SLAC-PUB-4203
February 1987
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PROPERTIES OF THE EM CASCADE*

A Tutorial Utilizing High Resolution 3D Color Graphics

by

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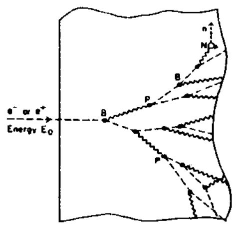
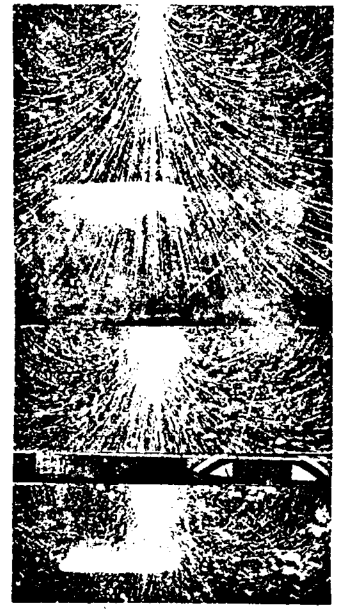


Invited talk presented at the 20th Midyear Topical Meeting on Health Physics Radiation Generating Machines, Reno, Nevada, February 8-12, 1987

*Work supported by the Department of energy contract, DE-AC03-76SF00515.

INTRODUCTION

The following is an electromagnetic cascade shower photographed in a cloud chamber at the California Institute of Technology many years ago (in R. B. Leighton, Principles of Modern Physics (McGraw-Hill, 1959)).



Neutral particles, such as photons, cannot be seen, but the sign and energy of the charged particles can be shown rather nicely by applying an external magnetic field, as in this example.

- A few interesting things can be gleaned from this picture:
  - Starting with just <u>one</u> particle, many more are produced.....and hence the name "shower".
  - But, the particles that are produced have much lower energies (as dictated by energy conservation principles).
  - The particles seem to be produced in "pairs" that are oppositely charged, at least at the start of the shower.
  - The shower can be easily <u>regenerated</u> in the forward direction by simply placing high density material in its path—*e.g.*, the two lead plates shown.
  - But, this regeneration may be caused by neutral particles.....since the charged particles seem to get "swept out" of the forward direction by the magnetic field (look carefully at the upstream and downstream sides of either plate).
  - If the magnetic field were not applied, the shower would be <u>very forward directed</u> indeed.

- The purpose of this lecture is not only to better understand exactly what is taking place in pictures such as this, but also:
  1. to review the various mechanisms of electron-photon transport in general,
  2. to demonstrate some of the problems (and benefits) of EM showers,
  3. and to provide some "rules of thumb" for the accelerator health physicist to use in the field.

- To make it easier to understand what is taking place in the individual interactions—as well as the shower multiplication process itself—we will make use of the EGS4 Monte Carlo program to "simulate events".

- We will show you the events <u>and</u> some showers using newly developed methods:
  - The IBM 5080 Color Graphics Display System.
  - SLAC Unified Graphics coupled to EGS4.

- The rest of this lecture will be as follows:
  1. A review of the fundamental interactions of electrons and photons (the ones that are <u>important</u> to shower physics).
  2. The shower description itself, including a simple model, an advanced analytic model, and Monte Carlo approaches.
  3. Examples of real, as well as simulated, EM showers.
  4. Some "rules of thumb" thrown in along the way.

PHOTON INTERACTIONS

Major, Minor, and Negligible Processes

(Fano's classification scheme)

CLASSIFICATION OF PHOTON INTERACTIONS

Interaction with \ Type of Interaction	ABSORPTION	SCATTERING: ELASTIC (Coherent)	SCATTERING: INELASTIC (Incoherent)
	A	B	C
I ATOMIC ELECTRONS	[Photoelectric Effect] τ_{pe} ~ Z^4 (low energy), ~ Z^5 (high energy)	Rayleigh Scattering $\sigma_R \sim Z^2$ (low energy limit)	[Compton Scattering] $\sigma \sim Z$
II NUCLEONS	Photonuclear Reactions (γ, n), (γ, p), (γ, f), etc. $\sigma_{pn} \sim Z$ ($h\nu \gtrsim 10$ MeV)	Elastic Nuclear Scattering	Nuclear Resonance Scattering
III ELECTRIC FIELD OF SURROUNDING CHARGED PARTICLES	[Pair Production] a. [Field of Nucleus] $\kappa_n \sim Z^2$ ($h\nu \geq 1.02$ MeV) b. Field of Electron $\kappa_e \sim Z$ ($h\nu \geq 2.04$ MeV)	Delbruck Scattering	
IV MESONS	Photomeson Production $h\nu \gtrsim 140$ MeV		

Key: "major" = boxes, "minor" = underlined, "negligible" = others

- Elastic scattering (KE conserved)
- Inelastic scattering (KE not conserved). Note: Compton scattering is inelastic because energy is needed to overcome binding energy of electron to atom; however, Compton scattering kinematics treated as if elastic because B.E. is usually relatively small.

Major Processes & Total Cross Sections

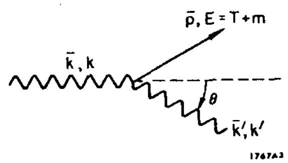
- Mass attenuation coefficient is a measure of the interaction cross section

$$\frac{\mu}{\rho} = \frac{1}{\rho}\left[\tau + \sigma + \sigma_R + \kappa\right] \quad (\mathrm{cm}^2\mathrm{g}^{-1})$$

- Rayleigh (coherent) scattering is shown above because it is usually included in tabulations of μ/ρ, and because it can be important in some radiation transport problems (*e.g.*, scattering of synchrotron radiation inside a beam pipe)
  - — More important at high-Z
  - — 60-70% of Rayleigh scattering confined to small angles: *e.g.*, Pb—4° (1 MeV) & 30° (100 keV)
- Rayleigh scattering not important process in EM showers, but can be important in other radiation transport problems.
- Important γ interactions:
  - — Photoelectric effect
  - — Compton scattering
  - — Pair production
- Most important photon interaction is pair production (*i.e.*, high energy e^+ and e^- make bremsstrahlung)

