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THE MANY DIMENSIONS OF T. D. LEE*

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Approximately thirty years ago, at about the time of the downfall of parity, or should I say its elevation to the more fundamental but only approximate symmetry of CP, the great American novelist and playwright Thorton Wilder turned sixty. On that occasion Supreme Court Justice Felix Frankfurter sent him a wire with the following message: "Welcome to the great decades". Today I also say:

> "Welcome T.D. to the great decades! Having beaten you there by two months, I can report that—at least so far—it's o.k. And there are some special advantages to look forward to. In a few years we can go to the movies at a cut rate and ride the buses and subways at half fare."

There are two very special pleasures of being a physicist: first of all, we enjoy sailing on one of the great adventure voyages of the human mind as we seek to discover what we are made of and what holds us together. And, secondly, we sail on this great adventure in the company of such wonderful friends, such marvelous people as the community here today; and it is a special pleasure to count Tsung Dao and Jeannette Lee as one's close friends.

Occasions like today have elements of fun as well as of seriousness. They provide great opportunities and wonderful excuses for reminiscing—which I will now proceed to do.

For scientific reminiscing, let me remind you of what the world was made of—or so we thought when T.D. and others of us were graduate students forty years ago. The basic nuclear glue was known. Sometimes we called it mesons and sometimes mesotrons. And I recall sitting in the University of Illinois Union for afternoon coffee with my two professors, Sid Dancoff and Arnold Nordsieck, the day the news arrived that the nuclear and cosmic ray mesons, the π and μ, were different—which led Nordsieck to suggest that we view this new development with great caution because one of those two presumed elementary particles would surely go away in short order—otherwise things would be too complicated!

Our understanding of elementary particles and processes has advanced enormously since then by any measure, but we have yet to match the masterful description of the world as presented early in the 1950's by Amos and Andy. I was reminded of their theory several weeks ago by a retrospective on their once-popular radio and television series that was shown on public television very late one evening out in California. Back in those times—when T.D. was studying

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Invited talk presented at the Symposium Commemorating the Thirteenth Aniversary of Parity Nonconservation and Celebrating the Sixtieth Birthday of T. D. Lee, New York, Nov. 22, 1986 viscosity, turbulence, statistical theories of equations of state and phase transitions, and the motion of slow electrons in polar crystals—Kingfish was explaining to Amos that the world was really made of protons, neutrons, fig newtons, and morons!

That was back around 1951—of course, long before one had learned about fig newtinos, smorons, and all those other Zuminos!

And there are many reminiscences of other happy occasions with T.D., away from physics. I don't remember when I first met T.D. but I do remember clearly, and I still savor the memory of, our first visit to Paris in the summer of 1958, following the Rochester conference—which that year, for the first time left Rochester to be held at that upstart new laboratory in Geneva. T.D. and I drove together from CERN to Paris to be greeted by a fantastic tour guide for our first visit to that lovely city. Our guide was David Pines, who had spent the year there and was already an accomplished bon vivant and a discriminating gourmet. Under David's tutelage we set a record on our first day in Paris that I believe still merits recognition in the Guinness Book of Records. It is theoretically possible to break our record, but I do not believe that in fact—on the basis of actual experiment—the record is vulnerable. On that day, T.D., David, and I ate a Michelin two-star lunch <u>and</u> a three-star dinner! After that we didn't have to—nor did we want to—eat again for the rest of the week! [Note added in proof: David Pines reports that on a subsequent occasion he and his wife Suzy did partake of two three-star meals on the same day! So much for that record.]

T.D. himself has had, as we well know, an enormous impact on the addiction of the American physics community to gourmet Chinese cuisine.

