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ANALYSIS OF ORBIT PERTURBATIONS OF THE SLC ARCS*

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1. INTRODUCTION

The Stanford Linear Collider (SLC) is the first linear collider to be built. The repetition rate of a linear collider is much lower than that of a circular collider. To recover the luminosity, the emittance and beam size at collision point have to be one or two orders of magnitude smaller than a conventional collider. Therefore, for the SLC, the small beam emittance created from the damping ring has to be kept small by all possible means to preserve the luminosity potential for the physics experiment.

To minimize emittance growth of electron and positron beams in the SLC Arcs, bending magnets with very strong quadrupole components have to be used. In addition, a sextupole component is included to eliminate the second order chromatic and geometric aberrations. Consequently, in the presence of magnet misalignments, both orbit error and optical distortion can be generated. This report will concentrate on the generation and correction of orbit errors. The optical distortions are treated in another paper in this conference.[1]

Our analysis is applicable to any periodic lattice structure with periodic arrangement of beam monitors and correctors. The unique feature of SLC Arcs is that the orbit errors are corrected by moving magnets. We will establish a general stability criterion for the orbit correction in Sec. 2. In Sec. 3, the rms orbit errors and corrector strengths will be calculated. Finally, in Sec. 4, the formulation will be applied to the design of SLC to obtain estimates for residual orbit distortion after correction.

2. STABILITY CRITERION

Consider a transport line which consists of periodic cells. Let the beam position monitors and the orbit correctors be located with the same period as the cells and let the BPM's and the corrector distributions interlace each other as shown in Fig. 1.

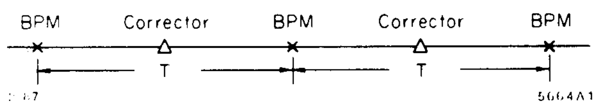

Fig. 1. Periodic array of monitors and correctors in a periodic lattice.

Consider the n-th cell in the transport line. Let errors in this cell be such that they produce an orbit displacement e_n and angle e'_n at the end of the cell where the n-th BPM is located. The orbit at the entrance to the n-th cell has been corrected so that it has a zero displacement with an error in angle x'_{n-1}.

At this point, we leave the type of orbit correctors open, except that its strength in the n-th cell will be specified by D_n. The orbit at the end of the n-th cell is then given by

$$\begin{pmatrix} x_n \\ x'_n \end{pmatrix} = T \begin{pmatrix} 0 \\ x'_{n-1} \end{pmatrix} + D_n \begin{pmatrix} d \\ d' \end{pmatrix} + \begin{pmatrix} e_n \\ e'_n \end{pmatrix} \tag{1}$$

where T is the 2×2 transfer matrix for the cell, d and d' are the orbit responses to the corrector at the end of the cell.

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If the corrector is represented as a short dipole kick, then D_n is the kick angle and d, d' are the 12- and 22- elements of the transfer matrix from the corrector to the monitor.

Eq. (1) can be rewritten as two equations

$$x_n = t_{12} x'_{n-1} + d D_n + e_n \quad , \tag{1a}$$
$$x'_n = t_{22} x'_{n-1} + d' D_n + e'_n \quad . \tag{1b}$$

The corrector strength D_n is determined by the condition that $x_n = 0$. This gives, using Eq. (1a),

$$D_n = \frac{t_{12} x'_{n-1} + e_n}{d} \quad . \tag{2}$$

Substituting into Eq. (1b), then gives the angle at the exit of the n-th cell

$$x'_n = A x'_{n-1} + B_n \tag{3}$$

where

$$A = t_{22} - \frac{d'}{d} t_{12} \quad , \tag{4a}$$

$$B_n = e'_n - \frac{d'}{d} e_n \quad . \tag{4b}$$

Equation (3), showing the angular divergence of the orbit at the BPM's after orbit correction, contains two terms. The first term comes from the propagation of the residual angular divergence upstream of the cell under consideration. The second term is the noise contribution from errors in the cell. Furthermore, the first term is "damped" by the factor A per cell (when $|A| < 1$). As a result the orbit, containing a damping on the one hand and a noise on the other hand, as a balance between those two terms will acquire an equilibrium value.

