SLAC – PUB – 4170 REV.
August 1987
(T)

$e^+e^- \to \tau^+\mu^-$ Due to Slepton Mixing

MICHAEL J. S. LEVINE*

Stanford Linear Accelerator Center

Stanford University, Stanford, California, 94305

and

Department of Physics,

Stanford University, Stanford, California, 94305

ABSTRACT

We consider a softly-broken supersymmetry model in which explicit flavour mixing occurs in the supersymmetric sector. Attention has been restricted to the leptonic sector, and a model-independent approach has been adopted. For the reaction $e^+e^- \to Z^0 \to \tau^+\mu^-$ at Z^0 energies, it is found that, for reasonable parameters, $\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$ is $\mathcal{O}(10^{-7} - 10^{-6})$.

Submitted to *Physical Review D*

⋆ Work supported by the Department of Energy, contract DE-AC03-76SF00515.

Thus far violation of lepton family number conservation has not been observed. Despite this, in many extensions of the standard model we find that lepton number is broken to some extent.[1] This can occur through massive neutrino oscillations,[2] exotic family-blind particles,[3] or violation of lepton number itself.[4]

In this paper we focus on the supersymmetric "standard" model with soft supersymmetry-breaking terms.[5] The supersymmetric partners of the leptons (*i.e.* sleptons) are given explicit, arbitrary, mass terms in the Lagrangian and allowed to mix with arbitrary angles. The resultant calculations are model-independent, and parameters predicted by specific models may be readily inserted. A more detailed description of the model will be published in a subsequent paper[6] along with further results.

The Lagrangian for leptons assumed here is

$$\mathcal{L} = \mathcal{L}_{SGWS} + \mathcal{L}_{Break}$$

where $\mathcal{L}_{SGWS}$ is the standard supersymmetric electroweak Lagrangian with two Higgs doublets and $\mathcal{L}_{Break}$ contains the explicit soft supersymmetry-breaking terms. The superfield content is given in Table 1.

Note that in a supersymmetric theory, a minimum[7] of two Higgs doublets must be used to provide masses for the u and d quarks. The reason for this is that the conjugate of the Higgs supermultiplets cannot be employed in the superpotential.[8]

The most general soft supersymmetry-breaking terms which can be added are[9]

$$\mathcal{L}_{Br} = -\tfrac{1}{2}M\tilde{W}^0\tilde{W}^0 - \tfrac{1}{2}M'\tilde{B}^0\tilde{B}^0 - \mu\tilde{\psi}^0_{H_1}\tilde{\psi}^0_{H_2} - M\tilde{W}^+\tilde{W}^- + \mu\tilde{\psi}^-_{H_1}\tilde{\psi}^+_{H_2}$$
$$+ m_i\tilde{\ell}^*_i\tilde{\ell}_i + m_{i,j}\tilde{\ell}^*_i\tilde{\ell}_j + \hat{m}_i\tilde{\nu}_i\tilde{\nu}_i + \hat{m}_{i,j}\tilde{\nu}_i\tilde{\nu}_j + h.c. \qquad (1a)$$

where i and j run over six sleptons ($\tilde{e}_{L,R}$, $\tilde{\mu}_{L,R}$ and $\tilde{\tau}_{L,R}$) and three sneutrinos ($\tilde{\nu}_{e,\mu,\tau}$). We use notation based upon that of Haber and Kane.[10] Here $v_1 = \langle H_1{}^0 \rangle$ and $v_2 = \langle H_2{}^0 \rangle$ are the vacuum expectation values of the two scalar Higgs fields. It is convenient to define the angle θ_v by $\tan\theta_v \equiv \frac{v_1}{v_2}$. In addition there are also terms which are the supersymmetric analogues of the $W^0 H^0_i H^0_j$ and $W^\pm H^\mp_i H^0_j$ vertices:

$$\mathcal{L}_{Mix} = \frac{i}{2}\left(g'\tilde{B}^0 - g\tilde{W}^0\right)\left(v_1\tilde{\psi}^0_{H_1} - v_2\tilde{\psi}^0_{H_2}\right)$$
$$- \frac{ig}{\sqrt{2}}\left[v_1\tilde{W}^+\tilde{\psi}^-_{H_1} + v_2\tilde{W}^-\tilde{\psi}^+_{H_2}\right] + h.c. \qquad (1b)$$

Typically the terms involving M, M', μ and θ_v will induce mixing among the charged Higgsinos and gauge fermions (gauginos) in conjunction with similar mixing between the neutral states. In the minimal model with two Higgs doublets we will have two charged states, the "charginos", $\tilde{\chi}^+_i$, and four neutral states, the "neutralinos", $\tilde{\chi}^0_i$. (If a Higgs singlet is included then the number of neutralinos becomes five.)

In general the left and right sleptons will mix together. This leads to a number of effects including enhancement of anomalous magnetic moments. It is only global lepton family number conservation which would seem to prevent $\tilde{e}$,

$\tilde{\mu}$ and $\tilde{\tau}$ from mixing. If we permit this mixing to occur then we expect to find small effects in the non-supersymmetric sector such as radiative muon decay. These could, of course, be in *addition* to other non-standard effects, such as those alluded to earlier. In general, then, we have mixing between six charged slepton states (involving 15 real angles and 10 complex phases) and three neutral sneutrino states (assuming no $\tilde{\nu}_R$ in the theory).

In order to reduce the burgeoning number of parameters somewhat we will consider the case where only the two 'heaviest' generations mix significantly with little left-right mixing. Thus the only mixings are between:

$$\begin{aligned} &\tilde{\mu}_L \text{ and } \tilde{\tau}_L \text{ with angle } \theta_L \\ &\tilde{\mu}_R \text{ and } \tilde{\tau}_R \text{ with angle } \theta_R \\ &\tilde{\nu}_\mu \text{ and } \tilde{\nu}_\tau \text{ with angle } \theta_\nu \end{aligned} \tag{2}$$

So that

$$\begin{aligned} \tilde{\ell}_{L_1} &= \tilde{\mu}_L \cos\theta_L + \tilde{\tau}_L \sin\theta_L \\ \tilde{\ell}_{L_2} &= -\tilde{\mu}_L \sin\theta_L + \tilde{\tau}_L \cos\theta_L \qquad etc. \end{aligned}$$

with $\tilde{\ell}_{L,R_1}$, $\tilde{\ell}_{L,R_2}$ and $\tilde{\nu}_{1,2}$ the physical mass eigenstates.

We consider the reaction $e^+e^- \to \tau^+\mu^-$ at a centre of mass energy equal to the Z^0 mass. We may therefore ignore all channels except those which involve a real, on-shell, Z^0 and restrict our attention to $Z^0 \to \tau^+\mu^-$. The contributing diagrams are illustrated in Fig. 1. The resultant matrix element is given by

$$\mathcal{M}^{\omega} = \mathcal{M}_L \xi^{\omega}{}_L + \mathcal{M}_R \xi^{\omega}{}_R \tag{3}$$

$$\xi^{\omega}{}_{L,R} = \bar{u}^{\mu}(p_1{}')\gamma_{\mp}\gamma^{\omega}\gamma_{\pm}v^{\tau}(p_2{}') \qquad \gamma_{\pm} = \tfrac{1}{2}\left(1 \pm \gamma_5\right) \tag{3a}$$