Compton Scattering

- Incident γ is scattered by loosely bound (*i.e.*, "virtually free") outer-shell electron.
- Inelastic process—at least some energy is required to overcome binding energy of electron.
- Incoherent process—scattering elements (*i.e.*, electrons) are virtually free and scatter independently of one another.
- First order approximation: assume electron is free.....simple two-body kinematics.
- When will this fail? (Answer: low-energy, high-Z).
- Compton kinematics:



$$\frac{1}{k'} - \frac{1}{k} = \frac{1}{mc^2}\left(1 - \cos\theta\right)$$

where k and k' are the incoming and scattered photon energies, θ is the angle of the scattered photon, and mc^2 is the electron rest mass energy.

Compton Scattering (cont.)

- Another way to write formula:

$$k - k' = \frac{\alpha(1 - \cos\theta)}{1 + \alpha(1 - \cos\theta)} = T$$

where $\alpha = k/mc^2$.

- The differential probability, $d\sigma/d\Omega$, for a photon to make a Compton collision is given by the Klein-Nishina formula:

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{r_o^2}{2}\left(\frac{k'}{k}\right)\left[\frac{k}{k'} + \frac{k'}{k} - \sin^2\theta\right] \quad \mathrm{cm}^2\,\mathrm{sr}^{-1}\,\mathrm{elec}^{-1}$$

where θ is the polar angle, r_o is the classical electron radius.

- Compton cross section sometimes denoted as ${}_e\sigma$ to let us know that units are barns/electron (or cm^2/electron). That is, $\sigma = Z\,{}_e\sigma$.
- Nice set of tables and figures by Nelms (NBS) that make it easy to quickly look up energies angles of scattered photons and electrons—great for looking at "trends".
- Azimuthal symmetry—*i.e.*, any angle ϕ about the direction of motion is equally probable.

Compton Scattering (cont.)

- Polar angle dependence:

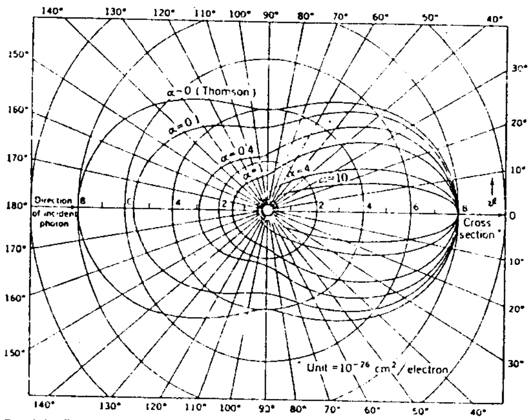


Fig. 2.3 The number of photons scattered into unit solid angle $d(\sigma)/d\Omega$, at a mean scattering angle ϑ, Eq. (2.8). [From Davisson and Evans (D12).]

- Attenuation coefficient (Compton component):

$$\sigma = Z \int \frac{d\sigma}{d\Omega} d\Omega \quad \text{(barns/\underline{atom})}$$

- Absorption Component of Compton Coefficient:

$$\frac{d\sigma_a}{d\Omega} = \frac{d\sigma}{d\Omega} \frac{T(\theta)}{k}$$

where T/k is the fraction of the energy that the electron gets (a "weighting factor"). Integrate to get $\sigma_a = \frac{\bar{T}}{k}$. Easy to show that $\sigma = \sigma_a + \sigma_s$.

Photoelectric Effect

- This process occurs between a photon and an atom (not with an electron).
- Threshold energy (called "binding energy").
- Jump discontinuities in the cross section (called "edges").

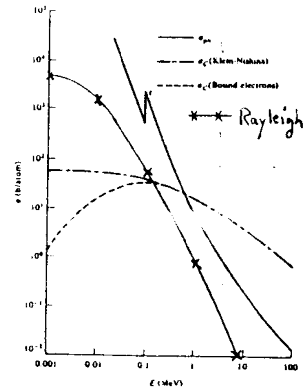


Figure 3.5 Comparison of incoherent-scattering and photoelectric-effect cross sections for photon interactions in lead. Units are barns per atom.

- About 80% of the PE interactions involve K-shell (because most tightly bound, making momentum conservation with nucleus "easier").
- $\tau \approx Z^4/k^3$ for low-k and $\tau \approx Z^5/k$ for high-k.
- What happens after a PE interaction?
  - Outer shell electrons fill inner shells (de-excitation).
  - Followed by emission of a) fluorescent radiation, b) Auger electron, c) both.
  - Fluorescent yield approaches 0 (Auger dominates) at low-Z and 1 (fluorescence dominates) at high-Z.

Pair Production

- Mechanism by which a γ is transformed into an e^+e^- pair (also known as materialization).
- Cannot occur in free space (*i.e.*, vacuum) because it violates conservation of momentum-energy.
- Usually occurs in the field of a nucleus (with $E_{th} = 2m = 1.022$ MeV).
- Can also occur in the field of an electron (with $E_{th} = 4m = 2.044$ MeV) and then given the special name "triplet production". Minor process except at low-Z (*e.g.*, at 50 MeV contribution is 10% in carbon and 1% in lead).
- Attenuation coefficient:

$$\kappa \approx \left(\frac{Z^2}{\ln k}\right) \quad (\text{for low} - \text{k})$$

$$\kappa \approx Z^2 \left(\frac{7}{9X_o}\right) \quad (\text{for high} - \text{k})$$

where X_o is the radiation length for the material (a constant).

