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CHANNELED PARTICLE ACCELERATION BY PLASMA WAVES IN METALS

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ABSTRACT

We present a solid state accelerator concept utilizing particle acceleration along crystal channels by longitudinal electron plasma waves in a metal. Acceleration gradients of order 100 GV/cm are theoretically possible. Particle dechanneling due to electron multiple scattering can be eliminated with a sufficiently high acceleration gradient. Plasma wave dissipation and generation in metals are also discussed.

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Presently existing high energy particle accelerators are limited to acceleration gradients of order 10 MV/meter. This implies that to achieve ultrahigh energies exceeding several TeV would require great distances. In recent years there has been an increased interest in the high-gradient linear acceleration of changed particles.[1,2] One concept which promises very high gradients is the plasma accelerator.[3] In this scheme longitudinal plasma oscillations with phase velocities near the speed of light provide large electric fields which are intended to accelerate particles to high energy over a short distance. Gradients G of order $\sqrt{n}$ V/cm are theoretically possible, where n is the electron number density in units of cm^{-3}. Typical laboratory gas plasma densities are in the range 10^{14}–10^{18} cm^{-3} corresponding to maximum gradients of 10 MV/cm–1 GV/cm.

However, a high gradient is not the only requirement for linear colliders. Stability and emittance requirements for the accelerating system are very stringent. Since the beams from two independent accelerators must collide at an interaction point, excessive transverse motion and emittance growth of the beams induced during acceleration must be avoided. One concern is that plasma accelerators may be prone to such beam instabilities due to plasma nonuniformities and multiple ion scattering.

To extend the plasma wave acceleration idea to very high gradients and reduce emittance growth, we explore in this paper a solid state accelerator concept in which particles are accelerated along atomic crystal channels by plasma waves in a metal. Conduction electrons in a metal form a very uniform high density plasma exhibiting longitudinal plasma oscillations.[4] Typical conduction electron densities are of order 10^{22} cm^{-3} corresponding to a maximum gradient of order 100 GV/cm. Although this gradient equals 10^3 V/Å, the metal can support such high fields because the ionization energy of the atomic core electrons is at least several times the plasmon energy $\hbar\omega_p \sim 10$ eV.

For phase velocities near the speed of light, the plasma wave number $k_p = \omega_p/c \simeq 5 \times 10^{-3}$ Å^{-1} is much less than the Fermi wave number $k_F \sim 1$ Å^{-1} in the metal, so plasmon damping is primarily due to interband transitions (electron transitions to unfilled bands) with the decay width Γ_p being typically 10^{-2}–$10^{-1}\hbar\omega_p \sim 0.1$–$1$ eV.[5] To use such plasma oscillations in solids to accelerate charged particles to very high energy is problematic since the radiation length for electrons and positrons is of order 1 cm, while the nuclear collision length for protons and antiprotons is of order 10 cm.

These problems can be substantially mitigated for positively charged particles by utilizing the channeling phenomenon in crystals.[6] Positively charged particles are guided by the average electric fields produced by the atomic rows or planes in the crystal. The particles make a series of glancing collisions with many atoms and execute classical oscillatory motion along the interatomic channels.[7] In contrast, negatively charged particles are attracted by the atomic nuclei and suffer large angle Coulomb scatterings resulting in rapid dechanneling. This suggests that it is possible to accelerate positively charged particles on plasma waves for considerable distances through channels in metallic crystals.