$$\begin{aligned}
\mathcal{M}_L = \frac{\mathcal{K}_\nu}{\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{2} \Bigg\{ & |V_{i1}|^2 \left[\Delta G(T_i^+, R_i^+) + \cos 2\theta_w \Delta G(T_i^+, 0)\right] \\
& + 2\sum_{j=1}^{2} V_{i1}^* V_{j1} \Bigg[\sqrt{S_{ij}^+}\, O_{ij}^{R}{}'\, \Delta\check{I}_{[1]}(T_i^+, S_{ij}^+, R_i^+) \\
& \qquad - R_i^+ O_{ij}^{L}{}'\, \Delta\check{I}_{[\widetilde{\mathbf{Z}}^2-\mathbf{Z}^2]}(T_i^+, S_{ij}^+, R_i^+) \\
& \qquad + O_{ij}^{L}{}'\, \Delta\check{G}(T_i^+, S_{ij}^+, R_i^+)\Bigg]\Bigg\} \\
- \frac{\mathcal{K}_L}{2\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{4} \Bigg\{ & |\tan\theta_w N_{H\,i1} + N_{H\,i2}|^2 \cos 2\theta_w \Delta\tilde{G}(T_L{}^0{}_i, R_i^0) \\
& - 2\sum_{j=1}^{4} (\tan\theta_w N_{H\,i1}^* + N_{H\,i2}^*)(\tan\theta_w N_{H\,j1} + N_{H\,j2}) \\
& \quad \Bigg[\sqrt{S_{ij}^0}\, O_H^{R}{}''_{ij}\, \Delta\check{I}_{[1]}(T_L{}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad - R_i^0 O_H^{L}{}''_{ij}\, \Delta\check{I}_{[\widetilde{\mathbf{Z}}^2-\mathbf{Z}^2]}(T_L{}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad + O_H^{L}{}''_{ij}\, \Delta\check{G}(T_L{}^0{}_i, S_{ij}^0, R_i^0)\Bigg]\Bigg\}
\end{aligned} \tag{3b}$$

$$\begin{aligned}
\mathcal{M}_R = \frac{2\mathcal{K}_R}{\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{4} \Bigg\{ & |\tan\theta_w N_{H\,i1}^*|^2 \cdot 2\sin^2\theta_w \Delta\tilde{G}(T_R{}^0{}_i, R_i^0) \\
& + 2\sum_{j=1}^{4} (\tan\theta_w N_{H\,i1})(\tan\theta_w N_{H\,j1}^*) \\
& \quad \Bigg[\sqrt{S_{ij}^0}\, O_H^{L}{}''_{ij}\, \Delta\check{I}_{[1]}(T_R{}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad - R_i^0 O_H^{R}{}''_{ij}\, \Delta\check{I}_{[\widetilde{\mathbf{Z}}^2-\mathbf{Z}^2]}(T_R{}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad + O_H^{R}{}''_{ij}\, \Delta\check{G}(T_R{}^0{}_i, S_{ij}^0, R_i^0)\Bigg]\Bigg\}
\end{aligned} \tag{3c}$$

$$K_\nu = \frac{i}{16\pi^2} e^3 \sin\theta_\nu \cos\theta_\nu$$

$$K_L = \frac{i}{16\pi^2} e^3 \sin\theta_L \cos\theta_L \tag{3d}$$

$$K_R = \frac{i}{16\pi^2} e^3 \sin\theta_R \cos\theta_R$$

$O_{H}{}''_{ij}$ and N_{Hij} arise from neutralino mixing at the vertices while O'_{ij} and V_{ij} arise from chargino mixing at the vertices. These are given in the appendix along with expressions for the mass eigenvalues. The ratios T, R and S and the various I and G functions are tabulated in Table 2.

We have assumed that the leptonic masses (m_μ, m_τ, m_{ν_μ} and m_{ν_τ}) are small compared with at least one of the supersymmetric particles in a subprocess, or with q^2 (which is $M_z{}^2$ in this case). Terms proportional to the lepton masses, which arise, for instance, from the Higgsino component of a vertex (chargino or neutralino) have thus been ignored. If a heavy fourth generation of leptons were involved this would not be the case.

The expression in (3) is quite complicated in general and rich in structure. The functions can be evaluated explicitly in terms of Dilogarithmic (Li_2) Functions, but this proves to be unilluminating. Some general properties should be noted. The functions $\Delta\tilde{G}$, $\Delta I_{[\mathrm{N}]}$ and $\Delta\check{I}_{[\mathrm{N}]}$ are, for the 'N' used, finite analytic functions of their arguments (which are ratios of masses squared) which go to zero as any of these masses tends to infinity. $\Delta G(T,R)$ and $\Delta\check{G}(T,S,R)$ are analytic but diverge logarithmically as $T \to \infty$, and hence as $m_{\tilde{\nu}}$ or $m_{\tilde{\ell}} \to \infty$. (See definitions of Table 2.) In both the neutralino and chargino sectors these terms always cancel among themselves in these limits. Thus the matrix element remains finite for any value of the twelve superparticle masses which are involved. As the mass of any particle becomes large, the contributions from subprocesses involving that particle vanish, in agreement with the Carazzone-Appelquist decoupling theorem. The matrix element also is finite for any momentum of the Z^0 , which is on shell in this case.

The largest cross-sections occur when the mass-splitting between $\tilde{\ell}_{L,R_1}(\tilde{\nu}_1)$ and $\tilde{\ell}_{L,R_2}(\tilde{\nu}_2)$ is large. In many models[11] we may restrict ourselves to the case where the relative mass-splitting is small, *i.e.*

$$\delta_{\tilde{\ell}_L} \ll 1, \qquad \delta_{\tilde{\ell}_R} \ll 1, \qquad \delta_{\tilde{\nu}} \ll 1$$

where

$$\delta_{\tilde{\ell}_{L,R}} \equiv \frac{{m_{\tilde{\ell}_{L,R_1}}}^2 - {m_{\tilde{\ell}_{L,R_2}}}^2}{{m_{\tilde{\ell}_{L,R_1}}}^2} \qquad\qquad \delta_{\tilde{\nu}} \equiv \frac{{m_{\tilde{\nu}_1}}^2 - {m_{\tilde{\nu}_2}}^2}{{m_{\tilde{\nu}_1}}^2} \tag{4}$$

In this limit we can write

$$\Delta F(T_i^+, S_{ij}^+, R_i^+) = \delta_{\tilde{\nu}} F'(T_i^+, S_{ij}^+, R_i^+) \tag{5a}$$

$$\Delta F({T_{L,R}}^0{}_i, S_{ij}^0, R_i^0) = \delta_{\tilde{\ell}_{L,R}} F'({T_{L,R}}^0{}_i, S_{ij}^0, R_i^0) \tag{5b}$$

$$\Delta F \equiv F(\tilde{\ell}_1) - F(\tilde{\ell}_2) \doteq \delta_{\tilde{\ell}} F'(\tilde{\ell}_1) \tag{6a}$$

$$F'(T,S,R) \equiv T\frac{\partial F(T,S,R)}{\partial T} = \frac{\partial F(T,S,R)}{\partial \ln T} \tag{6b}$$

In this case we may replace each ΔI and ΔG term in (3) by $\delta_{\tilde{\ell},\tilde{\nu}} I'$ and $\delta_{\tilde{\ell},\tilde{\nu}} G'$. (See Ref. 6 for these functional forms).

Given the matrix element, $\mathcal{M}^{\omega}$, in (3) we may derive[6]

$$\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = \frac{\left[|\mathcal{M}_L|^2 + |\mathcal{M}_R|^2\right]}{8\pi\alpha(\cot^2 2\theta_w + \tan^2\theta_w)} \tag{7}$$

Note that if any of the supersymmetric particles has a mass of less than $\frac{1}{2}M_Z$ that real production can occur and that $\mathcal{M}_L$ and $\mathcal{M}_R$ are complex. It is the modulus which is important in (7).