- RULE OF THUMB: Mass attenuation coefficient is equal to $7/9X_o$ (cm^2/g) at high energies. If you know μ, you can determine X_o, or vice versa. Once you have X_o, then you immediately know the bremsstrahlung cross section too (*i.e.*, $1/X_o$)!
- Account for triplet production by $Z^2 \rightarrow Z(Z+1)$.
- Angular distribution of pairs involves a very complicated $d^2\sigma/dEd\Omega$, containing a common "scaling" term called the characteristic angle, $\theta_{pair} = m/k$, which can be quite useful (*e.g.*, for 100 MeV, $\theta_{pair} = 0.5/100 \approx 0.3°$).

Pair Production (cont.)

- Energy distribution within the pair:

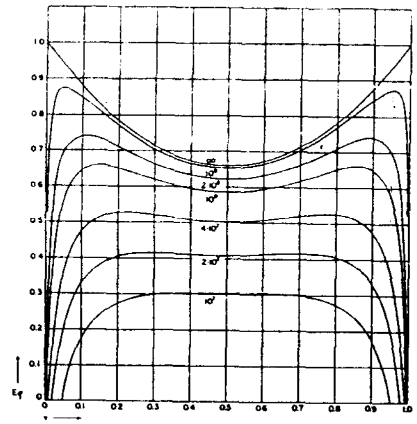
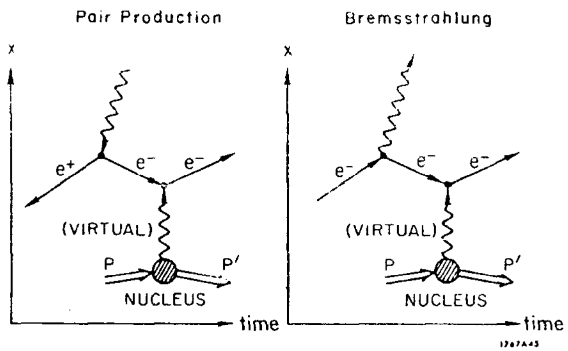


Fig. 2.19.2. Differential probability of pair production per radiation length of lead for photons of various energies. Abscissa: $v = (E' + m_ec^2)/E$; ordinate: $E\varphi_{pair}(E,E')$. The numbers attached to the curves indicate the energy E of the primary photon. From Rossi and Greisen (RB41.1).

- All energy splits kinematically possible.
- Slight trend towards 50-50 split at low energies.
- At high energies, tendency for either the e^+ or the e^- to have most of the energy.
- In general, a broad spectrum. For quick calculations one usually assigns the energy equally to each.

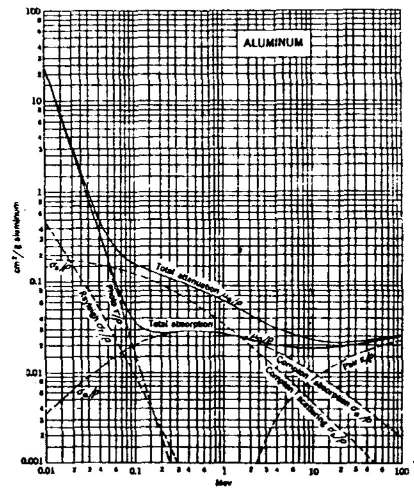
Pair Production (cont.)

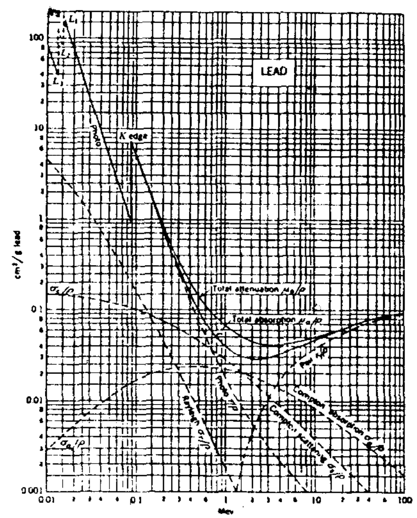
- Important connection between bremsstrahlung and pair production, demonstrated by the following Feynman diagrams:



- Reverse arrowheads and you change particles to their antiparticles: e^+ is simply an e^- going backwards in time.
- Hence, pair production and bremsstrahlung are essentially the same process.
- Pair production is the most important photon process at high energies; therefore, bremsstrahlung will be the most important process for electrons ($\pm$) at high energies.
- Is the basis of Approximation A of analytic shower theory (namely, neglect all the other processes except pair production and bremsstrahlung).

Photon Interaction Summary





Photon Interaction Summary (cont).

- Relative importance of three major interactions:

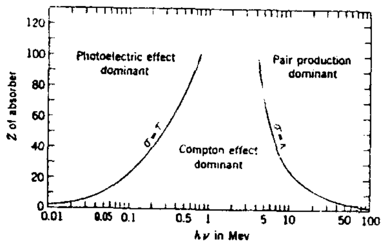


Fig. 1.1 Relative importance of the three major types of γ-ray interaction. The lines show the values of Z and $h\nu$ for which the two neighboring effects are just equal.

- Ask the following questions:
  - — Where is lead?
  - — Where is concrete?
  - — Where is the human body?
- Also note—EM showers will be most important to the right of the $\sigma = \kappa$ line.

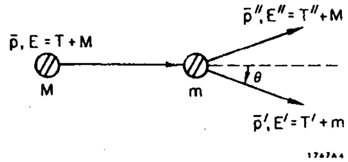
CHARGED PARTICLE INTERACTIONS

Three Major Processes

- Classify by interaction with:
  - — Collision with an atom as a whole.
  - — Collision with an orbital electron.
  - — Radiative collision (*i.e.*, bremsstrahlung).
- Mode of interaction depends on energy and distance of approach:
  - — Large distance (relative to atomic dimensions). Atom reacts to field of passing particle via the Coulomb force. The net result is excitation or ionization of the atom. Called a "soft collision" (as well as "distant collision").
  - — Distance ≈ order of atomic dimensions. Orbital electron reacts to moving charged particle. Called a "hard collision" (as well as "knock-on" process). The secondary electron is called a delta-ray and ionization of the atom certainly results.
  - — Distance smaller than atomic radius. Electric field of nucleus is most important and causes deflection with emission of photon radiation called bremsstrahlung.
- First interaction treated quantum mechanically, the second using classical mechanics....leading to the Bethe-Bloch stopping power formula. Radiation process treated just as in the pair production case (using quantum mechanics).

