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On-shell scattering theory with empirical input*

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ABSTRACT

We accept from experiment either a two particle binding energy and scattering length (or scattering length and effective range) for each two particle s-wave channel in a three particle system. These observables can be supplemented, if need be, by phase shifts measured at higher energy. We show that our general on-shell scattering theory (which we here stop calling a "zero range theory" because that has led to confusion) makes unique, finite and unitary predictions for three particle elastic and rearrangement scattering, breakup and coalescence, and three particle bound state energies, using only this empirical input. Comparison with experiment of predictions drawn from this model would delimit the significant "off shell" and/or "three body force" effects that more detailed models must explain.

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I. INTRODUCTION

If we understood the forces between two hadrons, we should at least be able to use this knowledge to calculate the behavior of three hadron systems. Currently we lack this much understanding. For example, the nucleon-nucleon scattering amplitudes are known up to, and in some cases well beyond, pion production threshold; yet, from this knowledge we cannot predict the binding energy of the triton or ^3He, or their electromagnetic form factors. Even the n-p capture cross section at threshold differs by 10% from the model-independent[1] Bethe–Longmire prediction. The reason is, as we learned long ago from the Wick explanation[2] of Yukawa's meson theory,[3] that the coupling of the uncertainty principle to special relativity entails the creation of mesonic degrees of freedom at short distance. Nuclear physicists usually assume that these hidden degrees of freedom can be approximated by a "potential", but there is no unique way to define such a potential once the short range non-locality implied by the Wick–Yukawa mechanism is taken seriously.

Faced with this ambiguity, it is important to have clear experimental criteria for determining what new information is contained in three hadron observables that is not already predictable using two hadron observables. Starting from the "Fixed Past—Uncertain Future" interpretation of quantum mechanics,[4] it was proposed that such a reference theory might be provided by calculating three particle amplitudes using only two particle on shell scatterings.[5] Once a way of doing this has been developed, the theoretically ambiguous mixture of mesonic (and quark?) effects—designated but not defined by the terms "off shell effects" and "three body forces"—could be unambiguously separated by subtracting the prediction based solely on two particle observables from experimental results. Three attempts to articulate such a theory failed.[6–8]

Our first paper which gave a rigorous mathematical answer to the problem was entitled "Zero Range Scattering Theory. I."[9]—a title which has turned out to be misleading. What most people seem to think of as a "zero range theory" (see ZRST I, Ref. 9 for early history) is the zero range limit (often modeled by a δ-function) of some non-relativistic phenomenological potential used as the interaction term in the Schroedinger, Lippmann–Schwinger, Alt–Grassberger–Sandhas or Faddeev equations. This approach can be used to unify the infinite accumulation of three body bound states identified in the Efimov and Thomas effects,[10] but does *not* lead to a well defined three body theory based only on two body observables. In contrast ZRST I took the "zero range limit" in the sense of taking the off-shell behavior in the Kowalski-Noyes functions to zero, which was shown to be equivalent to a boundary condition at zero radius. The same limit in the three body system then was shown to define unique and unitary three body equations using as input only two body bound state poles and a dispersion-theoretic representation of the two body amplitudes in the physical region. To avoid future confusion we will from now on refer to *our* "zero range scattering theory" as an "on-shell scattering theory".

The difficulty with ZRST I was that it applied only to models of the two body phase shifts which had no singularities when analytically continued to negative energies other than the bound state poles, thus apparently excluding the "left hand cuts" due to meson exchange or the popular non-relativistic potential models. In this paper we will show that this difficulty can be circumvented (at least in systems with weakly bound or virtual states close to two-particle threshold such as the two nucleon system) by lumping these singularities in a single pole whose parameters are calculable from the two particle threshold parameters in each channel; the usual "effective range model" provides the starting point from which these models can be generalized without going outside the framework of unique connection between the model and two particle physical observables.