An immediate concern in such an acceleration scheme is beam loss through gradual dechanneling.[8] The transverse momentum of channeled particles increases due to collisions with electrons in the channel.[9] Dechanneling occurs when a particle's transverse kinetic energy $K_\perp = E\psi^2/2$, where $E = \gamma mc^2$ is the total particle energy and ψ is the channeling angle, allows it to overcome the channel potential energy barrier V_c ($\simeq ze\ 10^2$–10^3 volts for a particle of charge ze). This defines the critical channeling angle $\psi_c = (2V_c/E)^{1/2}$. The increase in the angular divergence per unit length for a channeled particle due to multiple scattering can be written as

$$\frac{d\langle\psi^2\rangle}{d\ell} = \frac{\psi_c^2}{2\ell_d} \quad , \tag{1}$$

where $\ell_d = \Lambda E/ze$ is the characteristic dechanneling length. The dechanneling constant Λ is typically 1–10 μm/MV, so high energy particles can channel considerable distances in a crystal. For example, a 1 TeV proton could channel of order 1 m in a metallic crystal like tungsten.[10]

Particle dechanneling in a solid state accelerator is modified by the fact that a channel's normalized rms acceptance $\varepsilon_{cn} = (1/2)\gamma a\psi_c$ increases with energy. Here a is the axial channel radius. This effect can compensate for the increase in the channeled particle emittance due to multiple scattering if the acceleration gradient G is high enough. Our earlier calculation[11] did not correctly include this effect resulting in a more pessimistic estimate for particle dechanneling than will be given here. The channel acceptance is the phase space area available to particles for channeling motion. Each channel acts like a smooth focusing accelerator with betatron focusing function (wavelength/2π of transverse oscillations)

$$\beta_F = (a^2E/2\mathrm{V}_c)^{1/2} = a/\psi_c \quad . \tag{2}$$

Multiple scattering in a transverse focusing system randomly excites betatron oscillations leading to a growth in the invariant (or normalized) rms emittance $\varepsilon_n = \gamma\varepsilon = \gamma\sigma^2/\beta_F$, where ε is the unnormalized rms emittance, and σ^2 is the spatial divergence.[12] The emittance growth per unit length can be written as

$$\left(\frac{d\varepsilon_n}{d\ell}\right)_{\text{scatt.}} = \frac{\gamma\beta_F}{2}\frac{d\langle\psi^2\rangle}{d\ell} = \frac{zea\psi_c}{4\Lambda mc^2} \quad . \tag{3}$$

Initially trapped particles remain channeled indefinitely provided that

$$\left(\frac{d\varepsilon_n}{d\ell}\right)_{\text{scatt.}} \leq \frac{d\varepsilon_{cn}}{d\ell} = \frac{zeGa\psi_c}{4mc^2} \quad , \tag{4}$$

or equivalently $G \geq \Lambda^{-1}$.

Integration of Eq. (3) yields for the final emittance after acceleration from γ_i to γ_f,

$$\varepsilon_{nf} = \varepsilon_{ni} + \frac{a\psi_c}{2\Lambda G}\sqrt{\gamma_f}\left(\sqrt{\gamma_f} - \sqrt{\gamma_i}\right) \quad . \tag{5}$$

At high energy $(\gamma_f \gg \gamma_i)$, $\varepsilon_{nf} \simeq a\psi_c\gamma_f/2\Lambda G$, and accelerated particles oscillate asymptotically with mean square amplitude $\sigma_f^2 = \beta_F\varepsilon_{nf}/\gamma_f = a^2/2\Lambda G$ about the channel axis. A charged particle channeling through a crystal naturally emits radiation as it oscillates transversely in a channel.[13] The maximum attainable energy in such a channeling accelerator occurs when the radiative energy loss approaches the energy gain from the acceleration gradient. The radiative loss per unit length in a smooth focusing system is given by[12]

$$\left(\frac{dE}{d\ell}\right)_{\text{rad}} = -\frac{2(ze)^2}{3}\sigma^2\left(\frac{\gamma}{\beta_F}\right)^4 \quad , \tag{6}$$

where σ is the rms oscillation amplitude. Using $\sigma = \sigma_f$ calculated above for the amplitude and $V_c/a \simeq ze\ 10^2$ V/Å yields for the maximum energy

$$E_{\max} \simeq \left(\frac{m}{m_p}\right)^2 (\Lambda G)^{1/2} \left(\frac{G}{z^3\ 100\ \text{GV/cm}}\right)^{1/2} 10^5\ \text{TeV} \quad , \tag{7}$$

where m_p is the proton rest mass.[14]