Hard Collisions

- Two-body kinematics apply because orbital electron is essentially free (*i.e.*, $T' >> B.E.$).



- Recoil kinetic energy of the delta ray is:

$$T' = 2m \frac{p^2 \cos^2\theta}{[m + (p^2 + M^2)^{1/2}]^2 - p^2 \cos^2\theta}$$

where M and p are the rest mass and the momentum of the incident charged particle (electron, muon, proton, alpha, etc.).

- Maximum K.E. transferred to delta ray is:

$$T'_{max} = 2m \frac{p^2}{[m^2 + M^2 + 2m(p^2 + M^2)^{1/2}]}$$

obtained by setting $\theta = 0$.

- Above OK for all charged particles, but we only care about electrons and positrons for EM showers (*i.e.*, M=m).

Hard Collisions (cont.)

- The maximum K.E. transferred to the delta ray by an electron or positron is therefore

$$T'_{max} = T$$

which provides a problem in the case of the electron: *i.e.*, how does one tell the scattered "primary" electron from the delta ray? (it is easy for positrons!)

- Answer: By convention:

$$T'_{max} = \frac{T}{2} \quad \text{(electrons)}$$

whereas

$$T'_{max} = T \quad \text{(positrons)}$$

by kinematics.

- The delta ray is always the electron with the lower of the two energies in a hard collision of two electrons.
- The electron-electron process is called a Møller collision, whereas the positron-electron process is called a Bhabha collision.
- The Møller probability (cross section) is given by:

$$\phi_{col}(T, T')\, dT' = 2Cm \frac{T^2}{(T - T')^2 (T')^2} \left[1 - \frac{T'}{T} + \left(\frac{T'}{T}\right)^2\right]^2$$

Hard Collisions (cont.)

- The Bhabha probability (cross section) is given by a similar expression (*e.g.*, see various textbooks: Evans, or Kase & Nelson, or Rossi, or Fitzgerald, Brownell & Mahoney).

- The important thing to observe is that all such equations (including heavy charged particles) contain a common factor. That is, for $T' << T'_{max}$ we get what is called the Rutherford formula:

$$\phi_{col}(T,T')\,dT' = \frac{2Cm}{\beta^2}\frac{dT'}{T'^2} \quad (\text{cm}^2\text{g}^{-1})$$

where

$$C = \frac{\pi N_o Z r_o^2}{A} = 0.150\frac{Z}{A} \quad (\text{cm}^2\text{g}^{-1})$$

and where the units are now shown (note: same as the photon attenuation coefficient, μ).

- What's important to note here is that
  - Cross section goes as Z/A; hence, mass stopping power will also go as Z/A.
  - Energy transfer to delta rays goes as $\frac{1}{T'^2}$; hence, low-energy delta ray production is much more probable than high-energy (*e.g.*, 10 MeV delta rays are produced 100 times less abundantly than 1 MeV delta rays).

Collision Loss—Soft and Hard

- Two contributions to the total picture of collision loss:

$$\left.\frac{dT}{dx}\right|_{col} = \left.\frac{dT}{dx}\right|_{col}^{S} + \left.\frac{dT}{dx}\right|_{col}^{H}$$

  - First component is due to "soft" (aka "distant") collisions (calculated by Bethe using quantum mechanics) and provides all of the excitation contribution and some of the ionization.
  - Second component is due to "hard" (aka "close") collisions and gives the remainder of the ionization contribution. This is also where all the delta ray energy transfer is located.

- The average mass stopping power is obtained in a very straightforward manner using:

$$\left.\frac{dT}{dx}\right|_{col} \equiv \int_{T_{min}}^{T_{max}} T'\phi_{col}(T,T')\,dT'$$

$$= \int_{T_{min}}^{H} T'\phi_{col}^{S}(T,T')\,dT' + \int_{H}^{T_{max}} T'\phi_{col}^{H}(T,T')\,dT'$$

- The first integral is done for us (for all particle types) by Bethe. The second integral we perform for each particle type and for whatever T_{max} we wish.

- For the unrestricted stopping power we choose $T_{max} = T$ (e^+) and $T_{max} = T/2$ (e^-).

Stopping Power & LET

- The unrestricted mass stopping power for electrons and positrons is then:

$$\left.\frac{dT}{dx}\right|_{col}^{Unres} = \frac{2Cm}{\beta^2}\left\{\ln\left[\frac{\tau^2(\tau+2)}{2(I/m)^2}\right] - F^{\pm}(\tau) - \delta\right\}$$

where $F^{\pm}$ are simple algebraic expressions involving the ratio, $\tau = T/m$ (*e.g.*, see Eqns. 3.15 and 3.26 of Kase & Nelson).

- The quantity δ is a subtractive term that corrects for the polarization of the medium (aka the density effect).
- The restricted mass stopping power is now fairly easy to explain—one simply "restricts" the upper limit of integration in the second integral above. That is, if we let

$$T_{max} \rightarrow T_{esc} = m\Delta$$

be the K.E. of a delta ray which just escapes the region of interest, we can obtain another formulation (*e.g.*, see Eqns. 3.17-19 of Kase & Nelson) such that

$$\left.\frac{dT}{dx}\right|_{col}^{Res} \leq \left.\frac{dT}{dx}\right|_{col}^{Unres}$$

- The region of interest, of course, can be an ion chamber, a cell, a tumor, etc.—*i.e.*, by selecting a value for Δ, we are "localizing" or "restricting" the energy loss to regions surrounding the track of the moving charged particle.

Stopping Power & LET (cont.)