The need for a unique reference calculation of the three nucleon system using *only* two particle observables, which initiated this research over two decades ago, still persists. This was made clear at Few Body XI[11] and the satellite Workshop.[12] For example, both the Sendai[13] and the Iowa–Los Alamos[14] triton calculations showed that once the triton binding energy was fixed other three nucleon observables fell into place. No one explained which of the available ill defined parameters should be picked to adjust in order to achieve these results, let alone how to give empirically testable criteria for distinguishing among them. This led Redish to remark[15] that what we badly need is a "shape independent" approximation for the three nucleon system capable of identifying observables which *are known to be* sensitive to mesonic degrees of freedom; of course this is could be one application of the approach this author asked for at Few Body V a decade and a half ago.[5]

Results presented at the Workshop and at Few Body XI made the problem acute. Sasakawa's result[12] that the new Bonn potential almost fits the triton *without* any three-body force has been achieved independently by the Basel–Bonn–Los Alamos group.[16] They attribute the discrepancy between their results and numbers obtained using older models to the fact that the new model has more central and less tensor force; this makes sense. However, it seems unlikely that this has been achieved without some *on-shell* (phase shift) differences between this model and earlier "fits" to the two nucleon data. Again, whether this new result is due to on-shell or off-shell differences between the new potential and earlier models cannot be unambiguously determined using currently available techniques.

Two papers presented at the Workshop made a new start on the low energy threshold problem. Adhikari[17] used a $1/r^2$ long range potential, cut off at $R_{Efi} >> \hbar/m_\pi c$, to simulate the Efimov effect in the $n-d$ doublet state, and a short range ($R_{nuc} \simeq \hbar/m_\pi c$) "nuclear" potential to fix the threshold parameters. In this way he succeeded in reproducing not only the "Phillips plot" connecting a_2 and ϵ_t but the peculiar (from a "local potential" point of view) energy dependence of $k\ ctn\ \delta_2^{nd}$ close to threshold. For those who are not experts in this field, I add the comment that the "received wisdom" is that no simple "potential model" can reproduce the observed energy dependence of $k\ ctn\ \delta_2^{nd}$; accepted explanations require explicit three-particle dynamics[18]. More remarkable is that when Adhikari simply added the p-d coulomb potential to this model, he could reproduce the "Phillips plot" connecting a_2^{pd} to the binding energy of ^3He in *quantitative* agreement with the Iowa-Los Alamos calculations. The experts present at WORK could not accept these detailed quantitative results as relevant because they were not "derived" from a multi-particle "model" in any sense they were willing to accept.

Hasegawa[19] confined himself to the $n-d$ system, but came closer to recognizable 3-particle dynamics by using three "effective range" formulae (for $tan\ \delta/k$) related to the "direct", "knockon" and "pickup" components of the standard Faddeev channel decomposition. His energy dependence of

$$(k\ ctn\ \delta_2^{nd})^{-1} = \frac{1}{k\ ctn\ \delta}^{direct} + \frac{1}{k\ ctn\ \delta}^{knockon} + \frac{1}{k\ ctn\ \delta}^{pickup}$$

is obtained[20] from variational calculations of the Faddeev components of the wave function, and does not relate these to a conventional "shape independent" formulation.

We mention both contributions because they led us to re-examine the reasons that caused us to abandon the program proposed[5] so long ago, which we have revived here.

In Chapter II we give the basic connection between the on-shell two body threshold parameters, the measured phase shifts, and the dispersion-theoretic representation of the two body amplitude needed for our three body theory. In Chapter III we show that the lumped singularity in the two body amplitudes which replaces the "left hand cut" can be prevented from destroying the unitarity of the three body theory by making an appropriate subtraction in the integral equations. We show that this prescription introduces a short-range repulsive effective interaction in the three body system due to particle exchange analagous to the long range attractive interaction which Barton and Phillips[18] derived from the single nucleon exchange mechanism. This has the effect of shielding the short range behavior of the system where meson exchange effects [or their simulation by some sort of (necessarily off shell) "potential"] bring in degrees of freedom our model is designed to ignore. Chapter IV sketches how the theory might be applied, with a focus on the three nucleon system.

II. THE TWO BODY EMPIRICAL INPUT

We have shown[9] that in our on-shell limit, the three-body Faddeev amplitudes M_{ab} driven by two body s-wave scatterings depend only on the spectator momenta $\vec{p}_a$ and that the two particle on-shell amplitudes factor out, allowing the representation (which holds[9] for *any* three particle theory on shell):

$$M_{ab}(\vec{p}_a,\vec{p}_b;z) = t_a(z-\tilde{p}_a^2)\delta_{ab}\delta(\vec{p}_a-\vec{p}_a^0) + t_a(z-\tilde{p}_a^2)Z_{ab}(\vec{p}_a,\vec{p}_b;z)t_b(z-\tilde{p}_b^2)\ . \quad (2.1)$$

