detector technology, new computing capabilities and, of course, a lot of money. However, we wish to address here an entirely different question. It is not enough to have the money and the technology, the accelerator, the detectors and the computers. We need to be able to study interesting physics and that requires a sufficient number of events of the interesting types.

If string theory is right (or if Grand Unified Theories are right), we expect cross sections to continue to reflect a pointlike behavior at energies between 1 TeV and 1 PeV. The cross section for $e^+ + e^- \rightarrow \mu^+ + \mu^-$ at $E = 1$ PeV would then be approximately $10^{-43} cm^2$. Cross sections for other processes such as $e^+ + e^- \rightarrow W^+ + W^-$ will be somewhat larger but of the same general order of magnitude. In order to accumulate, say, only 1000 events per year in any given such process, one will then need a luminosity of $10^{39} cm^{-2} sec^{-1}$(!). One should never underestimate future technology, but such a luminosity really appears to be beyond any reasonable extrapolation. In particular, if we simply scale the preliminary parameters recently discussed as a possibility for an electron collider at the 1 TeV range, [44] we will find that a beam size of the order of $10^{-8} cm$(!) is required for such a luminosity in a linear collider, assuming everything else remains the same as in the 1 TeV parameters.

The situation will, of course, be very different if some substructure is uncovered anywhere between 1 TeV and 1 PeV. All cross sections involving the particles whose substructure is unraveled will become much larger than the above estimates. A substructure at 1, 10, 100 TeV, respectively, would yield $e^+ + e^- \rightarrow \mu^+ + \mu^-$ cross sections of the order of 10^{-33}, 10^{-35}, $10^{-37} cm^2$, respectively, allowing experimentation at much lower luminosities.

Do we have the right to expect that "physics will be good to us"? We do not know. At least in two cases in the past, cross sections ended being much larger than the then current standard predictions. Deep inelastic electron scattering was expected by many to show no events. Hadronic final states in e^+e^- collisions were expected to be few and far between. In both cases, experiments showed that the existence of pointlike constituents led to cross sections which were much larger than otherwise predicted. It is ironic that we now have to base our hopes for a large cross section on the option of "losing" that same pointlike behavior. If that does not happen, the best detectors may not be able to detect anything by the time our present graduate students reach retirement age.

Our agenda for the next few years is then to continue benefiting from existing machines, start exploiting the Fermilab collider, SLC and TRISTAN, continue building LEP and HERA, continue planning the SSC and possible Large Hadron Colliders and Large Linear Colliders, continue to search for the new theory of the physics beyond the standard model, develop new accelerator physics ideas, *and, last but not least, pray for a decent cross section at high energies*!

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