- So what is LET?
  - Numerically, LET and stopping power are the same—they are calculated using the formulas discussed above, albeit LET is usually expressed in units of MeV/cm (*i.e.*, linear stopping power).
  - LET_{Δ} is the restricted stopping power and LET_{∞} is the unrestricted stopping power.
  - The ∞ subscript is symbolic and denotes the fact that the K.E. of the delta ray takes on its maximum:

$$\Delta \rightarrow \frac{T_{max}}{2m} \quad \text{(electrons)}$$
$$\Delta \rightarrow \frac{T_{max}}{m} \quad \text{(positrons)}$$

- Or, as one leading expert has stated:

> *"...the term LET was intended to draw attention to the energy deposited in the medium rather than that lost by the charged particle and in addition was intended to apply to energy losses that could be considered as 'local'."* H. H. Rossi in **Radiation Dosimetry. Volume I** (p.47)

Bremsstrahlung

- Caused by deceleration of the charged particle under the influence of the electric field of the nucleus.
- Generally get a change in direction.....albeit very small. To be more specific, $d^2\sigma/dEd\Omega$ is "scaled" in terms of a characteristic angle, $\theta_{brem} = m/E$, where E is the energy of the incident electron. (*e.g.*, for 100 MeV, $\theta_{brem} = 0.5/100 \approx 0.3°$).
- Remember the Feynman diagrams relating bremsstrahlung and pair production?
- Influenced by distance from nucleus:

— No screening case:

$$10^{-13}\text{cm (nucleus)} < \text{distance} < 10^{-8}\text{cm (atom)}$$

(electric field not affected by atomic electrons).

— Complete screening case:

$$10^{-8}\text{cm (atom)} < \text{distance}$$

(electric field most affected by atomic electrons).

- The higher the energy the more the screening, so that the complete screening formula for the radiation cross section usually applies at the energies involved in EM showers.

Bremsstrahlung (cont.)

- Consider the high energy cross section (complete screening)

$$\phi^n_{rad}(T,k)\,dk = 4\alpha\frac{N_o}{A}Z^2r_o^2\frac{dk}{k}\left\{\left[1+\left(\frac{E'}{E}\right)^2 - \frac{2}{3}\frac{E'}{E}\right]\right.$$
$$\left.\times\left(\ln 183Z^{-1/3}\right)+\frac{1}{9}\frac{E'}{E}\right\} \quad (\text{cm}^2\text{g}^{-1})$$

where $E - E' = k$ (Note $\frac{1}{k}$-dependence).

- The average radiative mass stopping power is obtained from (assuming $T \approx E >> m$):

$$\left.\frac{dT}{dx}\right|_{rad} \equiv -\int_0^T k\phi^n_{rad}(T,k)\,dk \qquad (\text{MeV cm}^2\text{g}^{-1})$$
$$= -4\alpha\frac{N_o}{A}Z^2r_o^2\left[\left(\ln 183Z^{-1/3}\right)+\frac{1}{18}\right]T$$
$$= -KT$$

where K is a constant. This equation integrates to

$$T(x) = T_o e^{-Kx}$$

If we set the above equation equal to $T_o e^{-1}$ and solve for x, we get the distance in which the electron loses 1/eth its energy due to bremsstrahlung processes (only)—*i.e.*, a definition called the radiation length.

Bremsstrahlung (cont.)

- Formula for radiation length, X_o:

$$\frac{1}{X_o} = 4\alpha\frac{N_o}{A}Z^2r_o^2\left[\left(\ln\ 183Z^{-1/3}\right)+\frac{1}{18}\right]$$
$$(\mathrm{g\,cm^{-2}})$$

- Very important shower concept:
  1. Bremsstrahlung—most important process for HE electrons with distance measured in radiation lengths.
  2. Pair production—most important process for HE photons.
  3. Bremsstrahlung and pair production are same process. Hence, pair production distance must also be measured in radiation lengths.
  4. Therefore, EM showers are best measured in radiation length units.

- $Z^2 \rightarrow Z(Z+1)$ in all equations, to account for bremsstrahlung in the field of an electron.

- Collision *vs* radiative loss:

$$\left.\frac{dT}{dx}\right|_{col} \approx \frac{Z}{A}\ln T$$
$$\left.\frac{dT}{dx}\right|_{rad} \approx \frac{Z^2}{A}T$$

Bremsstrahlung (cont.)

- Collision processes dominate at low energies and radiative processes dominate at high energies.

- The cross-over point is called the critical energy, ϵ_o—*i.e.*, that energy where radiative and collision losses are equal.

- Z-dependence (approximate formula):

$$\epsilon_o \approx \left(\frac{800}{Z+1.2}\right)\quad \text{(MeV)}$$

- Useful critical energies and radiation lengths (crude):

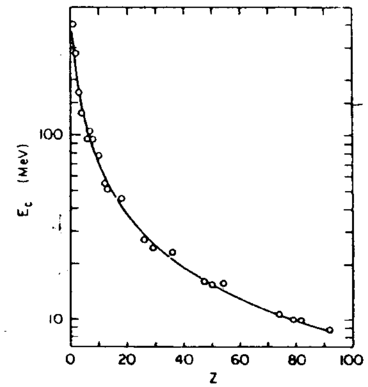
Water (or Air) $\begin{cases}\epsilon_o = 100 & \text{MeV}\\ X_o = 36 & \text{cm}\end{cases}$

(scale to air by density)

Al $\begin{cases}\epsilon_o \approx 60 & \text{MeV}\\ X_o = 9 & \text{cm}\end{cases}$

Fe (or Cu) $\begin{cases}\epsilon_o = 30 & \text{MeV}\\ X_o = 1.5 & \text{cm}\end{cases}$

Pb $\begin{cases}\epsilon_o = 10 & \text{MeV}\\ X_o = 0.56 & \text{cm}\end{cases}$



- Interesting observation: Critical energy tells us approximately what energy to expect the "onslaught of EM cascade showers"

Above 10 MeV in a high-Z Clinac target, but not really until 100 MeV in humans.

Bremsstrahlung (cont.)

