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THE SUPERSYMMETRIC WESS–ZUMINO ACTION AND U(1) GAUGE FIELDS*

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ABSTRACT

The supersymmetric effective action is constructed which when varied under gauge groups containing explicit U(1) factors reproduces the mixed gauge field contribution to the SU(N) anomaly while being U(1) invariant. It constitutes a generalization of the supersymmetric Wess–Zumino action. The form of the supersymmetric mixed anomaly is also discussed.

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The presence of U(1) gauge fields can significantly alter the short distance behavior of field theories containing fermions coupled to non-abelian gauge fields which are associated with additional internal or space- time symmetries. In the case of internal symmetries, new mixed U(1) and Yang-Mills field contributions to the non-abelian anomaly occur [1-3]. For local Lorentz invariance in four dimensional space-time, while pure gravitational field contributions are required to vanish[4], mixed field anomalies can occur[5-6]. When the internal symmetries are spontaneously broken or in the presence of torsion in the local Lorentz case, the associated Nambu-Goldstone modes or the additional degrees of freedom associated with the torsion can be utilized to construct an effective action whose variation reproduces the anomaly structure of the underlying theory[7]. The part of this action which reproduces the mixed gauge field anomaly provides an extension of the original effective action of Wess and Zumino which reproduces the pure non-Abelian gauge field contribution to the anomaly.

For supersymmetric theories, the Wess-Zumino action whose variation produces the pure non-abelian gauge field contribution to the SU(N) anomaly has previously been constructed[8] as a functional of the anomaly. The explicit form for the anomaly, which was not required in obtaining the structure of the Wess-Zumino term, has subsequently been subject to numerous investigations [9-19]. The purpose of this paper is to extend the supersymmetric Wess-Zumino action to the case where explicit supersymmetric U(1)gauge fields are present. The resultant effective action constitutes a supersymmetric generalization of that found in the ordinary field theoretic case[8]. The explicit form of the mixed supersymmetric gauge field anomaly will then be discussed.

The U(1) invariant but SU(N) anomalous effective action will be constructed in terms of the supersymmetric (SUSY) SU(N) gauge fields, A^i, the SUSY U(1) gauge field B, and (anti-) chiral Nambu-Goldstone boson superfields $(\bar{g})g$ which are elements of the SUSY SU(N) group. This effective action reproduces the anomaly structure of the underlying theory that describes the SUSY gauge interactions of (anti-) chiral matter superfields, $(\bar{\phi})$ ϕ, transforming as the m-dimensional representation λ^i of SU(N), while carrying U(1) charge q, and m SU(N) singlet (anti-) chiral matter superfields, ($\bar{S}$) S, each with U(1) charge -q. (We set q=1 in what follows.) In addition, these matter superfields might also carry non-trivial color quantum numbers and thus interact with the SU(N)xU(1) invariant vector supergluon field, V, of the presummed confining color group. We also assume that the SU(N) symmetry is spontaneously broken (either as a result of condensation arising from the color interactions or due to a perturbative Higgs effect) resulting in the appearance of the Nambu-Goldstone (anti-)chiral superfields $(\bar{g})g$. On the other hand, the U(1) symmetry is preserved.

Defining $(\bar{\Lambda}^i)\Lambda^i$ as the SU(N) (anti-)chiral gauge parameters and $U = e^{-i\Lambda.\lambda}$, $\bar{U} = e^{i\bar{\Lambda}.\lambda}$ as SU(N) group transformation matrices with $\Lambda.\lambda = \Lambda^i\lambda^i$, $\bar{\Lambda}.\lambda = \bar{\Lambda}^i\lambda^i$, then the SU(N) SUSY gauge transformations of the fields are defined to be

$$\phi_U = U\phi, \quad \bar{\phi}_U = \bar{\phi}\bar{U}$$

$$S_U = S, \quad \bar{S}_U = \bar{S}$$

$$e^{2A_U.\lambda} = \bar{U}^{-1}e^{2A.\lambda}U^{-1}$$

$$B_U = B$$

$$g_U = gU^{-1}, \quad \bar{g}_U = \bar{U}^{-1}\bar{g}. \tag{1}$$

For infinitesimal SU(N) gauge variations these reduce to

$$\delta_N(\Lambda, \bar{\Lambda})\phi = -i\Lambda.\lambda\phi, \quad \delta_N(\Lambda, \bar{\Lambda})\bar{\phi} = i\bar{\phi}\bar{\Lambda}.\lambda$$