T.D. also had something to do about another gourmet occasion, which I regret to say never came to pass. Not everyone was immediately persuaded that the bold—or wild—suggestion of parity violation offered a sensible resolution to the $\tau - \theta$ puzzle. It would be false to suggest that I was readily enchanted by that proposal when I first heard it, but I remember arriving at Stanford in the summer of 1956 and trying to explain it to Felix Bloch. I worked hard at it—and gave it my best shot. Physics discussions with Felix were never easy or casual. At the end of our session Felix countered that he would eat his hat if this idea were confirmed by experiment. Felix did have a hat, which he rarely wore. I frequently reminded him of his obligation and offered a variety of appropriate seasonings. But long after the beautiful experiments that you heard about this morning had convinced him that parity, as he had known it, was violated, Felix failed to pay up on his bet.

As one looks over the broad landscape of physics it is truly very hard to find a region or a territory that has not been touched by T.D.'s incisive and enduring contributions, by his technical strength and virtuosity, or by his physical insights.

But let me remind you that the many dimensions of T.D.'s contributions as a teacher and statesman of physics are almost as impressive as his research achievements. First, as all of us who have enjoyed and learned from his lectures and seminars know, he excels in the art of classroom teaching, in giving theoretical seminars, conference talks, and lecture series at summer institutes. He has been a superb teacher of us all. Additionally, T.D. has been more than the inspiration—he has been a personal teacher of an entire generation of young Chinese scientists as he has provided exceptional leadership and energy in order to help China recover from the damage to their science of the Cultural Revolution. In 1979 he lectured to some 1200 young Chinese physicists who assembled in Beijing to hear him hold forth on all modern physics—particle physics and statistical mechanics. This led to his classic book—PARTICLE PHYSICS AND INTRODUCTION OF QUANTUM FIELD THEORY—published in 1981. Therein you can find all of modern particle physics, circa 1981, assembled in 850 or so clear and concisely written pages, including symmetry theory, QCD and gauge theory, quark confinement, chiral theory, quark models and high energy processes, weak-electromagnetic unification and the like. The only defect in this volume is that it is written in the unforgivable East Coast metric of x, y, z and ict. Maybe at the start of his seventh decade T.D. will now outgrow that kid stuff of i's and convert to the superior West Coast metric. T.D. and I were both raised with x, y, z and ict notation in Wentzel's book on quantum field theory, but I do think it's time for T.D. to learn better. He should set that as one of his goals for the great decades ahead. By the way T.D., we're also waiting for you to write up for publication in English the other half of your 1979 lectures in China on statistical mechanics!

T. D., the teacher and educator, has also made it possible for more than 700 of China's brightest young physicists to come to our universities for their graduate education under what he calls the CUSPEA program (whatever that stands for), but we know them as Lee scholars. This is a program to give these scholars a superb modern training in physics while the education system in China is still rebuilding, so that upon returning home they can themselves be the professors for their succeeding generation of students. Does anyone doubt the tremendous importance of that investment in developing the latent talent and skills of such a great pool of brainpower for the future of man's culture and of the civilization of our planet, as well as for our understanding of nature? Without a doubt, I rate that program, dollar for dollar, as one of the most valuable investments a nation can make in its future.

And there is yet a fourth dimension to T.D.'s teaching activities. He has been the personal tutor of China's maximal leaders in helping them lead China to take great strides back to the scientific frontiers. Let me describe to you T.D.'s simple, sensible, and successful explanation to Deng Xiaoping of the need for postdoctoral research support and fellowships for the Lee scholars when they return home to China. As he described it to Deng, when teaching students in college, professors must pose the problems to them and also provide the correct answers. When students advance to the stage of writing a Ph.D. thesis, it is the task of the professor to pose the question and provide the problem—but not to know the answer ahead of time. That is to be determined by student's own research. What is important for the postdoctoral training is for young scientists to develop the capacity to pose problems on their own, as well as find their own answers. Deng Xiaoping apparently understood that explanation very well, and China has now started a program to make it possible for their young postdoctoral fellows to develop their own new research programs in physics.