In order to get a feeling for the range of $\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$ we examine (3) in two extreme gaugino limits. The first is the "supersymmetry" limit in which we eliminate the gaugino supersymmetry-breaking terms by letting $M', M, \mu \to 0$ and let $v_1 = v_2$. Thus the only supersymmetry-breaking terms are the explicit slepton mass terms. Then (3) reduces to

$$\begin{aligned}\mathcal{M}^{\omega}{}_{SUSY} = \frac{\mathcal{K}_\nu \xi^{\omega}{}_L}{\sin^2\theta_w \sin 2\theta_w} & \Big\{ \Delta G(T_1^+, R_1^+) + \cos 2\theta_w \Delta G(T_1^+, 0) \\ & - \cos 2\theta_w \Delta \bar{I}_{[1]}(T_1^+, R_1^+) \\ & + 2\cos^2\theta_w \left[R_1^+ \Delta \bar{I}_{[\tilde{Z}^2 - Z^2]}(T_1^+, R_1^+) - \Delta\bar{G}(T_1^+, R_1^+)\right] \Big\} \\ -2\mathcal{K}_L \xi^{\omega}{}_L \cot 2\theta_w & \Big\{ \Delta\tilde{G}(T_L{}^0{}_1, R_1^0) + \cot^2 2\theta_w \Delta\tilde{G}(T_L{}^0{}_2, R_2^0) \Big\} \\ +2\mathcal{K}_R \xi^{\omega}{}_R \tan\theta_w & \Big\{ \Delta\tilde{G}(T_R{}^0{}_1, R_1^0) + \tan^2\theta_w \Delta\tilde{G}(T_R{}^0{}_2, R_2^0) \Big\}\end{aligned}$$

with

$$\Delta\bar{I}_{[\mathrm{N}(\mathrm{Z},\tilde{\mathrm{Z}})]}(T,R) = \Delta\check{I}_{[\mathrm{N}(\mathrm{Z},\tilde{\mathrm{Z}})]}(T, S \equiv 1, R) \qquad \bar{G}(T,R) = \Delta\check{G}(T, S \equiv 1, R) \tag{8}$$

where the physical masses are now

$$m_{\tilde{\chi}_i^+} = (M_W, M_W) \qquad m_{\tilde{\chi}_i^0} = (0, M_Z, M_Z, 0) \tag{8a}$$

and the relevant mass ratios are

$$T_1^+ = \frac{{m_{\tilde{\nu}_1}}^2}{{M_W}^2} \qquad {T_{L,R}}^0{}_1 = \infty \qquad {T_{L,R}}^0{}_2 = \frac{{m_{\tilde{\ell}_{L,R_2}}}^2}{{M_Z}^2}$$
$$R_1^+ = \frac{1}{4\cos^2\theta_w} \qquad R_1^0 = \infty \qquad R_2^0 = 1/4 \tag{8b}$$

$\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$ is plotted in this limit in Fig. 2. Since experiments rule out charged supersymmetry partners of mass less than about 24 GeV,[12] and in some instances much stronger limits have been placed,[13] we see that for light slepton masses with large mass-splitting that $\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$ could be as large as 3×10^{-6}. The significance of this will be discussed shortly. In order to achieve such a value parameters conducive to a large cross-section have been selected. We have allowed the sleptons to mix maximally so that $\theta_\nu = \theta_L = \theta_R = \frac{\pi}{4}$. We have further assumed maximal mass-splitting between the two slepton sectors, *i.e.* $m_{\tilde{\nu}_1}$,$m_{\tilde{\ell}_{L_1}}$ and $m_{\tilde{\ell}_{R_1}}$ are relatively light but $m_{\tilde{\nu}_2}$, $m_{\tilde{\ell}_{L_2}}$ and $m_{\tilde{\ell}_{R_2}}$ are large and decouple.

Another extreme limit is the "unmixed" limit in which the Higgsino and Gaugino sectors have been disentangled from one another. There are several ways of achieving this. Here we consider the limiting case $M \to \infty, M' \to 0$ and $\mu \to 0$ (θ_v arbitrary). The physical mass states become

$$M_{\tilde{\chi}_i^+} = (M \to \infty, 0) \qquad\qquad M_{\tilde{\chi}_i^0} = (M \to \infty, 0, 0, 0). \tag{9}$$

Note that $\tilde{\chi}_1^0 = \tilde{W}_3^0$, $\tilde{\chi}_2^0 = \tilde{B}^0$ and thus are purely gaugino whereas $\tilde{\chi}_{3,4}^0$ are purely Higgsino. Thus $T^0{}_{L,R_2} \equiv \frac{m^2_{\tilde{\ell}_{L,R_1}}}{{M_{\tilde{\chi}_2^0}}^2} \to \infty$ and $R_2^0 \equiv \frac{q^2}{4{M_{\tilde{\chi}_2^0}}^2} \to \infty$. The matrix element is given by

$$\mathcal{M}^\omega{}_{Unmixed} = \frac{2\mathcal{K}_R\xi^\omega{}_R}{\cos^2\theta_w}\tan\theta_w \Delta\tilde{G}({T_R}^0{}_2, R_2^0) - \frac{\mathcal{K}_L\xi^\omega{}_L}{2\cos^2\theta_w}\cot 2\theta_w \Delta\tilde{G}({T_L}^0{}_2, R_2^0) \tag{10}$$

Note that this result is independent of θ_v. The corresponding cross-section is plotted in Fig. 3. Again the sleptons are assumed to have maximal mixing and mass-splitting. Note, however, that in this limiting case we have a massless chargino (a charged Higgsino). Since experimentally the mass limit for new charged particles is ~ 24 GeV, we conclude that this case is merely illustrative.

The question arises as to whether such a process would be observable at SLC or LEP. These machines will operate as Z-factories and so are ideally suited for such a search. We see from Fig. 2 that, at best, the cross-section for $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\mu^-$ will be $\sim 3 \times 10^{-6}$ that of $e^+e^- \rightarrow Z^0 \rightarrow \mu^+\mu^-$. Thus we expect

$$BR(Z^0 \rightarrow \tau^+\mu^-) \lesssim \mathcal{O}(10^{-7}) \tag{11}$$

where we have used the relevant standard branching ratios[14]

$$BR(Z^0 \rightarrow \bar{\tau}\tau) \simeq BR(Z^0 \rightarrow \bar{\mu}\mu) \simeq BR(Z^0 \rightarrow \bar{e}e) \simeq 3\% \tag{12a}$$

We also note that

$$BR(\tau \rightarrow \mu\bar{\nu}\nu) \simeq BR(\tau \rightarrow e\bar{\nu}\nu) \simeq 17.5\% \tag{12b}$$

The experimental signature for such a decay would be μ and τ back-to-back with the μ having $E = \frac{M_Z}{2}$ and no missing energy. The principal background will be from $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^-$ followed by $\tau^\pm \rightarrow \mu^\pm\bar{\nu}\nu$, with the μ having nearly all of the τ momentum. The τ will travel approximately 2.5 mm before decaying in this case and, since the τ and μ are nearly collinear, the kink in the track will be unobservable. Clearly the number of μ, $N(\Delta\varepsilon)$, produced in the energy range from $\frac{M_Z}{2} - \Delta\varepsilon$ to $\frac{M_Z}{2}$ is of paramount importance. We find that

$$N(\Delta\varepsilon) = \frac{3-\alpha}{9+\alpha} \cdot 24 \frac{\Delta\varepsilon^2}{M_Z^2} \tag{13}$$

where α measures the degree of polarization ($\alpha = 0$ for the unpolarized case; $\alpha = 1$ for complete polarization). At SLAC we expect $\alpha \approx 1$ and thus $N_{pol}(\Delta\varepsilon) \doteq \frac{24}{5}\frac{\Delta\varepsilon^2}{M_Z^2}$. From this and (19b) we see that we need $\Delta\varepsilon \lesssim 0.1\%(\frac{M_Z}{2})$ to achieve a background of 10^{-7} or less which would compare with (11). Thus for muons with $E \approx \frac{M_Z}{2}$ we need $\frac{\Delta p}{p} \approx 0.1\%$.