- Another interesting observation: Photoneutron production (and photoactivation) starts at a threshold (*e.g.*, $E_{th} \approx 7 - 20$ MeV). But, the EM shower doesn't really "get going" until beam energies near the critical energy.

  If your copper beam stopper (e.g., a Faraday cup) for your 20 MeV linac is getting activated, then maybe its time to switch to an aluminum one!

- Fractional energy loss (per r.l.)—if we define $t = x/X_o$, then from a previous equation:

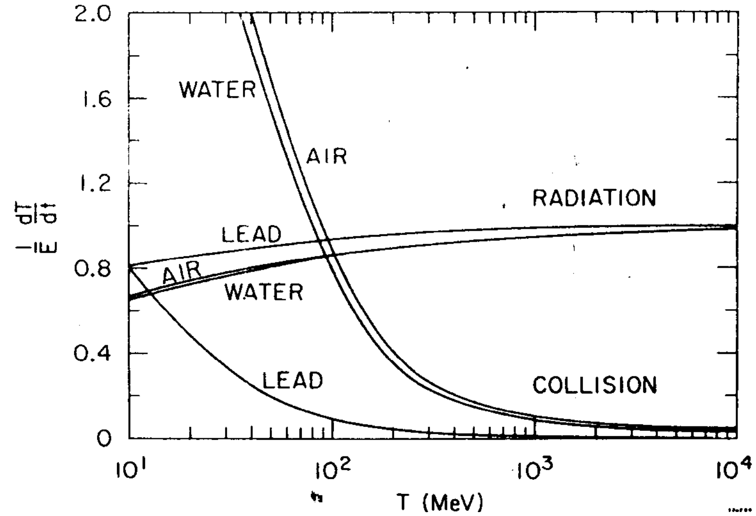
$$\frac{1}{T}\frac{dT}{dt}\bigg|_{rad} = \frac{1}{T}\frac{dT}{dx}\bigg|_{rad}\frac{dx}{dt} = \frac{1}{T}KT\frac{1}{X_o} = 1$$

which tells us:

...at high energies (e.g., > 100 MeV), where virtually all the energy losses are due to radiative processes, the fractional energy loss per radiation length is independent of absorbing material and particle energy, and in fact is almost identical to 1

Bremsstrahlung (cont.)

Fractional energy losses—collision *vs* radiative:



- Remaining processes—multiple scattering and positron annihilation.
  - Annihilation process......positron interacts with and electron, either at rest or in-flight ($\leq$ 10% of the time), and the result is two 0.511 MeV photons.
  - Multiple Coulomb scattering takes a little more discussion (next topic).

Coulomb Scattering

- When a charged particle passes in the neighborhood of a nucleus, it undergoes a change in direction, referred to as Coulomb scattering.
- Usually considered to be an elastic process because small probability of photon being emitted with energy comparable to KE of charged particle.
- Nucleus is much heavier than incident electron—acquires momentum but not significant KE.
- Contribution to scattering from atomic electrons is relatively small (10% for Al, 1% for Pb)—even though collisions with atomic electrons can account for a large share of the energy loss.
- General description of elastic scattering:
  - — Most scattering interactions result in very small deflections.
  - — Small net deflections are generally the result of a large number of very small deflections—*multiple scattering* component.
  - — Large net deflections are the result of a single large-angle scatter......superimposed on a number of very small deflections—*single scattering* component.
  - — *Plural scattering*—the intermediate case, connecting the other two components.

Coulomb Scattering (cont.)

- The Rutherford (single) scattering formula:

$$P(\theta)\, d\Omega = 4N_o \frac{Z^2}{A} r_o^2 \left(\frac{m}{p\beta}\right)^2 \frac{d\Omega}{\theta^4} \quad (\mathrm{cm}^2\mathrm{g}^{-1})$$

derived under the assumption that θ is small (*i.e.*, $\sin\theta \approx \theta$), although θ is still big when compared to multiple scattering (a summation of very small angles). Note also that the above equation is not defined at $\theta = 0$.

- Fermi-Eyges multiple scattering theory:
  - — Also known as Gaussian scattering (we shall see why).
  - — Start with the Fermi diffusion equation (beam along x):

$$\frac{\partial F(x,y,\theta_y)}{\partial x} = -\theta_y \frac{\partial F}{\partial y} + \frac{1}{W^2}\frac{\partial^2 F}{\partial \theta_y^2}$$

  where $W = 2p\beta/E_s$ and where $F(x,y,\theta_y)dy\, d\theta_y$ is the number of particles at x having lateral displacement (y, dy) and traveling at a projected angle $(\theta_y, d\theta_y)$.
  - — Symmetry about x-direction (*i.e.*, above equation could have been written $F(x,z,\theta_z)$ as well).
  - — Solved under the assumption of continuous energy loss. Collision (but not radiative) energy loss enters above equation via W in the form of range-energy (*i.e.*, $p\beta$) expressions (or tables).
  - — The Fermi-Eyges theory is a much heralded "tool" in medical physics nowadays, appearing in the literature in the form of the pencil-beam approach to patient dosimetry.

Coulomb Scattering (cont.)

— Angular distribution solution:

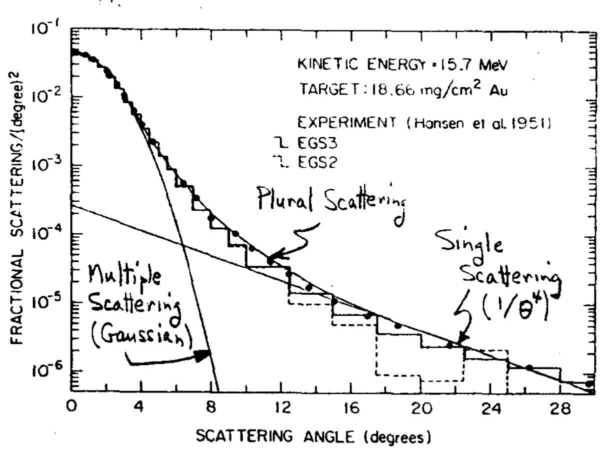
$$G(x,\theta_y)\,d\theta_y = \int_y F(x,y,\theta_y)\,dy\,d\theta_y$$
$$= \frac{1}{\sqrt{4\pi\theta_o^2}}\,e^{-\theta_y^2/2\theta_o^2}\,d\theta_y$$

where $\theta_o = 15\sqrt{x}/p\beta$ is the rms (projected) scattering angle, with $p\beta$ in MeV/c and x in radiation lengths.