But let me go back a decade earlier and describe a 1974 meeting of Tsung Dao with then Chairman Mao Zedong which marked a very early important step in the commitment of the Chinese government to the education of bright young scientists.

In the prologue to the book being published this year by the University of Washington Press containing his Danz Lectures, T. D. describes how he answered Chairman Mao's question and explained to him why the concept of symmetry was so important. In Chinese, as described by T. D., the word "symmetry" carries the meaning of a static concept—*i.e.*, "the beauty of form arising from balanced proportions". But in Mao's view, as recounted by Tsung Dao, the entire evolution of human societies is based on dynamic change. Dynamics is the only important element, not statics. Mao felt strongly that it also had to be true in nature. Therefore, he was quite puzzled that symmetry should be elevated to such an exalted place in physics.

To illustrate the deeper dynamical meaning of symmetry to Mao, T.D. describes how as he sat talking to him in his residence inside the Imperial Palace. T.D. put a pencil on a pad of paper resting on the end table that was placed between their two chairs. Let me quote T.D.'s description of his demonstration:

> "I put a pencil on the pad and tipped the pad towards Mao and then back towards me. The pencil rolled one way and then the other. I pointed out that at no instant was the motion static, yet as a whole the dynamic process had a symmetry. The concept is by no means static; it is far more general than its common meaning, and applicable to all natural phenomena from the creation of our universe to every microscopic subnuclear reaction."

T.D. recounts that Mao appreciated the simple demonstration and then asked more questions about the deep meaning of symmetry and also about other physics topics, expressing regret that he had not had the time to study science. In the end of the conversation Mao accepted T.D.'s proposal that the education of at least the very brilliant young students should be maintained, continued, and strengthened. This led, with the strong support of Zhou Enlai, to the elite "youth class", a special intensive education program for talented students from the early teens through college. It was established first at the University of Science and Technology in Anhui and later, because of its success, also at other Chinese universities.

Let me turn next to the many dimensions of T.D.'s research achievements. The numerous major honors T.D. has received are evidence enough of his enormous contributions. His mark can be found in just about every area of physics. In statistical mechanics we look to the work of T.D., Yang, and collaborators for understanding the nature of phase changes, the theory of cluster expansions and the low-temperature behavior of hard-sphere boson systems, and for the analysis of the general many-body problem in quantum statistical mechanics. In the area of weak interactions, one need only mention parity violation and all its ramifications, including high energy neutrino processes, experimental tests of CP, or T, invariance as well as various aspects of the heavy intermediate vector bosons. Most theorists have used the Lee model as a sandbox for some idea or other. T.D. has also made elegant studies of coherent states, or solitons, exploring bag models and abnormal vacuum states and their implications for high energy heavy-ion collisions. He has undertaken a major research program in lattice field theory, including the effects of gravity; and, in particular, he has developed a random lattice theory designed with a very elegant mathematical formalism to restore rotational symmetry properties to the lattice.

The day before I left Stanford last week to come East I received a package of four new papers by T.D. and collaborators which describe new possibilities for configurations of cold stellar matter based on soliton solutions in general relativity. They found coherent quantum states with masses up to as much as 10^{15} solar masses.

There isn't enough time today to describe all of T.D.'s contributions to physics—even if I could do it. So instead I took it as a personal challenge in preparing this talk today to find a topic in physics that T.D. has not solved or contributed to in his more than 200 published research papers—and to teach it to him. I have found such a topic and have been working on it in recent months with Dick Blankenbecler. This is the progress we have to report.

An important parameter in the design of very high energy electron colliders is the fractional energy loss due to bremsstrahlung as one beam pulse passes through the other pulse. This is known as beamstrahlung and has been treated by Himel and Siegrist by adapting a quantum treatment of synchrotron radiation by an electron in a uniform magnetic field given by Sokolov and Ternov. This adaptation necessarily involves several assumptions, in particular the approximation of the effects of the pulse by a uniform magnetic field in which the electron is orbiting as it radiates. In fact, the electron sees the rapidly approaching pulse in the collider frame of reference as transverse, mutually orthogonal electric and magnetic fields of equal strengths whose spatial dependence is determined by the distribution of charges in the pulse.