At the SLC the MARK II detector will have an energy resolution for e's and μ's of ~0.3%/GeV without a vertex detector and, perhaps, as low as 0.1%/GeV with the planned vertex detector.[15] For $\frac{M_Z}{2} \simeq 50$ GeV this means that $\frac{\Delta p}{p}$ ~5% and from (13) the background will swamp the signal by at least a factor of 200. It appears unlikely that detector momentum resolutions will be improved by the required two orders of magnitude in the near future (there remains the formidable problem of increased multiple scatterings as detector mass is added). Thus the process $e^+e^- \to Z^0 \to \tau^+\mu^-$ will not be experimentally observable at the emerging generation of machines. Furthermore, because of (11), when SLC achieves its eventual target luminosity of 10^6 Z^0/year, the $Z^0 \to \tau^+\mu^-$production rate will be at best ~0.1 event/year.

We can do somewhat better if we assume that the principal slepton mixing occurs in the $\tilde{e} - \tilde{\mu}$ sector. Under the same conditions described above (maximal mixing; large mass-splitting; Z^0 on shell) the production rate will be the same[16] (off-shell it will be somewhat larger due to additional box diagrams). The principal background will still be due to $Z^0 \to \tau^+\tau^-$with $\tau^\pm \to \mu^\pm\bar{\nu}\nu$ and $\tau^\pm \to e^\pm\bar{\nu}\nu$.

The decay $Z^0 \to \bar{\mu}\mu$ will not be a problem since, at these energies, muons will not decay in the detector. Now misidentification of a $\tau^+\tau^-$ pair as a back-to-back $\mu^\pm e^\mp$ requires that **both** the μ and e emerge with energies near $\frac{M_Z}{2}$. Assuming complete polarization we find that $N(\Delta p)$, the number of back-to-back $e-\mu$ pairs emerging with $E = \frac{M_Z}{2}$, to within the experimental momentum resolution Δp, is given by $N(\Delta p) \doteq 1.4 \left(\frac{\Delta p}{p}\right)^4$. Thus with $\frac{\Delta p}{p} \sim 5\%$ we find $N(\Delta p) \approx 9 \times 10^{-6}$, which corresponds to roughly 0.02 misidentified events per year. Consequently, under the most propitious circumstances, it may be possible to reduce the background to an acceptable level.

In conclusion, even in the most favourable scenarios, it appears unlikely that $e^+e^- \to \tau^+\mu^-$ will be observed at SLC within the first few years of operation if the sole contribution is from slepton family mixing. The production rate is simply insufficient and the background rate overly severe. The decay $e^+e^- \to \mu^+e^-$ is equally rare but the background problems may prove more tractable. The full parameter space of the gaugino-higgsino sector has yet to be explored. Since the cross-sections predicted in the limiting cases presented come within an order of magnitude of being experimentally interesting at the SLC, realistic symmetry-breaking parameters might exist which would increase the cross-section to observable levels. In any instance the projected luminosity available at LEP might render such searches feasible.

ACKNOWLEDGEMENTS

I would like to thank Howard Haber and Pat Burchat for illuminating discussions on various theoretical and experimental aspects respectively. Professor Haber's remarks concerning preliminary versions of this manuscript were particularly appreciated. I would like to especially thank my advisor, Professor Stan Brodsky, for his many patient hours of guidance. Without his efforts this project may never have been completed.

APPENDIX

Mixing Matrices and Mass Eigenvalues

The masses of the charginos ($\tilde{\chi}^{+}$) are given in terms of the supersymmetry breaking terms of (1) by

$$m_{\tilde{\chi}^{+}_{1,2}} = \tfrac{1}{2}\left\{\mathrm{M}_{+} \pm \mathrm{M}_{-}\right\}\eta_{\pm} \qquad \mathrm{M}_{\pm} = \sqrt{(M \pm \mu)^2 + 2M_W^2(1 \mp \sin 2\theta_v)^2}$$

with $\eta_{+} = 1$ and $\eta_{-} = \mathrm{sign}[M\mu - M_W^2 \sin 2\theta_v]$

From $\mathcal{L}_{Br} + \mathcal{L}_{Mix}$ in (1a,b) we can write the neutralino mass terms in the form:[17] $\mathcal{L}_{mass} = -\frac{1}{2}(\psi^0)^T Y \psi^0 + h.c.$ where $\psi^0_j = (-i\tilde{B}^0, -i\tilde{W}^0, \tilde{\psi}^0_{H_1}, \tilde{\psi}^0_{H_2})$. Y is diagonalized by the unitary matrix N_H via $M_D = N_H^* Y N_H^\dagger$ (we can choose N_H^* instead of N_H because we use $(\psi^0)^T$ instead of $(\psi^0)^\dagger$.) The *mass eigenstates* then satisfy $\tilde{\chi}^0_i = N_{Hij}\psi^0_j$ $(i = 1,2,3,4)$. The masses $m_{\tilde{\chi}^+_i}$ and $m_{\tilde{\chi}^0_i}$ have been plotted, for typical parameter values, in Fig. 4. The neutralino vertex matrices are given by N_H and by[10]

$$O^{L\,\prime\prime}_{H\,ij} = \tfrac{1}{2}\left(N_{Hi4}N^*_{Hj4} - N_{Hi3}N^*_{Hj3}\right)$$

$$O^{R\,\prime\prime}_{H\,ij} = -O^{L\,\prime\prime\,*}_{H\,ij}$$

The chargino vertex matrices are given by[18]

$$U_{ij} = \begin{pmatrix} \cos\phi_- & \sin\phi_- \\ -\sin\phi_- & \cos\phi_- \end{pmatrix} \qquad V_{ij} = \begin{pmatrix} \cos\phi_+ & \sin\phi_+ \\ -\sin\phi_+ & \cos\phi_+ \end{pmatrix} \qquad D > 0$$

$$U_{ij} = \begin{pmatrix} \cos\phi_- & \sin\phi_- \\ -i\sin\phi_- & i\cos\phi_- \end{pmatrix} \qquad V_{ij} = \begin{pmatrix} \cos\phi_+ & \sin\phi_+ \\ -i\sin\phi_+ & i\cos\phi_+ \end{pmatrix} \qquad D < 0$$

where

$$D = M\mu - M_W^2 \sin 2\theta_v$$

$$\tan\phi_\pm = \frac{\sqrt{{A_\pm}^2 + {B_\pm}^2} - A_\pm}{B_\pm}$$

$$A_\pm = (M^2 - \mu^2) \mp 2M_W^2 \cos 2\theta_v$$

$$B_+ = 2\sqrt{2}M_W (M\cos\theta_v + \mu\sin\theta_v)$$

$$B_- = 2\sqrt{2}M_W (M\sin\theta_v + \mu\cos\theta_v)$$

$$O_{ij}^{L\,\prime} = \begin{pmatrix} \frac{1}{2}\sin^2\phi_+ - \cos^2\theta_w & \frac{1}{2}\sin\phi_+\cos\phi_+ \\ \frac{1}{2}\sin\phi_+\cos\phi_+ & \frac{1}{2}\cos^2\phi_+ - \cos^2\theta_w \end{pmatrix}$$

$$O_{ij}^{R\,\prime} = \begin{pmatrix} \frac{1}{2}\sin^2\phi_- - \cos^2\theta_w & \frac{1}{2}\sin\phi_-\cos\phi_- \\ \frac{1}{2}\sin\phi_-\cos\phi_- & \frac{1}{2}\cos^2\phi_- - \cos^2\theta_w \end{pmatrix}$$

$$\text{for } D > 0$$

$$O_{ij}^{L\,\prime} = \begin{pmatrix} \frac{1}{2}\sin^2\phi_+ - \cos^2\theta_w & -{}^i\!/_2\sin\phi_+\cos\phi_+ \\ {}^i\!/_2\sin\phi_+\cos\phi_+ & \frac{1}{2}\cos^2\phi_+ - \cos^2\theta_w \end{pmatrix}$$

$$O_{ij}^{R\,\prime} = \begin{pmatrix} \frac{1}{2}\sin^2\phi_- - \cos^2\theta_w & {}^i\!/_2\sin\phi_-\cos\phi_- \\ -{}^i\!/_2\sin\phi_-\cos\phi_- & \frac{1}{2}\cos^2\phi_- - \cos^2\theta_w \end{pmatrix}$$

$$\text{for } D < 0\,.$$

REFERENCES

1. C. Costa and F. Zwirner, Riv. Nuovo Cimento:**9**, 1 (1986),

   S. Weinberg HUTP-80/A038 Presented at the First Workshop on Grand Unification, 1980.

   T.-P. Cheng and L.-F. Li, *Deeper Pathways in High Energy Physics*, Coral Gables (Ed. Kursunaglu *et al.*) 1977.