Here z is the three body energy normalized to zero at the lowest breakup threshold, $\tilde{p}_a^2 = p_a^2/2n_a$ with $n_a = m_a(m_b+m_c)/(m_a+m_b+m_c)$, and Z_{ab} satisfies the equation

$$Z_{ab}(\ \vec{p}_a, \vec{p}_b; z) - R_{ab}(\ \vec{p}_a, \vec{p}_b; z)$$

$$= \Sigma_{c=a\pm} \int d^3p_c' R_{ac}(\ \vec{p}_a, \vec{p}_c'; z) t_c(z - (\tilde{p}_c')^2) Z_{cb}(\ \vec{p}_c', \vec{p}_b; z) \tag{2.2}$$

$$= \Sigma_{c=b\pm} \int d^3p_c' Z_{ac}(\ \vec{p}_a, \vec{p}_c'; z) t_c(z - (\tilde{p}_c')^2) R_{cb}(\ \vec{p}_c', \vec{p}_b; z)\ , \tag{2.3}$$

where

$$R_{ab}(\ \vec{p}_a, \vec{p}_b; z) = \frac{-\bar{\delta}_{ab}}{p_a^2/2\mu_b + p_b^2/2\mu_a + \vec{p}_a \cdot \vec{p}_b/m_c - z} = R_{ba}(\ \vec{p}_b, \vec{p}_a; z)\ , \tag{2.4}$$

and $\mu_a = m_b m_c/(m_b + m_c)$, etc. Our problem is to find a well defined model for the on-shell two particle amplitudes $t_a(\tilde{q}_a^2) = t_a(q_a^2/2\mu_a)$ with $\vec{q}_a = (m_b\vec{p}_a - m_a\vec{p}_b)/(m_a + m_b)$ which satisfies two particle on-shell unitarity in the dispersion-theoretic sense, i.e., when analytically continued to negative energies in the manner required for the solution of these equations. Given two-body amplitudes which satisfy the unitarity condition in the three-body space we have already proved in ZRST I that the three-body amplitudes so calculated satisfy three body on-shell unitarity.

Using as our paradigm the familiar nucleon-nucleon s-waves, the usual shape independent approximation allows us[21,22] to represent the singlet amplitudes by

$$\frac{e^{i\delta_s} sin\delta_s}{q} = \tau_s^{\alpha,\beta}(q^2) = \frac{\beta - \alpha}{(\alpha + \sqrt{-q^2})(\beta - \sqrt{-q^2})}\ , \tag{2.5}$$

where (recall that $a_s < 0$)

$$\alpha = \frac{[\sqrt{1-2r_s/a_s}-1]}{r_s}\ ; \qquad \beta = \frac{[\sqrt{1-2r_s/a_s}+1]}{r_s}, \tag{2.6}$$

and the triplet amplitude by

$$\frac{e^{i\delta_t} sin\delta_t}{q} = \tau_t^{\gamma,\phi}(q^2) = -\frac{\phi+\gamma}{(\gamma-\sqrt{-q^2})(\phi-\sqrt{-q^2})}\ ; \tag{2.7}$$

with

$$\begin{aligned} \gamma &= \sqrt{2\mu\epsilon} = \frac{[1-\sqrt{1-2r_t/a_t}]}{r_t} \\ \phi &= \frac{[1+\sqrt{1-2r_t/a_t}]}{r_t} = \frac{\gamma}{\gamma a_t - 1}\ ; \end{aligned} \tag{2.8}$$

where we have used the shape independent "mixed effective range"

$$\rho(0,-\epsilon) = r_t = \frac{2}{\gamma}(1-\frac{1}{\gamma a_t})\ . \tag{2.9}$$

Since

$$\frac{2}{\pi}\int dk \frac{k^2}{(k^2+\alpha^2)(k^2+\beta^2)(k^2-q^2-i0^+)} = \frac{1}{(\alpha+\beta)(\alpha+\sqrt{-q^2})(\beta+\sqrt{-q^2})} \tag{2.10}$$

we can rewrite these amplitudes in the dispersion-theoretic form

$$\tau_s^{\alpha,\beta}(q^2) = \frac{2\beta(\beta-\alpha)}{(\beta+\alpha)(\beta^2+q^2)} + \frac{2}{\pi}\int dk \frac{sin^2\delta_k^{\alpha,\beta}}{(k^2-q^2-i0^+)}$$

$$\tau_t^{\gamma,\phi}(q^2) = \frac{2\phi(\phi+\gamma)}{(\phi-\gamma)(\phi^2+q^2)} - \frac{2\gamma(\phi+\gamma)}{(\phi-\gamma)(\gamma^2+q^2)} + \frac{2}{\pi}\int dk \frac{sin^2\delta_k^{\gamma,\phi}}{(k^2-q^2-i0^+)} \tag{2.11}$$