The challenge undertaken by Blankenbecler and me is to compute the energy loss more simply, more generally, and more accurately by a straightforward application of high energy scattering theory to the problem of radiation in the presence of a very strong field. In this case the field is the actual electromagnetic field of the pulse of positrons being traversed by a very energetic incident electron beam pulse.

Practical interest in this problem arises from the scaling laws governing the extension of electron-positron colliders from today's energy ceiling of 100 GeV at SLC and 150 GeV at LEP to up to the multi–TeV range. For linear colliders the increase of cost is roughly proportional to the increase of energy. In contrast, for storage rings of radius R, the cost is given by the sum of two terms

$$\$ \propto (\text{Energy})^4/\mathrm{R} + (\text{constant})\mathrm{R} \qquad (1)$$

where the first term measures the cost of the radiofrequency power required to replace the power radiated per turn by the circulating e^+e^- beams, and the second term represents the real estate cost of the ring.

The minimum of Eq. (1) occurs for radius R$\propto$ (Energy)^2 and shows that

$$\$ \propto (\text{Energy})^2 \text{ for storage rings}$$

whereas

$$\$ \propto (\text{Energy})^1 \text{ for linear colliders.}$$

The event rate for colliding beams is equal to the product of the interaction cross section and the luminosity which is defined by

$$\mathcal{L} = \frac{N_+N_-}{\pi B^2}\mathrm{f} \qquad (2)$$

where $N_+(N_-)$ is the number of positrons (electrons) in the two colliding pulses, B is the common radius of the pulses and f is the frequency with which pulses collide. In practice, f is much smaller for liner colliders ($\sim$ 100 pulses/sec) than for storage rings in which f depends on the radius R and the number of pulses circulating in the storage ring, and is typically $\sim 10^5$/second. Therefore, by (2), in order to maintain a comparable event rate, the total charge Ne in each pulse must be higher and the radius B must be smaller for linear colliders. This means that the electromagnetic fields of the pulses will be very strong and cannot be treated perturbatively. Design numbers for the SLC and notional parameters for a "super" linear collider are listed below.

TABLE I

	SLC	**"Super"**
Beam energy (γm)	50 GeV; $\gamma = 10^5$	5 TeV; $\gamma = 10^7$
Pulse radius (B)	10^{-4} cm	5×10^{-8} cm
Pulse length (ℓ_o)	10^{-1} cm	3×10^{-5} cm
Number of charges (N)	5×10^{10}	3×10^8
Pulse frequency (f)	120/second	100/second
Luminosity ($\mathcal{L}$)	10^{31}/cm^2 -sec	10^{33}/cm^2 -sec
$\mathcal{L}\sigma_{point\ QED}$	0.3 events/hour	0.1 events/day

First consider the application of classical physics to study beamstrahlung. For simplicity, we choose to work in the rest system of one pulse (say the positrons) and consider radiation from one incident electron as it traverses the pulse at impact parameter b. In this reference frame, the pulse length is (Lorentz) stretched to $L = \ell_0 \gamma$ and the incident electron energy is $p = 2\gamma^2 m$. Assuming the pulse is a uniform cylinder of radius B and length $L \gg B$ the transverse electric field acting on the incident electron is

$$e\vec{E}_\perp = \frac{-2N\alpha\vec{b}}{LB^2} \tag{3}$$

and the classical equation of motion is given by

$$\ddot{\vec{b}} = \frac{1}{p}\, e\vec{E}_\perp \tag{4}$$

In this calculation we treat the pulse as a fixed charge distribution which produces a static transverse electric field in its rest system. Therefore, we must limit our calculation to small changes in the electron's impact parameter b—*i.e.*, $\delta b \ll b$. This is known as small disruption of the beam. Otherwise, as the incident pulse of electrons is squeezed by the attractive field of the positron pulse, the radius of the positron pulse is likewise squeezed by the effect of the electron pulse. A proper treatment of these mutual focussing effects (which if large would set up betatron oscillations) would require a much more extensive and difficult analysis.

