

## LIMITATION OF LINEAR COLLIDERS FROM TRANSVERSE RF DEFLECTIONS\*

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Abstract

Offaxis beam trajectories in a linear collider produce transverse wakefield and chromatic effects which cause emittance enlargement. One cause for non-centered trajectories in the accelerating structures is radial RF fields which produce transverse deflections. Static deflections can be compensated by static dipole magnetic fields. However, fluctuations of the RF fields cause variations in the deflections which must be managed or limited<sup>1</sup>. Given the level of fluctuation of the phase and amplitude of an RF system, a limit on the allowable RF deflection can be calculated. Parameters such as the beam emittance, lattice design, RF wavelength and the initial and final beam energies influence the tolerances. Two tolerances are calculated: (1) one assumes that the wakefields are completely controlled, and that chromatic effects are the only enlarging mechanism (optimistic), and (2) the other assumes the limit is due to transverse wakefields without the aid of Landau damping (pessimistic).

RF Instability and Deflection

A particle during acceleration that encounters a net radial RF field will be deflected transversely. Radial fields can be generated by power and load couplers, accelerator misalignments, and many possible mechanical construction asymmetries, such as tilted irises<sup>2</sup>. Instabilities of the output of the RF source cause fluctuations in the deflections which in turn cause emittance growth from wakefield and chromatic effects. Observations at the SLC<sup>2</sup> show that both phase and amplitude fluctuations will change the transverse deflection, because the peak RF deflection, in most cases, is approximately 45° out of phase with respect to the peak accelerating phase. For this study, the RF amplitude stability, A, is taken to be 0.001, and the corresponding phase stability is 0.05 degrees.

The transverse momentum, P, given to the beam by a single RF unit producing an energy change  $E_{RF}$  is  $P = T E_{RF}/c$ , where T is the effective "tilt" angle of the accelerating field and c the speed of light. The deflection angle,  $\theta$ , of the beam due to the first RF unit depends upon the initial beam energy,  $E_o$ , and the distribution of the RF deflections. It is assumed here that the accelerating structure for a single RF source has a length short compared to the betatron wavelength.

$$\theta = \frac{P A c}{(E_o + E_{RF}/2)} = \frac{T E_{RF} A}{(E_o + E_{RF}/2)} \quad (1)$$

The factor A enters because we are interested only in the effects of the fluctuations. The manufacturing tolerances on angles in the accelerating structure are approximately T and the assembly tolerances of order  $T \lambda_{RF}/2$  ( $\lambda_{RF}$  = RF wavelength).

Limits from Chromatic Effects

A deflected beam will oscillate along the linac in the quadrupole lattice and eventually decohere from chromatic effects, because the beam has a spectrum of particle energies and, thus, a spectrum of betatron phase advances. An estimate of the emittance

growth can be made by assuming the beam decoheres totally. This case applies when the bunch intensity is low and wakefield effects are absent and also at high currents when Landau damping is used to control transverse wakefields inducing a temporarily large energy spread. Furthermore, the bunch length compression procedure ahead of the linac inevitably makes all longitudinal slices of the bunch have an internal energy spectrum which aids filamentation. The deflection and subsequent filamentation will produce a transverse density profile which depends non-linearly on the deflection angle. If the deflection angle is equal to the angular size of the beam at the location of the RF deflection then the effective emittance of the beam increases roughly 100%. This sets the limit. The deflection limit must be reduced by roughly the square root of the number of independent RF units. The reduction factor  $G = 0.1$  is used to account for many small contributions.

$$\theta \leq G \cdot \sigma_x = G [\epsilon_o/\beta_o]^{1/2} \quad (2)$$

where  $\epsilon_o$  is the emittance of the beam at low energy and  $\beta_o$  the average betatron function in the early part of the linac.

Deflections limits for several colliders are shown in Table 1. Results for other conditions can be scaled from these cases. Case 1 represents the SLC. The tolerances do not depend upon the linac length, the final energy, or the RF frequency. Clearly, high  $E_o$ , low  $E_{RF}$  and reduced  $\beta_o$  are desired.

Deflection Limits from Transverse Wakefield Effects

An RF deflection in the linac causes the bunch to oscillate, generates transverse wakefields and increases the emittance. One measure of the increase in the emittance is the transverse position of the particles in the beam one half the bunch length,  $\sigma_x$ , behind the bunch center<sup>3</sup>. A limit on the allowed RF deflection can be set by keeping the centroid position of those particles less than the design beam width  $\sigma_x$  at the end of the linac. Let  $x_m$  be the maximum transverse centroid position of the  $-0.5\sigma_x$  particles in the last few quadrupoles in the linac and  $\beta_f$  and  $E_f$  the betatron function and beam energy, respectively, at the location of  $x_m$ . Thus,

$$x_m = \theta F [\beta_o \beta_f]^{1/2} [E_o/E_f]^{1/2} \quad (3)$$

where F is the gain ratio of the centroid displacement at the end of the linac to the injection error as determined by a simulation program. F depends upon the bunch length, the RF frequency, the wake potentials, bunch intensity, quadrupole lattice and acceleration. The value of  $x_m$  is chosen to be less than  $\sigma_x$  at the end of the linac. Because of multiple errors, the reduction factor  $G = 0.1$  is again used.

$$x_m \leq G \sigma_x = G [\epsilon_f \beta_f]^{1/2} \quad (4)$$

where  $\epsilon_f$  is the emittance at the end of the linac.  $\epsilon_f = \epsilon_o E_o/E_f$ . Combining Eqs. 3 and 4 a limit on  $\theta$  can be obtained,

$$\theta \leq G [\epsilon_o/\beta_o]^{1/2}/F \quad (5)$$

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**TABLE 1**

Allowed transverse RF deflections from only chromatic effects for several colliders. The initial beam energy is  $E_o$  (GeV), the invariant beam emittance  $\gamma\epsilon$  ( $10^{-7}$  r-m), and the acceleration per RF unit  $E_{RF}$  (GeV).  $\beta_o = 10m$  and  $G = 0.1$ . The RF source is assumed to be stable to 0.001 in amplitude and 0.05 degrees in phase.  $\theta_G$  ( $\mu$  rad) is the maximum allowed deflection angle,  $P_C$  (MeV/c) the maximum transverse momentum, and  $T_G$  (mrad) the allowed effective tilt angle of the accelerating field. The transverse wakefield effects are assumed to be controlled completely and do not enter this calculation. The lattice details and final beam energy, likewise, need not be known.

