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INDIRECT SEARCHES FOR ULTRA-HEAVY QUARKS IN RARE DECAYS*

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ABSTRACT

Existing accelerators will not allow the direct discovery of quarks with masses $\gtrsim M_W$. However such heavy objects can—via loop effects—have a significant impact on some rare decays, mainly of K and B mesons. Using a hierarchical model for the KM matrix we analyze the prospects for such an indirect search for ultra-heavy quarks.

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1. Introduction:

Adding a fourth quark-lepton family represents the mildest extension of the electro-weak Standard Model. A main motivation for doing so derives from our lack of understanding why there are families in the first place.

From PETRA data one concludes that the masses of a new quark doublet and of a new charged lepton have to exceed 20 GeV [1].UA1 data on W decays actually allow us to place more stringent limits on m_E : $m_E \gtrsim$ 40-50 GeV [2].It is unlikely that existing accelerators will allow the direct discovery of quarks (and leptons) with a mass close to M_W or beyond. Then it is natural to ask which processes could yield indirect evidence for such heavy fermions. It is our judgment—as explained in detail in chapters 3 and 4—that there are four classes of reactions that offer the best chance to succeed in this endeavor. (a) $K^+ \to \pi^+ \nu\bar{\nu}$ decays; (b) $Z^0 \to q'\bar{q}+$ h.c. transitions with $q \neq q'$; B^0 decays involving (c) mixing and (d) CP violation. The prospects for success range from reasonable (a,c) to not hopeless (b,d).

Statements like these have been made before, also by us in earlier papers [3].Yet we undertake to go beyond the existing literature by presenting a more specific analysis.

The impact ultra-heavy fermions have on low energy processes depends both on their masses and their couplings to the light quarks; this topic will be addressed next.

2. Guestimates on quark masses and the 4×4 mixing matrix:

In the following we denote the fourth family quark [lepton] doublet by (t', b') $[(L, \nu_L)]$. There are various theoretical arguments suggesting some upper limits on fermion masses:

(1) From the present experimental bound on the ρ parameter that relates the strength of neutral and charged currents one infers [4]

$$m_t, |m_{t'} - m_{b'}| \lesssim 180 \ GeV \ . \tag{1}$$

(2) In the Standard Model all fermions receive their mass from their couplings to the same Higgs doublet field that generates the mass for gauge bosons; in that case one can make sensible statements only about fermions with

$$m_t, m_{b'}, m_{t'} \lesssim 500 \ GeV \ . \tag{2}$$

For fermions with a larger mass experience Yukawa couplings of order one—therefore they cannot be treated perturbatively [5].

(3) Imbedding the Standard Model into a GUT and studying the low-energy behavior of the solutions to the resulting renormalization group equations suggests [6]

$$m_L \lesssim 90 \ GeV, m_{b'} < m_{t'} \lesssim 250 \ GeV \ . \tag{3}$$

The uncertainties become even larger when one considers the 4×4 quark mixing matrix: three more angles and two more phases are introduced about which nothing is known for sure. To make any progress towards semiquantitative predictions we follow a theoretical bias: we adopt the Wolfenstein parametrization [7] for the 3×3 mixing matrix as starting point and – following the method of Rosen [8] – generalize it to the 4×4 case, as it was done by Hayashi *et al.* [9] Although we employ the specific representations given there for our analysis, we believe that the pattern thus obtained is of a rather typical and general nature. As in ref. [9] we will always consider two cases for the mixing of the fourth family to the first three; namely

$$|V(t'b', t'b, t's, t'd)| \sim \begin{cases} (1, \lambda, \lambda^2, \lambda^3) \\ (1, \lambda^2, \lambda^3, \lambda^4) \end{cases} \qquad \text{scenario} \quad \begin{matrix} A \\ B \end{matrix}$$

with $\lambda = \sin\theta_c$. Two comments on updating the parameters of the Wolfenstein representation are appropriate:

(i) $|V(bc)| = A\lambda^2$; using [10] $|V(bc)| = 0.045 \pm 0.008$ one obtains

$$A = 0.85 \pm 0.15\ . \tag{4}$$

(ii) The supposedly conservative bound [10] $|V(bu)|/|V(bc)| < 0.19$ leads to

$$\rho^2 + \eta^2 < 0.68\ . \tag{5}$$

3. Indirect Evidence for Ultra-Heavy Quarks without CP Violation.

(A) $K^+ \to \pi^+ \nu\overline{\nu}$*:*

Following Inami *et al.* [11] one obtains

$$BR(K^+ \to \pi^+\nu\overline{\nu}) = 1.43 \times 10^{-5} \sum_j \left| \sum_i V^*(is)V(id)D(x_i, y_j) \right|^2 \tag{6}$$

where the summation $j[i]$ runs over the lepton [quark] families and $x_i = (m(q_i)/M_W)^2$, $y_j = (m(\ell_j)/M_W)^2$. The functions $D(x_i, y_j)$ denote the contributions from the various loop processes.

For three families one obtains [12]

$$\left.\begin{matrix} 3.2 \times 10^{-11} \\ 3.7 \times 10^{-11} \\ 4.2 \times 10^{-11} \end{matrix}\right\} \leq BR(K^+ \to \pi^+\nu\overline{\nu}) \leq \left\{\begin{matrix} 1.0 \times 10^{-10} \\ 3.4 \times 10^{-10} \\ 7.4 \times 10^{-10} \end{matrix}\right. \quad \text{for} \quad m_t = \left\{\begin{matrix} 40\ GeV \\ 100\ GeV \\ 160\ GeV \end{matrix}\right. \tag{7}$$

We have checked that our choice of parameters does not violate any bounds imposed by $K_L \to \mu^+\mu^-$. Ignoring the top contribution one finds $BR(K^+ \to \pi^+\nu\overline{\nu}) \sim 3 \times 10^{-11}$. The top contribution becomes more important with increasing m_t; thus the present uncertainties in $V(ts)$ and in particular in $V(td)$ have a larger impact and the range for the predicted value of $BR(K^+ \to \pi^+\nu\overline{\nu})$ is widened.

Additional parameters enter with a fourth family:

(i) the masses for the charged [neutral] lepton $L[\nu_L]$; we use as typical values:

$$m_{\nu_L} = 0 \ , \quad m_E = 50\ GeV \tag{8}$$

(ii) the mass for the new charge 2/3 quark and its KM angles. We use

$$m_{t'} = 200\ GeV$$

$$\begin{aligned} V(t'd) &= B[(\gamma - \alpha) + i(\delta - \beta)] \times \begin{cases} \lambda^3 \\ \lambda^4 \end{cases} & \text{scenario} \begin{matrix} A \\ B \end{matrix} \\ V(t's) &= B[(\alpha + i\beta) \times \begin{cases} \lambda^2 \\ \lambda^3 \end{cases} & \text{scenario} \begin{matrix} A \\ B \end{matrix} \end{aligned} \tag{9}$$

These forms are obtained from the requirement of the 4×4 mixing matrix being unitary (see ref. [9] for details). $B, \alpha, \beta, \gamma, \delta$ are new parameters entering in analogy to A, ρ, η; nothing is known about their values since no phenomenological need for a fourth family has surfaced so far. Yet for consistency reasons—the mixing matrix is expanded in powers of λ—we require

$$|B|, \alpha^2 + \beta^2, \gamma^2 + \delta^2 \lesssim 1 \ . \tag{10}$$

We then find

$$\left.\begin{matrix} 3.7 \times 10^{-11}[3 \times 10^{-11}] \\ 6.1 \times 10^{-11}[3.6 \times 10^{-11}] \\ 9.6 \times 10^{-11}[4.3 \times 10^{-11}] \end{matrix}\right\} \lesssim BR^{(4)}(K^+ \to \pi^+ \nu \bar{\nu}) \lesssim \left\{\begin{matrix} 1.9 \times 10^{-9}[1.4 \times 10^{-10}] \\ 2.8 \times 10^{-9}[4.2 \times 10^{-10}] \\ 3.8 \times 10^{-9}[9.1 \times 10^{-10}] \end{matrix}\right. \tag{11}$$

when employing mixing scenario $A[B]$ for m_t = 40, 100, 160 GeV and $m_{t'}$ = 200 GeV.

A comparison of Eq. (7) and (11) shows that a fourth family can enhance the branching ratio for $K^+ \to \pi^+ \nu\bar{\nu}$ considerably. In chapter 5 we will present a more detailed evaluation of these numbers.