According to the classical equation of motion, the condition for small disruption can be expressed as

$$\frac{\delta b}{b} = \frac{1}{2pb}\,(eE_\perp)\,L^2 \approx \frac{y\ell_0}{2\gamma B} \ll 1 \tag{5}$$

where $y \equiv N\alpha/(mB)$ is a useful classical variable. Referring to the nominal 'super' parameters, we see that this restriction is satisfied since

$$\left.\frac{\delta b}{b}\right|_{super} \approx 6 \times 10^{-2}.$$

The approximation is not quantitatively valid for the SLC for which

$$\left.\frac{\delta b}{b}\right|_{SLC} \approx 0.4$$

and a more careful treatment is called for.

We can now calculate the fractional energy loss defined as the power radiated multiplied by the time required for the electron to traverse the pulse, divided by the incident energy:

$$\delta(b)_{classical} = \frac{2}{3}\, e^2\, (2\gamma^2)^4\, |\tilde{b}|^2 \cdot L \cdot \frac{1}{p}$$

$$= \frac{16}{3}\, \frac{\alpha^3 N^2 \gamma}{m^3 \ell_0 B^2} \left(\frac{b}{B}\right)^2$$

Averaging over the impact parameter gives

$$\delta_{classical} = \frac{8}{3}\, \frac{\alpha^3 N^2 \gamma}{m^3 \ell_0 B^2} \tag{6}$$

This classical result which excludes all effects of radiation back on the motion of the radiating electron is valid only for values of $\delta_{classical} \ll 1$. Although this is valid for the SLC with $\delta_{classical}(SLC) \lesssim 10^{-2}$, it is grossly in error for the notional "super" for which $\delta_{classical}(super) > 10^5$!

In applying high energy quantum mechanical scattering theory to this calculation we identify three length scales that are important in characterizing the electron's path and radiation pattern:

1. the coherence length of radiation, ℓ_{coh}, defined as the path length of the electron corresponding to its acquiring a transverse momentum ~ m from the electric field. Since the width of the photon radiation pattern is also ~ m, the radiation can be coherent only from a finite length of the path, namely

$$\ell_{coh} \sim \left|\frac{m}{eE_\perp}\right| \sim \frac{L}{2y} = \frac{\ell_0}{2y}\gamma \tag{7}$$

2. the radiation length, ℓ_{rad}, related by the uncertainty principle to the reciprocal of the longitudinal momentum transfer,

$$\ell_{rad} \sim \frac{1}{q_z} \sim \frac{p}{m^2} \tag{8}$$

   where the last relation corresponds to giving a transverse momentum $\sim m$ to the target pulse.

3. the graininess of the bunch—*i.e.*, the average separation of its particles expressed by

$$\ell_{grn} \sim \frac{L}{N}. \tag{9}$$

In all cases of interest, the radiation length ℓ_{rad} is much larger than the graininess; i.e.

$$\ell_{rad} \sim \frac{p}{m^2} = \frac{2\gamma^2}{m} \gg \frac{L}{N} = \ell_{grn}. \tag{10}$$