It is instructive to apply the results to the case when the corrector is a δ-function kick. We have

$$D_n = \text{kick angle} \quad ,$$

$$d = \sqrt{\beta_c \beta_m} \sin \psi \quad , \tag{5a}$$

$$d' = \sqrt{\frac{\beta_c}{\beta_m}} (\cos \psi - \alpha_m \sin \psi) \quad , \tag{5b}$$

where the subscripts c and m refer to the corrector and monitor locations, respectively; ψ is the phase advance from corrector to monitor. In addition, we have

$$t_{12} = \beta_m \sin \psi_{\text{cell}} \quad , \tag{6a}$$
$$t_{22} = \cos \psi_{\text{cell}} - \alpha_m \sin \psi_{\text{cell}} \quad , \tag{6b}$$

where ψ_{cell} is the phase advance per cell. Substituting Eqs. (5) and (6) into (4a), the coefficient A is found to be

$$A = -\frac{\sin(\psi_{\text{cell}} - \psi)}{\sin \psi} \quad . \tag{7}$$

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The coefficient A plays the role of a "damping constant." Note that Eq. (7) is independent of the β-functions. A strong damping (which is preferred) requires A closer to 0. This is achieved if $\psi_{\rm cell} - \psi$ (the phase advance from monitor to corrector) or a multiple of π and if ψ (the phase advance from corrector to the next monitor) is away from a multiple of π. For instance, one optimum arrangement is to have 90° cells and to have the correctors immediately downstream of the monitors, which is a very common practice.

The figure of merit indicating the quality of orbit correction is the rms orbit slope at the BPM's. As mentioned before, this rms value is determined by the balance between a damping effect and a noise diffusion effect. From Eq. (3), we have

$$x'_n = B_n + AB_{n-1} + A^2 B_{n-2} + \dots \quad . \tag{8}$$

Squaring the quantity and taking the expectation value, assuming no correlations among errors in different cells, we obtain

$$\begin{aligned} \langle x'^2 \rangle &= \langle B^2 \rangle \left(1 + A^2 + A^4 + \dots\right) \\ &= \begin{cases} \frac{\langle B^2 \rangle}{1-A^2} & \text{if } |A| < 1 \\ \text{diverges} & \text{if } |A| \geq 1 \end{cases} \end{aligned} \tag{9}$$

The orbit correction scheme breaks down if $|A| \geq 1$. What happens then is that the orbit slope at the BPM's continue to grow as shown in Fig. 2.

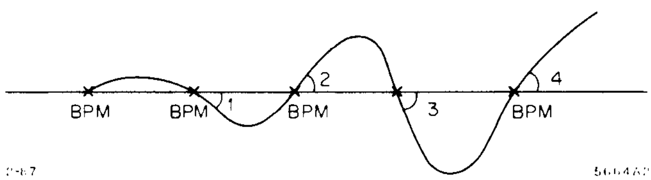

Fig. 2. Divergent orbit with increasing angle error if $A > 1$.

Equation (9) gives the value of $\langle x'^2 \rangle$ at the BPM's once A and $\langle B^2 \rangle$ are known. It is clear that to keep the rms orbit small, we need to make A not too close to 1 and to keep $\langle B^2 \rangle$ small.

To obtain an expression for $\langle B^2 \rangle$, we need to know the possible sources of errors. Let the k-th error have strength D_k and its orbit at the end of the cell be $e = D_k d_k$ and $e' = D_k d'_k$. Then we have

$$\langle B^2 \rangle = \sum_k \langle D_k^2 \rangle \left(d'_k - \frac{d'}{d} d_k\right)^2 \quad , \tag{10}$$

where the summation is over all possible sources of error in one cell. Note that if the error source is the same as that used as the corrector, it does not contribute to orbit error after correction since $d = d_k$ and $d' = d'_k$.