   E. Papantonopoulos *et al.*, J. Phys. G. **10**, 1 (1984).

   M. Soldate *et al.*, Fermilab Pub-86/61-T (1986).

2. J. D. Vergados, Ioannina PR0887 (1986) and Rep. 187 (1985).

   S. T. Petcov, Pre-print CENT-TH-4399/86 (1986).

   J. C. Romão *et al.*, Nuc. Phys. **B250**, 295 (1985).

3. G. Eilam and T. Rizzo, Preprint IS-J, 2170 (1986).

   T. K. Kuo and N. Nakagawa, Phys. Rev. **D32**, 306 (1985).

   G. Leontaris *et al.*, Phys. Lett. **162B**, 153 (1985).

4. Sally Dawson, Proc. of the Thirteenth Annual SLAC Summer Institute on Particle Physics, (1985).

   I-Hsiu Lee, Phys. Lett. **138B**, 121 (1984).

   L. J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984).

5. Soft breaking terms are those whose inclusion would not result in new quadratic divergences.

6. M. Levine, to be published.

7. Frequently a Higgs singlet, usually denoted as N, is added. If this is absent then the spontaneous symmetry-breaking of of $SU_2 \times U_1$ to $U_{1\ em}$ does not occur in the limit in which the (explicit) supersymmetry-breaking terms

vanish (the 'supersymmetry' limit). Since such terms are present in our model we shall consider only the minimal two-Higgs model.

8. The superpotential must be homogeneous in the superfields Φ_i. The appearance of a $\Phi\Phi^*$ term would violate Lorentz Invariance. This may be understood in the following way. Using the notation of Table 1 we generate lepton and down quark masses (at tree level) in the usual way via

$$\mathcal{L}^{\ell,u} = \ell^{ij}\hat{\mathbf{H}}_1\hat{\mathbf{L}}_i\hat{\mathbf{R}}_j + \; d^{ij}\hat{\mathbf{H}}_1\hat{\mathbf{Q}}_i\hat{\mathbf{D}}_j$$

Where i and j are generation indices, $Q = (u_L, d_L)$, and $D = (\bar{d})_L$. The SU(2) doublets are contracted in an SU(2)-invariant fashion (*i.e.* antisymmetrically into an SU(2) singlet). The leptons and down quarks then get mass when H_1^0 has a non-zero vacuum expectation value: $\langle H_1^0 \rangle = v_1$. In component language the term which does this is $H_1^0(d_L)_\alpha(\bar{d})_L^\alpha$ where α is a spinor index. Supersymmetry implies that the three terms in which any pair of the above fields is replaced by its superpartner also exists. This includes, for instance, the term $(\tilde{\psi}_{H_1}^0)_{L\alpha}\tilde{d}_L(\bar{d})_L^\alpha$. If we try to generate the mass of the u quark using $\hat{\mathbf{H}}_1^*$ via the term $u^{ij}\hat{\mathbf{H}}_1^*\hat{\mathbf{Q}}_i\hat{\mathbf{U}}_j$ [where $U = (\bar{u})_L$], as we would do in the standard model, we would find that, although all of the $SU(3) \times SU(2) \times U(1)$ quantum numbers are correct, there is a term like $(\tilde{\psi}_{H_1}^0)_{L\alpha}^*\tilde{u}_L(\bar{u})_L^\alpha$. Since $(\tilde{\psi}_{H_1}^0)_{L\alpha}^* = (\bar{\tilde{\psi}}_{H_1}^0)_{R\dot{\alpha}}$ we see that the spinor indices do not match. Thus we would not be forming a *Lorentz-invariant* object. Such a vertex would indicate a scalar decaying to one left-handed and one right-handed spinor which is helicity-forbidden since the spinors emerge back-to-back. In the down quark case we have two left-handed Weyl spinors emerging which conserves angular momentum. For further details

see, for instance, S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 162 (1981).

9. L. Giardello and M. T. Grisaru, Nucl. Phys. **B194**, 65 (1982).
   Also References 6 and 10.

10. H. E. Haber and G. L. Kane, Phys. Rep. **117**, 75 (1985).

11. These models assume (quite reasonably) that supersymmetry breaks spontaneously in a flavour-independent manner. If the breaking scale in not too large then small mass splitting follows naturally. This has not been assumed in this paper. In such scenarios the matrix elements are generally *at least* an order of magnitude smaller. The classic reference is:
    J. Ellis and D. V. Nanopoulos, Phys. Lett. **110B**, 44 (1982).

12. S. Komamiya, As presented at the Thirteenth Annual SLAC Summer Institute on Particle Physics (1985).

13. ASP has placed limits on the selectron mass and the wino mass of approximately 60 GeV (As reported by William Ford at the Fourteenth Annual SLAC Summer Institute on Particle Physics, 1986). When combined with the results of other groups they find that $m_{\tilde{e}} \gtrsim 84$ GeV. While such strong limits do not, as yet, exist for $m_{\tilde{\mu}}$ or $m_{\tilde{\tau}}$, and hence for $m_{\tilde{\ell}_{1,2}}$, it would be somewhat surprising to find $m_{\tilde{\mu}} \ll m_{\tilde{e}}$.

14. Particle Properties Data Booklet, April 1986. From Phys. Lett. **170B** (1986).

15. Pat Burchat, A. Weinstein, F. Gilman. Private Communications.
    Proposal for the MARK II at SLAC. CALT-68-1015.

16. This is not precisely true. The experimental limits on the process $\mu \to e\gamma$ place severe constraints on $\tilde{e} - \tilde{\mu}$ mixing in the neutralino sector. If we assume maximal mixing and large mass splitting then[6] we find that the *lightest* slepton has $m \gtrsim 1$ TeV and so decouples. Thus the charginos provide the sole contribution to $Z \to \mu^+ e^-$ (other than in exceptional circumstances). The matrix element is then proportional to the sneutrino mass splitting. Assuming that this is also large we find, for cases of interest, that the chargino sector contribution is generally much larger than that of the neutralino sector *even before the above constraint has been imposed.* Thus the removal of the neutralino contribution, while considerably simplifying calculations, would not greatly affect the final cross-section.

For the situation illustrated in Fig. 3(b), when applied to $Z \to \mu^+ e^-$, we find that imposing the above constraints on $m_{\tilde{l}}$ affects the results by, at most, 15%. Furthermore, in the region of interest, this change results in an *increase* in the cross-section (because the real parts of the neutralino and chargino contributions enter with opposite signs). Note that the preceding argument (except the quoted percentage) also follows in the cases of small mass-splitting and non-maximal mixing.