This justifies our making a smooth approximation for the charge distribution of the pulse, as we did in (3). This applies for both the SLC–like parameters corresponding to a dense pulse,

$$\frac{L}{N} \sim 2 \times 10^{-7} cm \ll B \sim 10^{-4} cm$$

and to those parameters quoted earlier as envisaged for a 'super' linear collider, corresponding to a dilute pulse,

$$\frac{L}{N} \sim 3 \times 10^{-6} cm \gg B \sim 5 \times 10^{-8} cm.$$

If the dimensionless ratio of the coherence length to the radiation length:

$$\frac{\ell_{coh}}{\ell_{rad}} \sim \frac{m\ell_0}{4\gamma y} \equiv C \tag{11}$$

is large, we are in the classical region, as appropriate for the SLC. In dimensional units $C \gg 1$ corresponds to the limit $\hbar \to 0$. In this regime the deflection of the electron orbit is negligible over a path length ℓ_{rad} and the form factor for radiation along the length ℓ_{rad} is unity. The result given in (6) can then be understood as radiation from L/ℓ_{rad} transverse slices of the pulse, each of thickness ℓ_{rad} and containing $N\ell_{rad}/L$ charges, with each slice radiating incoherently with respect to the others. Using (8), and introducing $d\sigma \propto \alpha^3 N^2 dk/(m^2 k)$ as the cross section for emission of a photon k by charge $N\alpha$, we find

$$\delta_{classical} \sim \frac{L}{\ell_{rad}} \frac{1}{\pi B^2} \int^{p} \frac{dk}{k} \left(\frac{k}{p}\right) \left(\frac{N\ell_{rad}}{L}\right)^2 \frac{\alpha^3}{m^2} \sim \frac{N^2 \alpha^3 \gamma}{m^3 \ell_0 B^2} .$$

In the quantum regime of small $C \ll 1$ as appropriate for the 'super', we calculate the form factor for the overlap of the radiation along the bending path ℓ_{coh} and find that it varies as $C^{4/3}$, clearly showing the diminishing overlap in this situation.

To formally carry out this calculation we must derive an expression for the matrix element for the emission of a photon during the scattering of an electron from a pulse of N positrons:

$$M = \left\langle \phi_f^{(-)} \middle| \vec{A} \cdot \vec{J} \middle| \phi_i^{(+)} \right\rangle , \tag{12}$$

where $\vec{A}$ is the photon field, $\vec{J}$ is the electron current and $\phi_f^{(-)}$ and $\phi_i^{(+)}$ are respectively the final (incoming) and initial (outgoing) scattering eigenstates of the electron in the static external field of the pulse.

Blankenbecler and I have now completed this analysis* (at the time of the talk it had not yet been completed) and I will just summarize the highlights:

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*SLAC–PUB–4186, January 1987.

1. The electron's scattering eigenstates must be constructed one order beyond the eikonal approximation in powers of 1/(energy). This corresponds to keeping terms of order 1/p in the phase of the wave function because they are of the same order of magnitude as the characteristic longitudinal momentum transfers as indicated in (8).
2. The wave function phase is of order $\sim N\alpha \gg 1$ so that some care is required in the formal analysis. We show that it is possible to do the matrix element integral over the impact parameter by the method of stationary phase and obtain the appropriate quantum corrections to the classical orbit.
3. After a series of manipulations we end up with remarkably simple scaling forms for the fractional energy loss and the power spectrum. In particular, the average energy loss obeys the scaling law $\delta = \delta_{classical} F(C)$ where $C = \frac{m\ell_0}{4\gamma y}$ as defined earlier and form factor F(C) has a simple limiting behavior

$$F(C) = 1 - \frac{4.77}{C} \qquad \text{for} \qquad C \gg 1$$

and

$$F(C) = 0.83C^{4/3}(1 - 2C^{2/3}) \qquad \text{for} \qquad C \ll 1$$

These results are useful for choosing parameters in the design of colliders with specified values of δ and of the radiated power spectrum. Our results quantitatively confirm the arguments of Himel and Siegrist and their adaptation of synchrotron radiation formulas to the collider in the extreme quantum limits $C \ll 1$.

Finally T.D., as a memento of this happy occasion, I want to present to you on behalf of all your Stanford friends and colleagues a SLAC beam tree inscribed in honor of your 60th birthday. It is an example of nature's beautifully artistic symmetry-breaking.