(B) $Z^0 \to q\bar{q}'$*:*

Flavor changing couplings of the Z^0 can be induced by quantum corrections. In ref. [13] a complete one-loop calculation for $Z^0 \to t\bar{c} + c\bar{t},\ b\bar{s} + s\bar{b},\ b'\bar{b} + b\bar{b}'$ was given with the dependence on the KM angles factored out. Using their results one finds

$$BR^{(3)}(Z^0 \to b\bar{s} + s\bar{b}) \sim 10^{-9} - 10^{-8} \tag{12}$$

for $m_t \sim$ 40-160 GeV. Adding a fourth family with $m_{t'} \sim$200-500 GeV would raise this number (in mixing scenario A) only to

$$BR^{(4)}(Z^0 \to b\bar{s} + s\bar{b}) \sim 4 \times 10^{-8} - 2 \times 10^{-7} \,. \tag{13}$$

There is just one case which offers some slight hope for observability:

$$BR^{(4)}(Z^0 \to b'\bar{b} + b\bar{b}') \leq \mathcal{O}(10^{-5}) \tag{14}$$

can hold in scenario A for $m_t \leq 500 GeV, m_{b'} \sim 50 - 70 GeV$.

4. Indirect Evidence for Ultra-Heavy Quarks in B Decays.

(A) $B^0 - \overline{B}^0$ *mixing:*

With three families one finds for the mass splitting due to $B_d - \overline{B}_d$ mixing:

$$x_d = \left.\frac{\Delta m}{\Gamma}\right|_{B_d} \sim 0.03F\,,\ 0.15F\,,\ 0.31F \tag{15}$$

for $m_t =$ 40, 100, 160 GeV, where

$$F = \frac{\mathrm{Re}(V(td))^2}{(0.01)^2} \frac{Bf_B^2}{(100\ MeV)^2}\,. \tag{16}$$

For $F \simeq 1$—a reasonable and not overly optimistic value—one obtains for the observable:

$$r_d = \frac{\Gamma(B_d \to \ell^+ X)}{\Gamma(B_d \to \ell^- X)} \simeq \frac{x_d^2}{2 + x_d^2} \sim 5 \times 10^{-4}\,,\ 0.01\,,\ 0.05\ . \tag{17}$$

In the case of B_s mesons much larger numbers are found

$$x_s = \left.\frac{\Delta m}{\Gamma}\right|_{B_s} \sim 1 - 4.5\,,\ 5 - 22\,,\ 10 - 45 \tag{18}$$

for m_t = 40, 100, 160 GeV, and therefore

$$r_s \simeq 0.33 - 0.91\,, \geq 0.92\,, \geq 0.98\ . \tag{19}$$

Scenario A[B] yields for the KM parameters of the fourth family

$$\begin{aligned} V(t'b)V^*(t'd) &= B^2((\gamma - \alpha) - i(\delta - \beta)) \times \lambda^4[\lambda^6] \\ V(t'b)V^*(t's) &= B^2(\alpha - i\beta) \times \lambda^3[\lambda^5]\ . \end{aligned} \tag{20}$$

Keeping $m_{t'}$ = 200 GeV fixed yields

$$r_d \sim 0.006[5 \times 10^{-4}]\,,\ 0.05[0.012]\,,\ 0.13[0.05] \tag{21}$$

i.e., scenario A leads to considerable enhancements.

A fourth family cannot enhance r_s in a discernible way since r_s is already almost as large as it can be: $r_s \leq 1$. Yet the t' contributions to Δm could interfere *destructively* with that from t quarks (if $\alpha > 0$ in particular) thus reducing x_s significantly. For example, for m_t = 40 GeV, $m_{t'}$ = 200 GeV, $\alpha = \beta = 1/\sqrt{2}$ one finds in scenario A:

$$x_s \sim 0.06 - 0.15\,, \quad r_s \sim 0.01$$

or for m_t = 100 GeV:

$$x_s \sim 2.6 - 12\,, \quad r_s \sim 0.77 - 0.99\ .$$

(B) CP Violation in SemiLeptonic B Decays:

$B^0 - \overline{B}^0$ mixing would allow us to search for CP violation in semileptonic B decays:

$$a_{SL} = \frac{\sigma(B^0\overline{B}^0 \to \ell^+\ell^+ + X) - \sigma(B^0\overline{B}^0 \to \ell^-\ell^- + X)}{\sigma(B^0\overline{B}^0 \to \ell^+\ell^+ + X) + \sigma(B^0\overline{B}^0 \to \ell^-\ell^- + X)} = \frac{\mathrm{Im}\, \frac{\Gamma_{12}}{M_{12}}}{1 + \frac{1}{4}\left|\frac{\Gamma_{12}}{M_{12}}\right|^2} \tag{22}$$

With three families one finds [14] for $m_t = 40$ GeV:

$$a_{SL}(B_d) \lesssim 0.01\,, \quad a_{SL}(B_s) \lesssim \text{few} \times 10^{-4}\ . \tag{23}$$

These numbers are distressingly small, in particular when one keeps in mind that for $m_t = 40$ GeV the number of like-sign dileptons coming from $B_d - \overline{B}$ events is expected to be tiny. The asymmetry drops rather quickly with increasing m_t.

Adding a fourth family brightens the prospects: with $m_t \sim$ 40-60 GeV, $m_{t'} \sim$ 200 GeV and the new KM parameters following scenario A, $a_{SL}(B_s)$ can get increased significantly

$$a_{SL}^{(4)}(B_s) \sim 0.01\ . \tag{24}$$

This asymmetry again drops quickly with increasing m_t.

(C) CP Violation in Non-leptonic Decays:

CP asymmetries can emerge also in non-leptonic B^0 decays [15]:

$$A_{NL} = \frac{\Gamma(\overline{B}^0(t) \to \overline{f}) - \Gamma(B^0(t) \to f)}{\Gamma(\overline{B}^0(t) \to \overline{f}) + \Gamma(B^0(t) \to f)} \simeq \sin \Delta mt\ \mathrm{Im}\ \frac{p}{q}\ \rho_f \tag{25}$$

where ρ_f denotes a ratio of transition amplitudes

$$\rho_f = \frac{A(\overline{B}^0 \to f)}{A(B^0 \to f)}\ . \tag{26}$$

Examples of interesting decay channels are (for a more complete list see ref. [15]: $B_d \to \psi K_s$, $B_s \to \psi\phi$. These decay channels are based on the quark transition $b \to c\bar{c}s$; in those cases one can show that $\frac{p}{q}\,\rho_f$ is—to a very high accuracy—a unit vector in the complex plane and can be expressed just in terms of KM parameters and quark masses. With three families one finds:

$$\frac{p}{q}\,\rho_f\bigg|_{B_d} = \frac{(V_{dt}V_{bt}^*)^2}{|V_{dt}V_{bt}^*|^2}\,\frac{(V_{bc}^*V_{sc})^2}{|V_{bc}^*V_{sc}|^2} \simeq \frac{(1-\rho-i\eta)^2}{(1-\rho)^2+\eta^2} \tag{27}$$

and thus

$$-\mathrm{Im}\,\frac{p}{q}\,\rho_f\bigg|_{B_d} \simeq \frac{2(1-\rho)\eta}{(1-\rho)^2+\eta^2} \sim 0.25-1 \tag{28}$$

using $\eta \geq 0.26$ as obtained from an analysis of ϵ_K in K_L decays, see ref. [9]. This is not a small number—yet the size of the observable asymmetry (Eq. (25)) depends also on Δm although that quantity *a priori* has nothing to do with CP violation. Using Eq. (15) one obtains for the three different top masses of 40, 100, 160 GeV:

$$A_{NL}^{(3)}(B_d) \sim (0.25-1)\sin[(0.03\,,\ 0.15\,,\ 0.31)t/\tau_B]\ . \tag{29}$$

In the presence of a fourth family these asymmetries can become considerably larger, mainly due to an increase in Δm

$$A_{NL}^{(4)}(B_d) \sim (0.25-1)\sin[(0.1\,,\ 0.3\,,\ 0.5)t/\tau_B]\ . \tag{30}$$

The situation is different in B_s decays. For with three families one finds

$$\frac{p}{q}\,\rho_f\bigg|_{B_s} = \frac{(V_{bt}^*V_{st})^2}{|V_{bt}^*V_{st}|^2}\,\frac{(V_{bc}^*V_{sc})^2}{|V_{bc}^*V_{sc}|^2} \simeq 1-2i\eta\lambda^2 \tag{31}$$

and therefore

$$\mathrm{Im}\,\frac{p}{q}\,\rho_f\bigg|_{B_s} \sim -\frac{1}{10}\,\eta \sim -0.026-0.08 \tag{32}$$

leading to asymmetries of not more than a few percent.