Then in the on-shell Faddeev equations we will have, in addition to the poles in $\tau(z-\tilde{p}^2)$, the terms $\frac{2}{\pi}\int dk \frac{sin^2\delta_k}{(k^2+p^2-z)}$. Clearly these are non-singular for $Re\ z < 0$

and give no more difficulty than the terms $\frac{2}{\pi}\int k^2dk\ t(\vec{q},\vec{k};\tilde{k}^2)t^*(\vec{k},\vec{q}';\tilde{k}^2)/(\tilde{k}^2+\tilde{p}^2-z)$ in the conventional theory. Above breakup threshold these terms will make our effective interaction complex, which is of course what would be expected from the "optical potential" phenomenology.

Written in this way, it would appear that we can extend the model to *arbitrary* empirical phase shifts simply by replacing the effective range approximation in the integral by a numerical integral, or analytic fit to observed phase shifts in the physical region. This is indeed the case [at least for models which, like the two-nucleon system have bound or virtual states (i.e., the deuteron and "singlet deuteron") close to two-body threshold] for any two body system whose phase shifts are elastic up to all relevant energies and have the property $\frac{2}{\pi}\int_0^\infty sin^2\delta/k^2=\Delta<\infty$. Then we can extend our model to arbitrary phase shifts in this class by defining

$$\tau_s^{\Delta_s}(q^2)=\frac{\Gamma_\beta}{(\beta^2+q^2)}+\frac{2}{\pi}\int dk\frac{sin^2\delta_k^{\Delta_s}}{(k^2-q^2-i0^+)}\ , \tag{2.12}$$

$$\tau_t^{\Delta_t}(q^2)=\frac{\Gamma_\phi}{(\phi^2+q^2)}-\frac{N_\gamma^2}{(\gamma^2+q^2)}+\frac{2}{\pi}\int dk\frac{sin^2\delta_k^{\Delta_t}}{(k^2-q^2-i0^+)}\ . \tag{2.13}$$

As was pointed out by Castillejo[23] this prescription would be inconsistent if we used the earlier definitions for the residues at the deepest lying poles. We define α,β,γ,ϕ as before in terms of the threshold parameters [Eqs. (2.6), (2.8)], which in effect makes a linear extrapolation of $q\ ctn\delta$ to negative energies, and note that $\tau(0)=-a$. Then a little algebra suffices to establish that the new pole residues are not given by the effective range formulae but by

$$\Gamma_\beta=\beta^2(-a_s-\Delta_s)=\frac{\beta}{\alpha}(\beta-\alpha)-\beta^2\Delta^s>0\ ,$$

$$\Gamma_\phi = \phi^2(a_t + \frac{N_\gamma}{\gamma^2} - \Delta_t) = \frac{\phi}{\gamma}(\phi - \gamma) + 2\frac{\phi^2(\phi+\gamma)}{\gamma(\phi-\gamma)} - \phi^2\Delta_t > 0 \ , \tag{2.14}$$

where the residues must be positive if these poles are not to represent deep lying bound states. Here we have used the effective range approximation for the reduced width at the physical bound state $N_\gamma^2 = \frac{2\gamma(\phi+\gamma)}{\phi-\gamma}$; an empirical value N_γ^2 would still be acceptable if the $\Gamma > 0$ constraint is satisfied.

Although we are not concerned with higher partial waves in this paper, we note that the phenomenology can be extended to include them, and in this case *requires* a phenomenological pole at negative energies. This is easy to see, because the threshold behavior of $sin\ \delta_\ell/q$ is proportional to $q^{2\ell}$; there is no analytic function having only the right hand cut with this behavior at threshold which vanishes like $1/q^2$ for large q. However a single pole at negative energies suffices to fix this up; given the parameters for an effective range approximation for $q^{2\ell+1}ctn\ \delta_\ell$, an appropriate model can readily be constructed.