Only for acceleration gradients $G \geq \Lambda^{-1} \simeq$ 1–10 GV/cm will significant beam fractions remain channeled over long distances in a crystal. In the case of a longitudinal plasma oscillation, this implies that a large amplitude wave with a gradient of order 100 GV/cm is desirable. This gradient corresponds to an energy density of order 10^8 J/cm^3. The plasma wave would occupy at most a transverse cross section of several square plasma wavelengths ($\sim 10^{-9}$ cm^2) over a long acceleration length in the crystal in order to keep the total energy in the plasma wave small and still maintain a uniform wavefront. Whether the energy contained in the plasma wave is sufficient to thermally damage the crystal depends on the relaxation time for converting plasmon energy to phonons.

After the original plasma wave ($\omega = \omega_p,\ \bar{k}_p = k_p\hat{z}$) decays via interband transitions, the excited electron states will in turn decay producing a plasmon gas with wave vectors ($|\bar{k}| \simeq k_p$) varying in direction. The plasmon gas can cause additional interband transitions but eventually electron-electron collisions will break up the plasmons as electrons are scattered out of synchronism.[5] The electron collision rate can be written approximately as $\Gamma_{ee}/\hbar \simeq 0.4[(k - k_F)/k_F]^2 E_F/\hbar$ when the electron wave number k is near the Fermi wavenumber k_F.[15] Here E_F is the Fermi energy which is typically less than the plasmon energy $\hbar\omega_p$. Electrons in a plasmon have wave numbers $k \sim k_F + k_p/2$, so the plasmon gas decays into a hot electron gas in about 10^{-10} sec. These superthermal electrons have energies of order 10^8 J cm^{-3}/10^{22} cm^{-3} $\sim$ 100 keV, but lose their energy at a rate of about 1 MeV/cm primarily through plasmon radiation and electron collisions. This distributes the energy of the original plasma wave radially about 1 mm among many thermal electrons which then heat the crystal by phonon emission ($\tau_{e^- - phonon} \sim 10^{-14}$–$10^{-15}$ sec). The plasma wave energy density thus decreases to about 10 J/cm^3 in 10^{-10} sec corresponding to a tolerable power input of 10^{11} W/cm^3 to the lattice. Crystal damage would occur for power inputs of order 10^{13} W/cm^3 in a 10^{-10} sec pulse.[16]

The generation of large amplitude plasma waves in a metal presumably requires an intense power source to supply the plasma wave energy in a short time without destroying the crystal. Certainly creative ideas for exciting such waves in a metal are needed. We briefly consider three possibilities, all of which are at best problematic when applied to metallic electron plasmas.

The laser beat-wave method[17] involves resonantly exciting the plasma wave by the ponderomotive force of two collinear beating lasers with frequency difference $\omega_1 - \omega_2 \simeq \omega_p$. In a metal this requires X–ray lasers with $\omega_{1,2} \gtrsim 10^{17}$ sec^{-1}. The plasmon decay width Γ_p results in the wave saturating at an amplitude $\alpha_p \equiv e\mathcal{E}_p/m\omega_p c \simeq \alpha_1\alpha_2\hbar\omega_p/2\Gamma_p$, where $\alpha_i = e\mathcal{E}_i/m\omega_i c$ are the normalized laser fields. To obtain a plasma wave with $\alpha_p \sim 10^{-2} - 1$ requires $\alpha_1\alpha_2 \gtrsim 10^{-4} - 10^{-2}$ or a laser intensity $I \gtrsim 10^{17} - 10^{19}$ W/cm^2. Since this intensity is to be delivered in a 10^{-14} sec pulse with a 10^{-9} cm^2 spot size, crystal survivability is questionable.