3. RMS ORBIT ERRORS AND CORRECTOR STRENGTHS

So far we have assumed that the orbit displacements at the BPM's are perfectly corrected. This will not be the case if the BPM readings have an rms error $\triangle$. To include this effect, a more complicated calculation similar to that of the previous section is carried out. The result is that instead of Eq. (9) we have

$$\langle x'^2 \rangle = \frac{\langle B^2 \rangle}{1 - A^2} + \langle \triangle^2 \rangle \left[\left(\frac{d'}{d}\right)^2 + \frac{K^2}{1 - A^2} \right] \tag{11}$$

where

$$K = t_{21} - \frac{d'}{d}\left(t_{11} - A\right) \quad . \tag{12}$$

Compared with Eq. (9), the rms orbit has acquired another term proportional to $\langle \triangle^2 \rangle$. To obtain a feeling for the coefficient K, consider the case of a δ-function corrector. We have

$$t_{21} = -\frac{1}{\beta_m}\left(1 + \alpha_m^2\right) \sin \psi_{\rm cell} \quad , \tag{13a}$$

$$t_{11} = \cos \psi_{\rm cell} + \alpha_m \sin \psi_{\rm cell} \quad , \tag{13b}$$

which gives

$$K = -\frac{1}{\beta_m} \frac{\sin \psi_{\rm cell}}{\sin^2 \psi} \quad . \tag{14}$$

To minimize the effect of BPM errors, we have to make $K^2/(1 - A^2)$ as small as possible. This requirement is not the same as the previous requirement that $|A|$ should be minimized. Depending on the magnitude of the BPM errors, therefore, some compromise has to be made. For example, if the corrector is immediately downstream of the monitor and $\psi_{\rm cell} = 90°$ as the example we mentioned before, we have $A = 0$ and Eq. (11) reads

$$\langle x'^2 \rangle = \langle B^2 \rangle + \langle \triangle^2 \rangle \frac{1 + \alpha_m^2}{\beta_m^2} \quad . \tag{15}$$

The corrector strength in the absence of BPM errors is given by Eq. (2). If the BPM errors are included, the rms corrector strength is found to be

$$\langle D^2_{\rm corr} \rangle = \frac{1}{d^2}\langle \triangle^2 \rangle + \left(\frac{t_{12}}{d}\right)^2 \langle x'^2 \rangle + \frac{1}{d^2}\langle e^2 \rangle \tag{16}$$

where $\langle x'^2 \rangle$ is given by Eq. (11) and

$$\langle e^2 \rangle = \sum_k \langle D_k^2 \rangle d_k^2 \quad .$$

4. APPLICATION TO SLC ARCS

A cell in the SLC Arc consists of two magnets. In this section we consider horizontal orbit correction with the arrangement shown in Fig. 3. The corrector is assumed to be moving the F-magnets horizontally.[2] The corrector strength D is chosen to be the amount of movement (in μm). The relevant parameters in the SLC design[3] are

$$\begin{aligned} t_{11} &= -2.812, \qquad t_{12} = 3.821 \\ t_{21} &= -1.877, \qquad t_{22} = 2.194 \\ d &= 5.514\ \mu\text{m}/1\ \mu\text{m movement} \\ d' &= 3.449\ \mu\text{rad}/1\ \mu\text{m movement.} \end{aligned}$$

The coefficient A is then found from Eq. (4a) to be

$$A = -0.196 \quad .$$

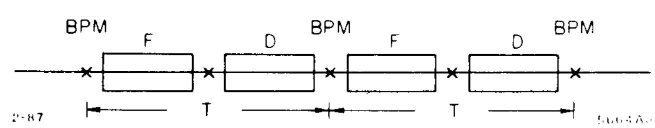

Fig. 3. SLC Arc orbit correction scheme. F-magnet serves as corrector in horizontal plane.