The smallness of the asymmetry in KM allowed B_s decays is due to the fact that on the leading level only the second and third family contribute. The addition of a fourth family is quite likely to change this situation very significantly via its impact on p/q:

$$\frac{p}{q} = \frac{M_{12}^{(4)}}{\left|M_{12}^{(4)}\right|}$$
$$M_{12}^{(4)} = M_{12}^{(3)}\left[1 + \frac{2V_{bt'}^{*}V_{st'}}{V_{bt}^{*}V_{st}}\,\frac{E(t,t')}{E(t,t)} + \frac{(V_{bt'}^{*}V_{st'})^2}{(V_{bt}^{*}V_{st})^2}\,\frac{E(t',t')}{E(t,t)}\right] \tag{33}$$

where $E(i,j)$ denotes the box contribution with internal quarks i,j and the KM parameters factored out. With $m_t = 40$ GeV and $m_{t'} = 200$ GeV one finds in scenario A:

$$M_{12}^{(4)} \simeq M_{12}^{(3)}[1 - 1.24(\alpha + i\beta) + 0.77(\alpha + i\beta)^2]\ . \tag{34}$$

Accordingly there exist possible scenarios with

$$\mathrm{Im}\ \frac{p}{q}\,\rho_f\Big|_{B_s} \sim \pm\frac{1}{2} \tag{35}$$

thus allowing for very large observable CP asymmetries of up to 50%.

As usual the prospects are less promising—but not hopeless—in scenario B:

$$M_{12}^{(4)} \simeq M_{12}^{(3)}[1 - 0.07(\alpha + i\beta) + 0.002(\alpha + i\beta)^2] \tag{36}$$

and therefore

$$\mathrm{Im}\ \frac{p}{q}\,\rho_F\Big|_{B_s} \lesssim 0.2\ . \tag{37}$$

5. Conclusions:

(i) It seems to us that a dedicated search for $K^+ \to \pi^+\nu\overline{\nu}$ has the best chance to reveal the existence of ultra-heavy fermions: with three families and a "light" top quark, *i.e.*, $m_t \sim 40$ GeV, $BR(K^+ \to \pi^+\nu\overline{\nu})$ cannot exceed the 10^{-10} level; a measured branching ratio larger than 10^{-10} therefore establishes the existence of ultra-heavy fermions, either a t or a t' quark. However a branching ratio below 10^{-10} does not necessarily rule against such heavy states.

(ii) Top quarks with $m_t \geq 100$ GeV can raise the observable $B_d - \overline{B}_d$ mixing strength, r_d, to and above the percent level; a t' can raise it even further to the 10-13% level, *i.e.*, to the present experimental bound. A weaker than predicted $B_s - \overline{B}_s$ mixing—*e.g.*, $v_s < 0.3$ [0.9] for $m_t \geq 40[100]$ GeV—would provide good evidence for a fourth family. Again, the absence of such signals cannot be invoked to rule against a fourth family.

The prospects for uncovering clear evidence for a fourth family appear only marginal in other reactions, at least at existing accelerators (a complementary discussion can be found in ref. 16.)

(iii) The branching ratio $Z^0 \to b'\overline{b} + \overline{b}b'$ could—with some luck—reach 10^{-5} which might not represent a completely hopeless challenge to experimental scrutiny.

(iv) A fourth family—but *not* an ultra-heavy top—could generate a 1% CP asymmetry in semileptonic B_s decays.

(v) A fourth family—but *not* an ultra-heavy top—will very naturally produce CP asymmetries in KM-favoured non-leptonic B_s decays that could be as large as 50%. It can also enhance the corresponding asymmetries in B_α decays.

Summary

Keeping the theoretical uncertainties and experimental limitations in mind one cannot guarantee that the existence of ultra-heavy quarks with $m_Q \geq 100$ GeV will be established in low-energy decays. Yet there is a good chance for this to happen.

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