Were it not for the poles at $q^2 = -\beta^2$ and $-\phi^2$, this "shape independent" model would already have met our requirements in ZRST I and we could have gone on to application of the theory long ago. The difficulty is that if we look at the implications in configuration space, these poles will generate an asymptotic wave function proportional to

$$\frac{e^{i\vec{p}_\omega\cdot\vec{r}}}{r}\frac{e^{-\omega\rho}}{\rho}Z(\vec{p}_\omega, \vec{p}_0; \tilde{p}^2_\omega(p_0^2) - \tilde{\omega}^2) \ , \tag{2.15}$$

where $\omega = \beta$ or ϕ, r is the distance of the spectator from the center of mass of the interacting pair, and ρ the relative coordinate of the interacting pair. Here $p^2_\omega(p_0^2) = \tilde{p}_o^2 - \epsilon_0 + \tilde{\omega}^2$, ϵ_0 is the binding energy of the lowest bound state and $\vec{p}_0$

is the momentum of the spectator in that channel incident on that bound state in the zero momentum frame.

The asymptotic form in (2.15) is just what we want for the bound state poles. These forms describe the elastic (or anelastic) outgoing scattered wave functions multiplied by the on-shell "bound state wave functions". But if we were forced to include asymptotic terms from these "ghost" or "virtual state" poles at $\omega^2 = \beta^2, \phi^2$ this would be a disaster. Much of the flux would go into these waves, because they are more deeply "bound" than the physical bound states. Even worse, the residue for these ghost poles is of opposite sign to that for a bound state poles. These ghosts would contribute negative probabilities to the unitarity sum and ruin the critical connection between elastic scattering and breakup which the theory is designed to maintain at all costs. Simply to say that such terms do not appear asymptotically in our three body theory, which was suggested in an earlier draft of this paper, looks dangerous. Fortunately, we have found it possible to define the three body amplitude in such a way that the ghost pole terms cannot contribute outgoing waves. They still make the correct contribution to the two-particle "final state scatterings", where they form a necessary part of the parameterization and are needed for on-shell three particle unitarity.

III. ON-SHELL THREE BODY EQUATIONS

It is instructive to think of our zero range equations (2.2) as multi-channel Lippmann–Schwinger equations with an off-diagonal effective interaction $R_{ab}(z)$ and a propagator $t_c(z - \tilde{p}_c^2)$. This stares one in the face when one keeps only the lowest bound state pole term in t_c and puts z on-shell (i.e., $z = \tilde{p}_0^2 + \tilde{q}_0^2 = \tilde{p}_0^2 - \epsilon_0$), giving a propagator inversely proportional to $p_c^2 - (p_c^0)^2 - i0^+$. This was realized in the relativistic context of single quantum exchange[24] and leads directly to the confined quantum model.[25] Sandhas has often remarked[26] that when the AGS technique is used to reduce the number of degrees of freedom and define effective interactions between the resulting clusters, what were the driving terms in the larger space become the propagators in the smaller space, while what were the propagators in the larger space become the effective potentials in the smaller space. In our simple environment we see that we can even get *free particle* propagators if we restrict ourselves to one bound state pole term in each channel.

The form of the effective interaction given by $R_{ab}(\vec{p}_a, \vec{p}_b; z)$ in Eq. (2.4) and exhibited kinematically in Fig. 1 is particularly interesting. In configuration space it is simply a Yukawa potential between the two constituents of the opening or closing vertex with range $\hbar/\sqrt{2\mu\epsilon}$, i.e., the radius of the appropriate bound state. Clearly when the binding energy goes to zero, the range goes to infinity, giving us an immediate way to understand the Efimov[27,28] effect. Further, it immediately justifies in terms of three body dynamics the phenomenological long range potentials used by Adhikari[17] to derive the $n-d$ and $p-d$ Phillips plots.

Barton and Phillips[18] used Fig. 1 as a dispersion theory diagram to derive the complicated low energy behavior of doublet $n-d$ scattering and the $n-d$ Phillips plot connecting this behavior with the triton binding energy, Our on-shell theory will reproduce and refine their results. Figure 1 also shows that our model is closely related to Lovelace's "pole approximation"[29] to the Faddeev equations, or if we replaced the constant vertex parameters by form factors, to Amado's "non-relativistic field theory" for the $n-d$ system.[30] The advantage we claim for our approach is that we have related it directly to the on-shell limit of the

Kowalski–Noyes representation in the standard Faddeev theory, and can—up to a point—relate the input to standard on-shell dispersion-theoretic amplitudes.