An immediate problem with beat–wave excitation is pump depletion as the lasers leave their energy behind in plasma waves. The laser-acoustic wave scheme avoids this problem by side-injecting a laser with frequency $\omega_0 \simeq \omega_p$ into a plasma containing an acoustic wave.[18] The laser is linearly polarized along the direction of the acoustic wave vector. The laser $(\omega_0, \bar{k}_0)$ and acoustic wave $(\omega_{ac}, \bar{k}_{ac})$ quasiresonantly excite forward and backward traveling plasma waves with $\omega = \omega_0 \pm \omega_{ac} \simeq \omega_p$ and $\bar{k}_p = \bar{k}_0 \pm \bar{k}_{ac} \simeq \pm\bar{k}_{ac}$. In a metal the plasma wave saturates at an amplitude $\alpha_p \simeq \alpha_0(\delta n_{ac}/n_0)\hbar\omega_p/2\Gamma_p$ where α_0 is the normalized laser field and $\delta n_{ac}/n_0$ is the acoustic wave density perturbation. To excite a plasma wave with $\alpha_p \sim 10^{-2} - 1$ requires $\alpha_0 \delta n_{ac}/n_0 \gtrsim 10^{-4} - 10^{-2}$ corresponding to an ultraviolet laser intensity of $10^{15} - 10^{17}$ W/cm^2 if $\alpha_0 \sim 10^{-2} - 10^{-1}$. The crystal may survive this high intensity because the energy would be primarily absorbed in plasmons and interband transitions and only later converted to lattice heat as discussed earlier.

The wakefield method for exciting plasma waves eliminates the need for lasers by employing a charged relativistic driving beam to leave behind a wake of plasma waves.[19] The ratio of the maximum accelerating wakefield to the maximum decelerating field experienced by the driver is called the transformer ratio, $R = |\, \mathcal{E}^+/\mathcal{E}^- \,|$. For a nonsymmetric finite length driver, R can be arbitrarily large.[20] In a metal collisional energy loss (1–10 MeV/cm) of the driver to electrons may destroy the crystal as the thermal electrons rapidly (10^{-14}–10^{-15} sec) heat the lattice by phonon emission. To excite a plasma wave with $\alpha_p \sim 10^{-2} - 1$ requires a driver charge density of order $10^{20} - 10^{22}$ e/cm^3. This yields a power input to the lattice of order $10^{18} - 10^{20}$ W/cm^3.

Independent of the method for exciting a plasma wave, similar considerations apply to the collisional energy loss by the accelerated beam. The thermal fracture threshold will presumably limit the maximum accelerated beam current density that the crystal can withstand to approximately 10^{11} A/cm^2 for a pulse length of order $\hbar/\Gamma_p \sim 10^{-14}$ sec. Although the channeling phenomenon and high acceleration gradient aid in maintaining the accelerated beam emittance over long distances, the collisional energy loss is a consequence of the collective nature of this solid state acceleration scheme. Certainly the scheme explored in this paper does not preclude other possibilities for accelerating particles in solids.

While completing the work described here, we discovered that several authors have discussed the acceleration of channeled particles by various types of fields in solids. We briefly mention these papers for the interested reader but do not claim the list to be complete. Kanofsky[21] discussed the use of masked laser fields to accelerate particles in a crystal and presented two beam dechanneling estimates. One of these estimates was identical to our more pessimistic dechanneling calculation in Ref. 11 which did not included the adiabatic damping of the beam emittance. Grishaev and Nasonov[22] suggested the acceleration of particles on a longitudinally polarized wave in a cubic crystal having a nonlinear optical susceptibility. Pisarev[23] analyzed the use of the longitudinal static polarization produced in a nonlinear crystal by optical waves to accelerate particles. Neither of these papers contained a discussion of dechanneling. Beloshitskii and Kumakhov[24] considered the use of the inverse Cerenkov effect in a crystal to accelerate particles with a laser. Beam dechanneling was briefly discussed in this paper with the suggestion that the decrease in beam emittance with increasing energy could trap channeled particles. Nasonov[25] suggested that particles could be accelerated by longitudinal optical phonons in an alkali halide crystal but did not mention dechanneling. In all these papers, the maximum acceleration gradients were estimated to be 0.1–1 GV/cm.

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