— Note that we got a Gaussian as promised.

— For fast electrons $p\beta \rightarrow T$, which is the case for most of the charged particles in the EM shower.

- A plot of the overall Coulomb scattering picture:



- A theory that contains all three features of scattering, called the Molière formulation, is quite often used in Monte Carlo work.

EM CASCADE SHOWERS

Overall Description of an EM Shower

"...in the interactions of high-energy electrons with matter only a small fraction of the energy is dissipated, while a large fraction is spent in the production of high-energy photons. The secondary photons, in turn, undergo materialization or Compton collision. Either process gives rise to electrons of energy comparable with that of the photons. These new electrons radiate more photons, which again materialize into electron pairs or produce Compton electrons. At each new step the number of particles increases and their average energy decreases. As the process goes on, more and more electrons fall into an energy range where radiation losses cannot compete with collision loses, until eventually the energy of the primary electron is completly dissipated in excitation and ionization of atoms".

B. Rossi, HIGH-ENERGY PARTICLES (Prentice-Hall, NY, 1952).

- We will look at some of the radiation transport approaches to the EM shower problem, including:
  — Analytic shower theory—a simple model to give the general features of cascades AND a terse introduction to the diffusion equation techniques.
  — Monte Carlo methods.
- Then we will look at some practical examples from the laboratory.

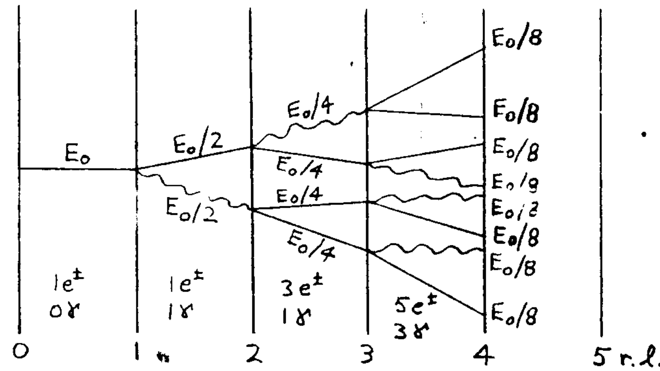
Analytic Shower Theory

- Logitudinal *vs* lateral spread:
  - — Bremsstrahlung and pair production angles are very small.
  - — Scattering angles small too (at least for low-Z materials).
  - — Shower develops essentially in the direction of the incident particle.
  - — Longitudinal development can be treated separately from lateral spread (the so-called "straight-ahead" approximation).
- Average behavior of showers of interest in shielding, dosimetry, heat deposition, radiation damage, etc.
- Statistical fluctuations about the average is of prime importance in detector design.
- Even the average behavior of showers represents a difficult mathematical problem (just to mention some of the early pioneers: Oppenheimer, Bhabha, Heitler, Landau, Serber, and Tamm).
- Approximations are necessary:
  - — Approximation A: Neglect collision processes and Compton effect and use asymptotic formula to describe bremsstrahlung and pair production.
  - — Approximation B: Same as Approximation A except that collision losses taken into account (a constant energy dissipation term).

Analytic Shower Theory (cont.)

- A very simple (but instructive) model

Consider the following sketch:



- — Assume that each electron (±) of energy greater than the critical energy, ϵ_o, goes exactly one radiation length.....and then makes a bremsstrahlung photon.
- — Let the scattered electron and the new photon each share the energy of the incoming particle.
- — Let each photon make pairs at the second radiation length and again split the energy equally, etc., etc.
- — After t(r.l.), the total number of particles will be:

$$N = N_{e^\pm} + N_\gamma = 2^t = e^{t \ln 2}$$

and the energy of each particle is:

$$E = E_o 2^{-t}$$

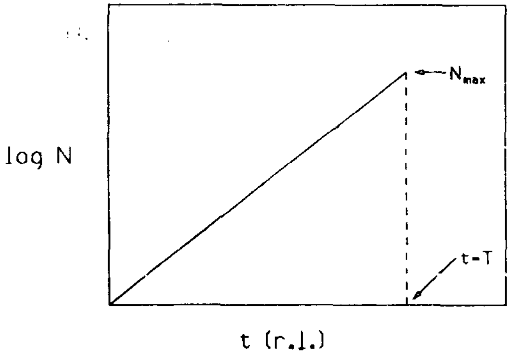
Analytic Shower Theory (cont.)

- Thus, the total number of particles with energy greater than E will increase exponentially with t from unity (at $t = 0$) to some maximum at

$$T(E) = \frac{\ln(E_o/E)}{\ln 2}$$

and then drop abruptly to zero. The number of these particles at the maximum will be:

$$N_{max} = E_o/E$$



- This discontinuous character is a consequence of the assumptions— *i.e.*, the model is too simple (would expect a smooth dependence of N on t).

Analytic Shower Theory (cont.)