As we have already noted, we cannot simply use our fit to the two body amplitudes in our on-shell equations because the "ghost" poles will not only contribute outgoing waves that look like deep lying bound states, but contribute them in such a way that they will introduce negative probabilities into the unitarity sum! However, the remedy is ready to hand. All we need do is to require that when the argument of Z_{ab} hits this value of momentum, the function be zero. Then there will be no singularity at the pole, and no two-particle outgoing wave; the model will yield only elastic scattering and anelastic scattering below breakup threshold, and three particle scattering with the usual (asymptotic) kinematic restrictions above breakup threshold. To force this constraint, we write the equation

$$Z_{ab}(\ \vec{p}_a^{\,\omega}\vec{p}_b;z) = 0 = R_{ab}(\ \vec{p}_a^{\,\omega},\vec{p}_b;z)$$

$$+\Sigma_{c=a\pm}\int d^3p_c^{\prime}R_{ac}(\ \vec{p}_a^{\,\omega},\vec{p}_c^{\,\prime};z)t_c(z-(\vec{p}_c^{\,\prime})^2)Z_{cb}(\ \vec{p}_c^{\,\prime},\vec{p}_b;z) \tag{3.1}$$

and subtract it from Eq. (2.2). We see that in effect this simply replaces the effective interaction R_{ab} by

$$R_{ab}^{\omega}(\ \vec{p}_a,\vec{p}_b;\vec{p}_a^{\,\omega},z) = R_{ab}(\ \vec{p}_a,\vec{p}_b;z) - R_{ab}(\ \vec{p}_a^{\,\omega},\vec{p}_b;z)\ . \tag{3.2}$$

At first sight this procedure appears somewhat arbitrary. Granted that the change in the equations only effects the value of the solution at energies far removed from those which represent physical scattering processes, one can still wonder what this prescription means. Fortunately we have already seen from the Barton–Phillips calculation that the first term corresponds to an attractive "potential" and indeed provides most of the binding energy of the triton treated as an $n-d$ system, even in an on-shell dispersion-theoretic treatment that neglects everything else! Consequently, the correction we prescribe for the interaction corresponds to a short range repulsion which sets in at negative (virtual) and positive (physical) energies where we would start to excite virtual pionic degrees of freedom in a more fundamental theory. At high energy we know that these pionic degrees of freedom look like a black (absorbing) region, which is well represented phenomenologically in the elastic channel by a "hard core" boundary condition at finite radius. We further know that in a relativistic model, this boundary condition evaluated using data above pion production threshold provides an intimate connection between behavior in the meson production region and the existence of the deuteron and "singlet deuteron" close to nucleon-nucleon threshold.[31] Our apparently arbitrary prescription in fact has some good physical reasoning behind it.

Since the input parameters for our model can be unambiguously computed from two particle observables measured (or measurable) in the laboratory we have met our original objective.[5]

IV. THE THREE-NUCLEON SYSTEM

Consider the n-d system driven by s-wave interactions described by the $n-n$ singlet, $n-p$ singlet and $n-p$ triplet s-wave shape independent effective range parameters. We have shown above how the departure of the phase shifts from this threshold behavior at higher energy, and in higher partial waves, can be included without going beyond experimentally accessible information about the two-nucleon system. We start by treating the two neutrons as formally distinguishable, although identical in mass and interactions. We assume that there is no n-n triplet interaction, as this would take us beyond s-wave input, but otherwise only impose the Pauli principle at the end of the analysis. Thanks to the identity of the masses and interactions in each of the two formally distinguishable $n-p$ systems, the equations reduce to three Faddeev channels, as we have demonstrated elsewhere.[32] Supplemented by the easily calculable[32] spin coefficients, the theory developed above can be applied directly. To make predictions, we must of course clothe the on-shell amplitudes Z_{ab} with the appropriate poles, two-particle scattering amplitudes and spin functions, add the Born terms, and compute the appropriate cross sections. We would then have a *predictive* theory for the $n-d$ system using only six on-shell (shape independent) parameters.

We outline a specific program for future research. First make a rough calculation of the triton binding energy predicted by the six input parameters. Since Barton and Phillips obtain most of the binding energy for the triton using only the deuteron pole and triplet scattering length (with a_t invoked only to get the correct pole residue) in a dispersion-theoretic calculation equivalent to keeping only the driving term R_{ab} in our equations, we anticipate that solving our on-shell equations would improve on their result. Next normalize these equations to fit a_2 and ϵ_t and use them to predict 3-nucleon observables at higher energy. This would define a minimal "shape independent" approximation for the three-nucleon system. When extended to include higher nucleon-nucleon partial waves our on-shell model could provide a unique reference theory that will tell us what experiments contain "off shell", "meson exchange" and "three body force" effects by separating them from those effects which can already be predicted within experimental error using known two-nucleon scattering data.