Case	$E_o$ GeV	$E_{RF}$ GeV	$\gamma\epsilon$ $\times 10^{-7}$	$\theta_G$ $\mu$ rad	$P_C$ MeV/c	$T_G$ mrad
1	1.2	0.25	300	3.60	4.70	18.9
2	1.0	0.25	30	1.20	1.40	5.60
3	1.0	0.50	30	1.20	1.50	3.10
4	1.0	1.00	3	0.39	0.59	0.59
5	2.5	0.25	30	0.78	2.10	8.20
6	2.5	0.50	3	0.25	0.68	1.40
7	5.0	0.50	30	0.55	2.90	5.80
8	5.0	1.00	3	0.18	0.96	0.96

**TABLE 2**

Allowed transverse RF deflections determined by transverse wakefield effects without Landau damping for several possible colliders.  $E_o$  is the initial beam energy in GeV,  $E_f$  the final beam energy in GeV,  $E_{RF}$  the acceleration per RF unit in GeV,  $\gamma\epsilon$  the invariant emittance ( $\times 10^{-7}$  r-m),  $\lambda_{RF}$  the RF wavelength in mm, L the Linac length in km, N the number of particles  $\times 10^9$ , F the wakefield effect enlargement factor,  $\theta_W$  the maximum allowed deflection angle in nrad,  $P_W$  the maximum transverse momentum (KeV/c), and  $T_W$  the allowed effective tilt angle ( $\mu$  rad) of the accelerating field. The RF source is assumed to be stable to 0.001 in amplitude and 0.05 degrees in phase.  $\beta_\theta = 10$  m, and  $G = 0.10$ .

Case	$E_o$ GeV	$E_f$ GeV	$E_{RF}$ GeV	$\gamma\epsilon$ $\times 10^{-7}$	$\lambda_{RF}$ mm	L km	N $\times 10^9$	F	$\theta_W$ nrad	$P_W$ KeV/c	$T_W$ $\mu$ rad
1	1.2	50	0.25	300	105	3	50	47	76	101	403
2	1.0	350	0.25	30	105	5	10	1	1238	1390	5570
3	2.0	350	0.25	30	50	3	10	15	58	124	496
4	1.0	350	0.25	30	33	3	10	305	4.1	4.6	18
5	5	350	0.25	30	25	3	10	619	0.9	4.6	18
6	1.0	350	0.10	30	200	10	10	1.1	1130	1182	12K
7	2.5	500	0.50	30	25	3	10	613	1.3	3.5	7.0
8	1.0	500	0.10	30	200	10	10	1	1238	1300	13K
9	5.0	1000	1.00	3	15	3	5	896	0.2	1.1	1.1
10	2.0	1000	0.50	3	25	5	5	134	2.1	4.6	9.3

This limit is more restrictive than the case with only chromatic effects by the factor  $F$ .

A simulation program has been used to calculate  $F$  for various examples. The reference accelerator is the SLAC 2856 Mhz disk loaded structure and its associated wakefields<sup>4</sup>. The bunch length must remain small so that a small energy spectrum can be obtained. Therefore,  $\sigma_z$  was set to 0.01 of the RF wavelength  $\lambda_{RF}$ . The bunch was moved in phase in all cases so that the first order effects of the longitudinal wakefield were cancelled by the curvature of the RF sine wave.  $\sigma_E/E \leq 0.3\%$ . The transverse wakefields were scaled with RF wavelength as  $(1/\lambda_{RF})^3$  per unit length and the longitudinal wakefields as  $(1/\lambda_{RF})^2$  as described in Ref.3. The iris to cylinder size ratio was not changed. The program tracks 26 longitudinal slices along the linac with a quadrupole spacing of 12.3 m. The betatron phase advance was held at 90° per cell. The bunch was launched with an angle  $\theta$ , the maximum deviation of the beam at the  $-0.5\sigma_z$  position in the last ten quadrupoles was recorded and the value  $F$  calculated from Eq. 3.

The allowable RF kick as determined by transverse wakefields relaxes rapidly with position down the linac. A given field error produces less of a deflection at higher beam energies, the wakefield effects are lower because the excursions are smaller, and the distance over which the wakefield-induced tails grow is shorter. Thus, the calculated tolerances are for the first few RF units.

The results for several colliders are shown in Table 2. Case 1 represents the SLC conditions. Clearly, the tolerances become tighter with lower  $E_o$ , shorter  $\lambda_{RF}$ , lower  $\gamma\epsilon$ , and larger  $E_{RF}$ .

Conclusions

Tolerances on RF deflections have been calculated for chromatic effects and for direct wakefield effects. The two tolerances are comparable for future colliders using long RF wavelengths. However, for short RF wavelengths, the wakefield tolerances become much more severe and assembly and manufacturing tolerances go beyond economic limits. Special techniques must be used to raise this limit such as Landau damping, a low frequency pre-accelerator or very short bunches.

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