When considering the process $Z^0 \to \tau^+ \mu^-$ current limits from $\tau \to \mu\gamma$ do not impose serious constraints on the neutralino sector. In the case of large mass-splitting and maximal mixing we find only that[6] $m_{\tilde{l}} \gtrsim 13$ GeV. In particular substantial effects persist in this case (arising from the neutralino sector) even when $m_{\tilde{\nu}_1} = m_{\tilde{\nu}_2}$.

17. J. F. Gunion and H. E. Haber, Nucl. Phys. **B272**, 1 (1986).

18. An equivalent formulation may be found in the appendix of R. M. Barnett and H. E. Haber, Phys. Rev. **D31**, 85 (1985).

Table 1. Fields Appearing in This Paper

Superfield	Ordinary Matter	Superpartners	Weak Isospin	y
		Gauge Multiplets		
$\hat{\mathbf{V}}$	$W^{\pm,0}$	$\tilde{W}^{\pm,0}$	Triplet	0
$\hat{\mathbf{V}}'$	B^0	$\tilde{B}^0$	Singlet	0
		Matter Multiplets		
$\hat{\mathbf{L}}_i$	ν_i, $\ell_{L_i}^-$	$\tilde{\nu}_i$, $\tilde{\ell}_{L_i}$	Doublet	-1
$\hat{\mathbf{R}}_i$	$(\ell_i^+)_L = (\ell_{R_i}^-)^*$	$(\tilde{\ell}_{R_i}^-)^*$	Singlet	2
$\hat{\mathbf{H}}_1$	${H_1}^0$, ${H_1}^-$	$\tilde{\psi}_{H_1}^0$, $\tilde{\psi}_{H_1}^-$	Doublet	-1
$\hat{\mathbf{H}}_2$	${H_2}^+$, ${H_2}^0$	$\tilde{\psi}_{H_2}^+$, $\tilde{\psi}_{H_2}^0$	Doublet	1

Table 2. Mass Ratios and Integral Functions

Mass Ratios

$$T_i^+ = \frac{m_{\tilde{\nu}_1}{}^2}{m_{\tilde{\chi}_1^+}{}^2} \qquad T_{L,R}{}^0{}_i = \frac{m_{\tilde{\ell}_{L,R_i}}{}^2}{m_{\tilde{\chi}_i^0}{}^2}$$

$$S_{ij}^+ = \frac{m_{\tilde{\chi}_j^+}{}^2}{m_{\tilde{\chi}_i^+}{}^2} \qquad S_{ij}^0 = \frac{m_{\tilde{\chi}_j^0}{}^2}{m_{\tilde{\chi}_i^0}{}^2}$$

$$R_i^+ = \frac{q^2}{4m_{\tilde{\chi}_i^+}{}^2} \qquad R_i^0 = \frac{q^2}{4m_{\tilde{\chi}_i^0}{}^2}$$

Integral Functions

$$\check{I}_{[\mathrm{N}]}(T,S,R) = \tfrac{1}{2}\int_0^1 dz \int_{-z}^{z} d\tilde{z} \frac{N(z,\tilde{z})}{(1-z)T + \frac{1}{2}\left[(1+S)z + (1-S)\tilde{z}\right] + \mathcal{R}(\tilde{z}^2 - z^2)}$$

$$\check{G}(T,S,R) = -\tfrac{1}{2}\int_0^1 dz \int_{-z}^{z} d\tilde{z} \ln\left[(1-z)T + \tfrac{1}{2}\left[(1+S)z + (1-S)\tilde{z}\right] + \mathcal{R}(\tilde{z}^2 - z^2)\right] - 3/4$$

$$G(T,R) = -\tfrac{1}{2}\int_0^1 dz \int_{-z}^{z} d\tilde{z} \ln\left[z(1-T) + 1 + \mathcal{R}(\tilde{z}^2 - z^2)\right] - 1/4$$

$$\tilde{G}(T,R) \equiv G(T,R) - G(T,R \equiv 0)$$

For any function, $\mathbf{F}$, of $T_i^{0,+}{}_{L,R}$

$$\Delta F \equiv F|_{\tilde{\ell}_1,\tilde{\nu}_1} - F|_{\tilde{\ell}_2,\tilde{\nu}_2}$$

FIGURE CAPTIONS

Fig. 1. Diagrams contributing to $Z^0 \to \tau^+\mu^-$.

(a) Chargino Diagrams.

(b) neutralino Diagrams.

Fig. 2. Ratio of cross-sections, $\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$, in "supersymmetry " limit.

(a) Varying all slepton masses equally. [Ignoring $m_{\tilde{\ell}_2}$ and $m_{\tilde{\nu}_2}$].

(b) Varying $m_{\tilde{\ell}_{L_1}}$ while holding all other mass parameters equal to the indicated values.

Fig. 3. Ratio of cross-sections, $\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$, in the "unmixed" limit. Vary $m_{\tilde{\ell}_{L_1}} = m_{\tilde{\ell}_{R_1}}$ assuming $m_{\tilde{\ell}_{L_2}} = m_{\tilde{\ell}_{R_2}} = \infty$.

Fig. 4. Chargino and neutralino masses for the indicated supersymmetry-breaking parameters.