The orbit errors are assumed to come from magnet misalignments. The misalignment of the F magnet (which is the corrector) does not contribute to orbit error after correction. The misalignment of the D magnet gives, using Eq. (10),

$$\langle B^2 \rangle = \langle D^2 \rangle \left(d'_D - \frac{d'}{d} d_D \right)^2 \quad . \tag{17}$$

Again using 2nd order TURTLE, we find

$$d_D = 1.702 \ , \qquad d'_D = 1.573 \ .$$

If the rms magnet misalignment is $\langle D^2 \rangle^{1/2} = 100$ μm, we have from Eq. (17)

$$\langle B^2 \rangle^{1/2} = 50.84 \ \mu\text{rad} \quad .$$

Substituting the values of A and $\langle B^2 \rangle$ into Eq. (9) gives

$$\langle x'^2 \rangle^{1/2} = 51.85 \ \mu\text{rad} \quad . \tag{18}$$

The orbit angle calculated above should be rather accurate. To translate it into a rough estimate of the orbit, note that the orbit has been corrected to zero at the monitor so that maximum orbit displacement tends to occur at the center of the D–magnets. Transferring $\langle x'^2 \rangle^{1/2}$ to the orbit in the D–magnets gives an orbit of 103.07 μm. A similar transfer gives an orbit of 59.6 μm and 79.3 μm at the center of the F–magnet and the gap between the two magnets, respectively. If we take the rms orbit to be the rms of the orbit at the BPM, the F and D magnets, and the gap, we obtain

$$\langle x^2 \rangle^{1/2} = 71.5 \ \mu\text{m} \quad . \tag{19}$$

To take into account the BPM errors, we use Eq. (11). Substituting the values into Eq. (12) yields

$$K = -0.241 \quad .$$

If we assume $\langle \triangle^2 \rangle^{1/2} = 100$ μm, then

$$\langle x'^2 \rangle^{1/2} = 84.68 \ \mu\text{rad} \quad . \tag{20}$$

The rms orbit error due to this $\langle x'^2 \rangle^{1/2}$ is about 116.8 μm. On top of this, there is a contribution from the BPM misalignments. Adding the contributions together gives an rms orbit error of

$$\langle x^2 \rangle^{1/2} = 153.7 \ \mu\text{m} \quad . \tag{21}$$

Comparing Eqs. (19) and (21), it is clear the BPM errors contribute significantly to the Arc orbit correction and must be taken into consideration. Furthermore, Eq. (21) shows that for 100 μm rms misalignment and 100 μm BPM error, the resultant rms orbit error is about 150 μm which are used in Ref. 1 to estimate the optical perturbations.

To find the rms corrector strength, one can use Eq. (16). The result is

$$\langle D^2_{\text{corr}} \rangle^{1/2} = 121.34 \ \mu\text{m}$$

for the needed rms magnet movements. This number becomes 110.65 μm if there are no BPM errors. The movement doubles if magnets are moved with one end fixed. The results of orbit errors and corrector strength after orbit correction are summarized in Table I.

Table I. Horizontal orbit error with 100 μm misalignments after corrections.

	BPM Error = 0	BPM Error = 100 μm
$\langle x' \rangle_{\text{rms}}$	51.9 μrad	84.7 μrad
$\langle x \rangle_{\text{rms}}$	71.5 μm	153.7 μm
$\langle D_{corr} \rangle_{\text{rms}}$	110.7 μm	121.34 μm

As a comparison, one might ask what if the D magnets are used as the horizontal correctors? Such a correction scheme does not work at all. Because then the damping factor $A = -1.337$ and the corrected orbit diverges as shown in Fig. 2. However, for the vertical orbit correction, the D magnets will be moved instead of the F magnets. Due to the symmetry between vertical and horizontal lattices, the vertical orbit error after correction should be smilar to that of the horizontal orbit.

Here we only discuss the case of random misalignments. The case of systematic misalignments or energy errors have been studied by T. Fieguth[4] and M. Sands.[5] Actual commissioning experiences on the perturbations and corrections of the orbits in North Arc are discussed in another paper presented at this Conference.[6]

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