CONCLUSIONS

In this paper we have developed a general three particle theory using only two particle observables, which could be readily extended to a system of three distinguishable spin-$\frac{1}{2}$ particles and applied to the low energy behavior of the three nucleon system. Our first approximation here uses only the six threshold two nucleon parameters as input. We claim that when this numerical work is extended to include higher partial waves and make predictions of three nucleon observables, *any* theory of the three nucleon system will then have the following tasks: (a) It must first fit the two nucleon data within experimental error (no extant models do even this) or (failing that) show that its departure from experiment in this respect has no *significant* qualitative or quantitative effect on the predictions it makes about three-nucleon observables—we believe our approach will assist others in this task. We predict that no *a priori* three-nucleon theory based solely on two-nucleon data will fit both a_2 and ϵ_t within experimental error. (b) Hence any theory will have the task of explaining why it fails (other than through inadequacy of the on-shell 2-nucleon data) and where one or more intrinsic 3-nucleon parameters *have* to be taken from experiment in order to resolve the discrepancy. (c) Having established these inherent limits of accuracy any theory must then find where its predictions (after normalization to a_2 and ϵ_t) differ significantly from those provided by our "shape independent" approach. All of us can then get on with the real job of trying to understand what the three nucleon system tells us that we do not already know, or can predict, from available two-nucleon data.

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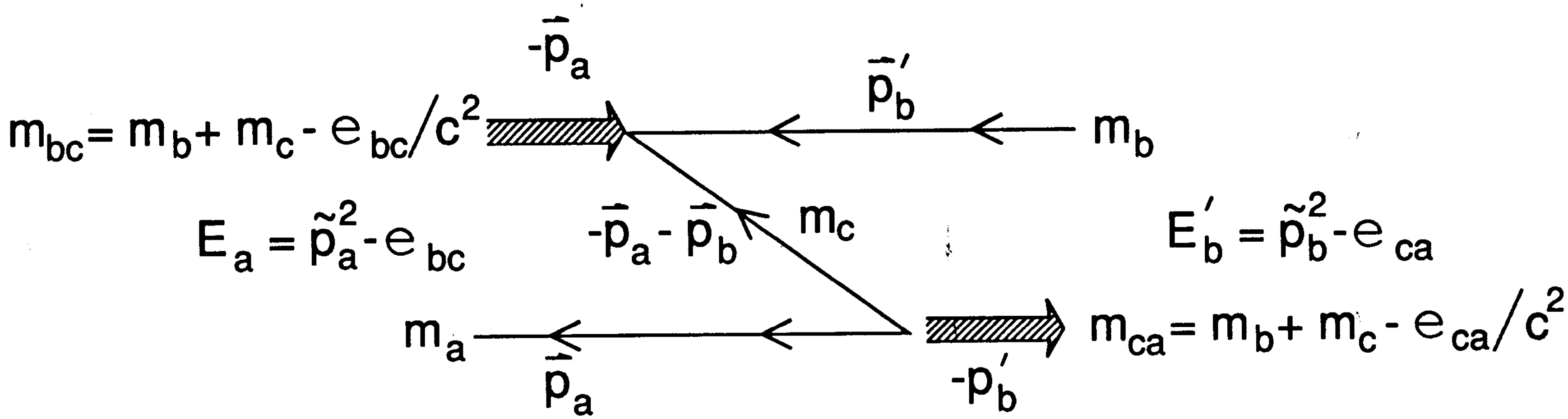
FIGURE CAPTION

FIG. 1. The kinematics of the three particle propagator $R_{ab}(\vec{p}_a(E_a), \vec{p}_b'(E_b'); E)$, and hence the (2,2) effective potentials, in the minimal on-shell model.



$$E = \frac{p_a^2}{2\mu_b} + \frac{p_b'^2}{2\mu_a} + \frac{\vec{p}_a \cdot \vec{p}_b'}{m_c}$$

$$\mu_b = \frac{m_c m_a}{m_c + m_a} \quad \mu_a = \frac{m_b m_c}{m_b + m_c}$$

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Fig 1