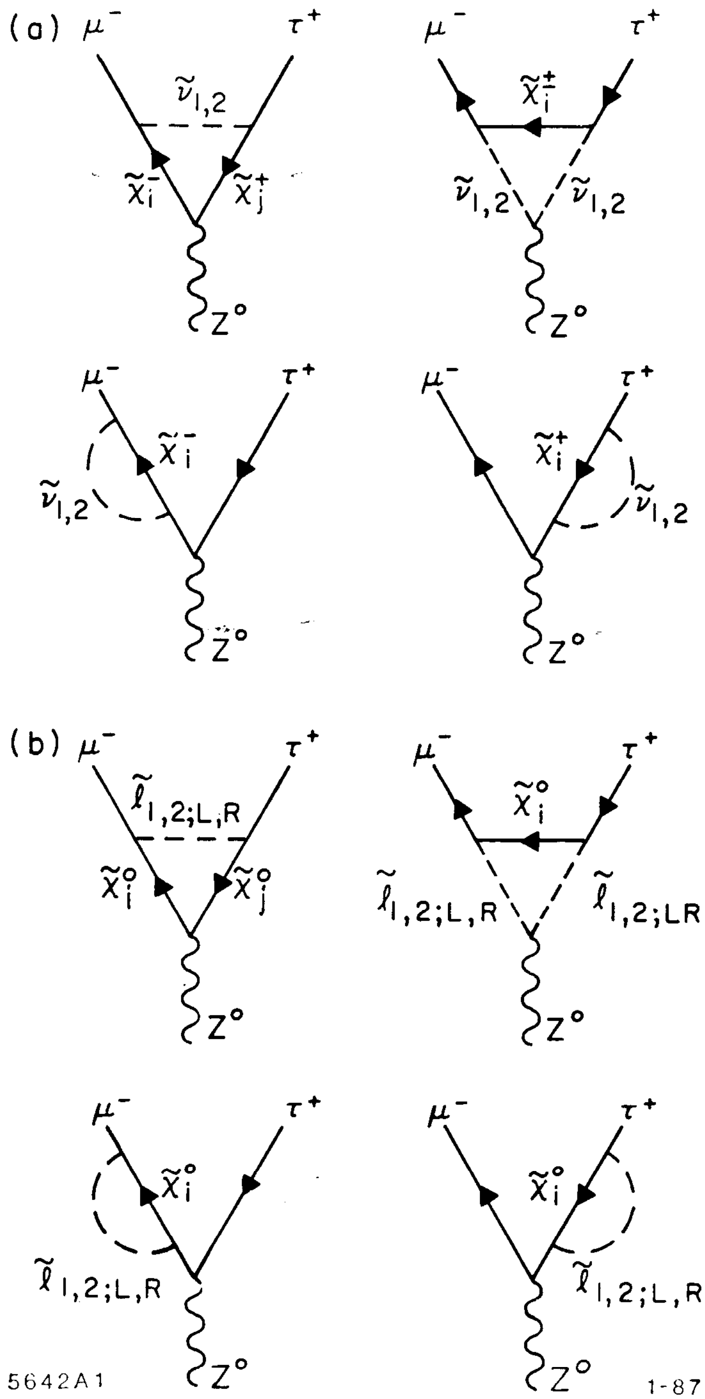

Fig. 1

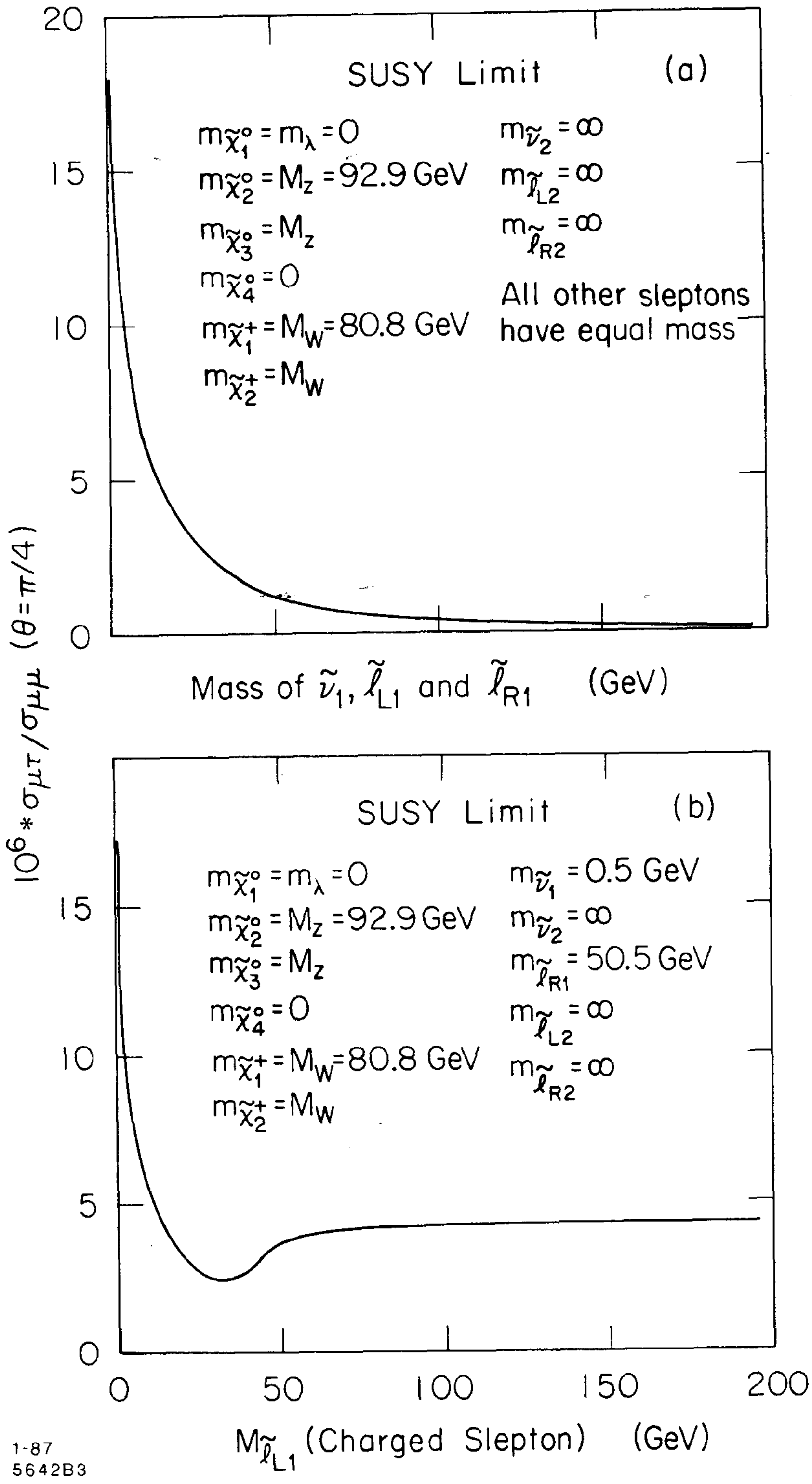
SUSY Limit
(a)
$m_{\tilde{\chi}_1^0} = m_\lambda = 0$
$m_{\tilde{\chi}_2^0} = M_Z = 92.9$ GeV
$m_{\tilde{\chi}_3^0} = M_Z$
$m_{\tilde{\chi}_4^0} = 0$
$m_{\tilde{\chi}_1^+} = M_W = 80.8$ GeV
$m_{\tilde{\chi}_2^+} = M_W$
$m_{\tilde{\nu}_2} = \infty$
$m_{\tilde{\ell}_{L2}} = \infty$
$m_{\tilde{\ell}_{R2}} = \infty$
All other sleptons have equal mass
Mass of $\tilde{\nu}_1, \tilde{\ell}_{L1}$ and $\tilde{\ell}_{R1}$ (GeV)
$10^6 * \sigma_{\mu\tau}/\sigma_{\mu\mu}$ $(\theta = \pi/4)$
SUSY Limit
(b)
$m_{\tilde{\chi}_1^0} = m_\lambda = 0$
$m_{\tilde{\chi}_2^0} = M_Z = 92.9$ GeV
$m_{\tilde{\chi}_3^0} = M_Z$
$m_{\tilde{\chi}_4^0} = 0$
$m_{\tilde{\chi}_1^+} = M_W = 80.8$ GeV
$m_{\tilde{\chi}_2^+} = M_W$
$m_{\tilde{\nu}_1} = 0.5$ GeV
$m_{\tilde{\nu}_2} = \infty$
$m_{\tilde{\ell}_{R1}} = 50.5$ GeV
$m_{\tilde{\ell}_{L2}} = \infty$
$m_{\tilde{\ell}_{R2}} = \infty$
0 5 10 15 20
0 50 100 150 200
$M_{\tilde{\ell}_{L1}}$ (Charged Slepton) (GeV)
1-87
5642B3