- Nevertheless.....the general features described by this simple model are expected to be approximately correct—*i.e.*, for energies greater than E:

1. Initially, $N_{e^\pm}$ or N_γ increases exponentially with t.
2. $N_{e^\pm}$ or N_γ goes through a maximum such that $t_{max} \propto \ln(E_o/E)$ (*i.e.*, t_{max} increases slowly with primary energy).
3. $N_{e^\pm}$(or N_γ) $\propto E_o/E$

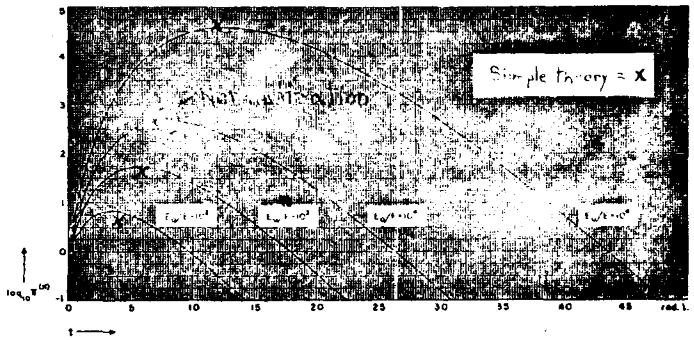


Fig. 5.10.1. The number of electrons of energy greater than E in a shower initiated by an electron of energy E_0, $\Pi^{(\pi)}(E_0,E,t)$, as a function of t. Computed for various values of E_0/E according to Approximation A. From Rossi and Greisen (RB41.1).

4. The shower curve for the total number of particles, irrespective of energy, has a maximum at a thickness of about $\ln(E_o/\epsilon_o)$.
5. Correspondingly, the total number of particles is approximately proportional to E_o/ϵ_o.

Analytic Shower Theory (cont.)

- Diffusion equations (some notation taken from B. Rossi). In what follows, the symbol γ stands for "photon" and π stands for "electron" (either + or -).

$$\pi(E',t) = \text{no. electrons in } dE' \text{about } E' \text{at } t$$
$$\gamma(E',t) = \text{no. photons in } dE' \text{about } E' \text{at } t$$

$$\phi_{\gamma\pi}(E',E) = \text{prob. per r.l. for photon of energy } E' \text{ to produce electron of energy } E \text{ in } dE$$
$$= 2\phi_{\text{pair}}(E',E) + \phi_{\text{com}}(E',E'-E)$$

$$\phi_{\pi\pi}(E',E) = \text{prob. per r.l. for electron of energy } E' \text{ to produce electron of energy } E \text{ in } dE$$
$$= \phi_{\text{rad}}(E',E'-E) + \phi_{\text{col}}(E',E)$$

$$\mu_\gamma(E) = \text{attenuation factor for photons}$$
$$= \mu_{\text{pair}}(E) + \mu_{\text{com}}(E)$$

- The differential equations that use this notation assume that one is only interested in the one-dimensional development of the shower (*i.e.*, no angular variables are involved). This is essentially the straight-ahead approximation.

Analytic Shower Theory (cont.)

- Diffusion equations (cont.):

$$\frac{\partial\pi(E,t)}{\partial t} = -\pi(E,t)\mu_\pi(E) + \int_E^\infty \pi(E',t)\phi_{\pi\pi}(E',E)\,dE' + \int_E^\infty \gamma(E',t)\phi_{\gamma\pi}(E',E)\,dE'$$

$$\frac{\partial\gamma(E,t)}{\partial t} = \int_E^\infty \pi(E',t)\phi_{\pi\gamma}(E',E)\,dE' + \int_E^\infty \gamma(E',t)\phi_{\gamma\gamma}(E',E)\,dE' - \gamma(E,t)\mu_\gamma(E)$$

- A pair of coupled integro partial differential equations that are solved using Mellin and Laplace transformation techniques (very messy indeed!).
- Solutions of the above are, at best, applicable to very limited physical situations. Just too many approximations are involved in setting up and solving them.
  - — Longitudinal only—no angular or radial information.
  - — Not all processes are accounted for.
  - — Only high energies cross sections are used.
- However, sometimes the solution of these equations is indeed quite useful—*e.g.*, photon track lengths.

Analytic Shower Theory (cont.)

- Track lengths and neutron yields:

— Photoneutron production is a relatively minor process within the EM cascade shower. Nevertheless, neutron production and radioactivation are of concern to the health physicist.

— Yields from targets can be obtained by integrating the photoneutron cross section and photon track length over the photon energy:

$$Y(E_o) = \frac{N_o \rho}{A} \int_0^{E_o} \sigma(k) \frac{dL(k)}{dk} \, dk$$

where N_o, ρ, and A are Avogadro's number, density, and atomic number, respectively, and E_o is the incident electron beam energy.

— For very thick targets the differential photon track length is obtained from the diffusion equations (Approx. A):

$$\frac{dL}{dk} = 0.572 \frac{X_o E_o}{k^2}$$

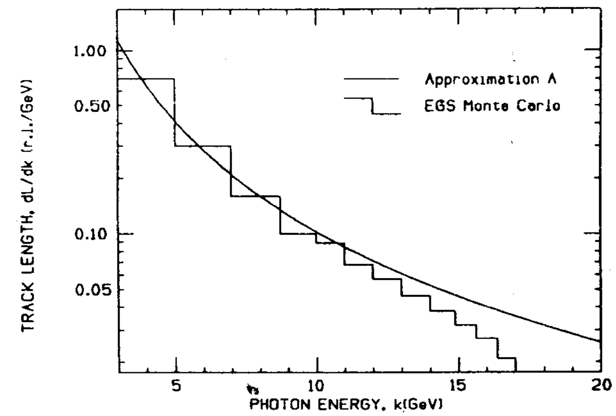
where L is in the same units as the radiation length, X_o.

- When using Approximation A, one assumes that the target is effectively "infinitely" thick—*i.e.*, all of the energy E_o is assumed to be deposited in the target.

- Approximation A works best at high energies.

Analytic Shower Theory (cont.)

- Comparison of Approximation A track length with EGS Monte Carlo results:



- The subject of photoneutron yields using Approximations A and B are discussed in great detail by Swanson (Health Physics 35 (1978) 353).

SUMMARY

- What we have tried to show in the formal part of this lecture is that electromagnetic cascade showers are governed by a number of important physical processes—the electron-photon interactions—which feed back and forth to each other in a multiplicative way.
- The remainder of this lecture will consist of a slide presentation showing various manifestations of EM showers.
- A bibliography is given at the end of these notes.

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