Fig. 2

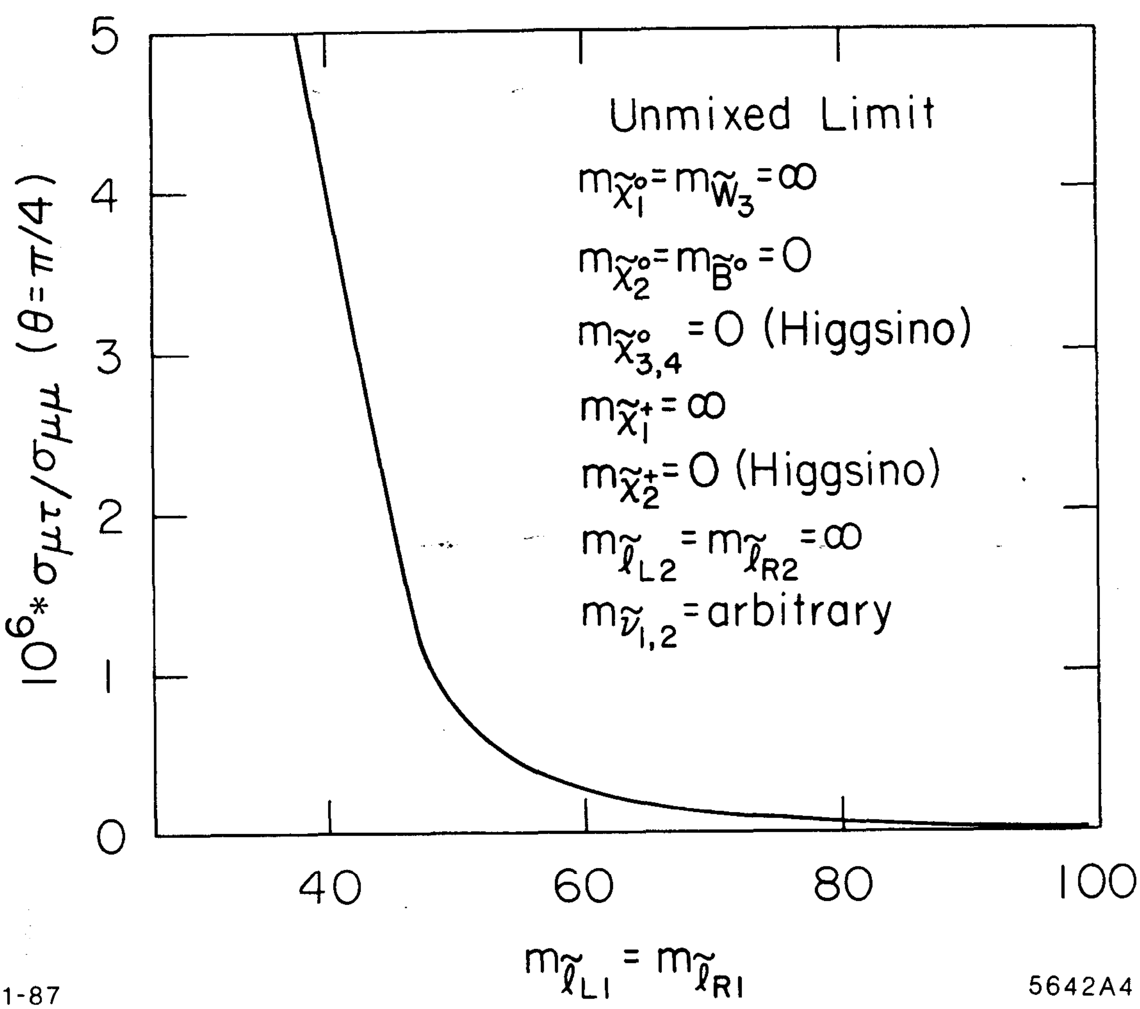
5
4
3
2
1
0
40
60
80
100
$10^6 * \sigma_{\mu\tau}/\sigma_{\mu\mu}$ $(\theta = \pi/4)$
Unmixed Limit
$m_{\tilde{\chi}_1^\circ} = m_{\tilde{W}_3} = \infty$
$m_{\tilde{\chi}_2^\circ} = m_{\tilde{B}^\circ} = 0$
$m_{\tilde{\chi}_{3,4}^\circ} = 0$ (Higgsino)
$m_{\tilde{\chi}_1^+} = \infty$
$m_{\tilde{\chi}_2^+} = 0$ (Higgsino)
$m_{\tilde{\ell}_{L2}} = m_{\tilde{\ell}_{R2}} = \infty$
$m_{\tilde{\nu}_{1,2}} =$ arbitrary
$m_{\tilde{\ell}_{L1}} = m_{\tilde{\ell}_{R1}}$
1-87
5642A4

Fig. 3

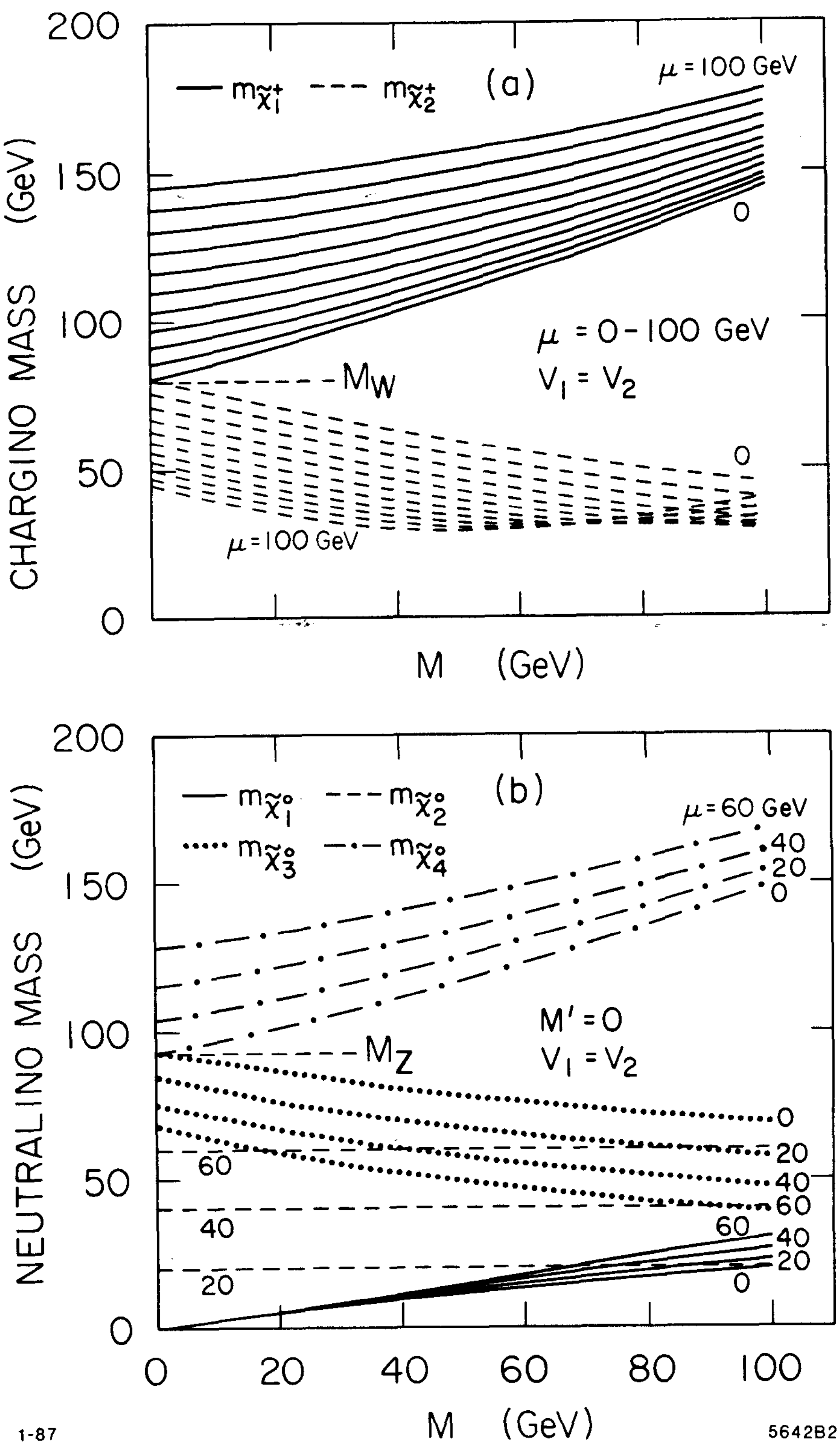
200
150
100
50
0
CHARGINO MASS (GeV)
(a)
$m_{\tilde{\chi}_1^+}$
$m_{\tilde{\chi}_2^+}$
μ = 100 GeV
0
μ = 0 − 100 GeV
$V_1 = V_2$
M_W
0
μ = 100 GeV
M (GeV)
200
150
100
50
0
NEUTRALINO MASS (GeV)
(b)
$m_{\tilde{\chi}_1^0}$
$m_{\tilde{\chi}_2^0}$
$m_{\tilde{\chi}_3^0}$
$m_{\tilde{\chi}_4^0}$
μ = 60 GeV
40
20
0
M′ = 0
$V_1 = V_2$
M_Z
0
20
40
60
60
40
20
60
40
20
0
0
20
40
60
80
100
M (GeV)
1-87
5642B2

Fig. 4