$$\delta_N(\Lambda, \bar{\Lambda})S = 0, \quad \delta_N(\Lambda, \bar{\Lambda})\bar{S} = 0$$

$$\delta_N(\Lambda, \bar{\Lambda})A^i = \frac{1}{2}(\bar{\Lambda}^j + \Lambda^j)f_{jik}A^k + \frac{i}{2}(\bar{\Lambda}^j - \Lambda^j)[A.t\coth A.t]_{ji}$$

$$\delta_N(\Lambda,\bar\Lambda)B = 0$$

$$\delta_N(\Lambda,\bar\Lambda)g = ig\Lambda.\lambda, \quad \delta_N(\Lambda,\bar\Lambda)\bar g = -i\bar\Lambda.\lambda\bar g, \tag{2}$$

where $(A.t)_{jk} = A^i(t^i)_{jk}$, with t^i being the adjoint representation matrices $(t^i)_{jk} = -if_{ijk}$. Analogously, we define U(1) (anti-)chiral gauge parameters $(\bar\Lambda_1)\Lambda_1$ and U(1) group transformation phases $U_1 = e^{-i\Lambda_1}, \bar U_1 = e^{i\bar\Lambda_1}$, so that the superfields transform as

$$\phi_{U_1} = U_1\phi, \quad \bar\phi_{U_1} = \bar\phi\bar U_1$$

$$S_{U_1} = SU_1^{-1}, \quad \bar S_{U_1} = \bar U_1^{-1}\bar S$$

$$A^i_{U_1} = A^i$$

$$e^{2B_{U_1}} = \bar U_1^{-1}e^{2B}U_1^{-1}$$

$$g_{U_1} = g, \quad \bar g_{U_1} = \bar g. \tag{3}$$

For infinitesimal U(1) variations this reduces to

$$\delta_1(\Lambda_1,\bar\Lambda_1)\phi = -i\Lambda_1\phi, \quad \delta_1(\Lambda_1,\bar\Lambda_1)\bar\phi = i\bar\Lambda_1\bar\phi$$

$$\delta_1(\Lambda_1,\bar\Lambda_1)S = i\Lambda_1 S, \quad \delta_1(\Lambda,\bar\Lambda_1)\bar S = -i\bar\Lambda_1\bar S$$

$$\delta_1(\Lambda_1,\bar\Lambda_1)A^i = 0$$

$$\delta_1(\Lambda_1,\bar\Lambda_1)B = \frac{i}{2}(\Lambda_1 - \bar\Lambda_1)$$

$$\delta_1(\Lambda_1,\bar\Lambda_1)g = 0, \quad \delta_1(\Lambda_1,\bar\Lambda_1)\bar g = 0 \tag{4}$$

The SU(N)xU(1) gauge and SUSY invariant underlying action Γ_0 is given by

$$\Gamma_0 = \int dV[\bar{\phi}e^{2A.\lambda+2B+2V}\phi + \bar{S}e^{-2B+2V}S]$$

$$+m\int dSSg\phi + m\int d\bar{S}\bar{\phi}\bar{g}\bar{S}. \tag{5}$$

(We employ the conventions of reference[20] .) The full quantum effective action, Γ, is defined as

$$e^{i\Gamma[A,B,g,\bar{g}]} = \int [d\phi][d\bar{\phi}][dS][d\bar{S}][dV]e^{iN_4[\Gamma_0]} \tag{6}$$

where the N_4 symbol denotes that we employ the manifestly supersymmetric version of the BPHZ subtraction procedure in order to perturbatively define the renormalized time ordered functions of the model[21] . This algorithm also maintains the U(1) invariance explicitly yielding the unbroken SUSY U(1) gauge Ward identity

$$\delta_1(\Lambda_1, \bar{\Lambda}_1)\Gamma[A, B, g, \bar{g}] = 0, \tag{7}$$

where the U(1) Ward identity functional differential operator is

$$\delta_1(\Lambda_1, \bar{\Lambda}_1) = \int dV[\delta_1(\Lambda_1, \bar{\Lambda}_1)B]\frac{\delta}{\delta B}. \tag{8}$$

On the other hand, the SUSY SU(N) gauge invariance is broken by the over-subtraction of the mass term. More specifically, the interactions $Sg\phi$ and $\bar{\phi}\bar{g}\bar{S}$ have power counting dimension 2 while the SUSY N_4 prescription dictates that these vertices are to be subtracted as if they were of dimension 3. Writing $g = e^{\pi.\lambda}, \bar{g} = e^{\bar{\pi}.\lambda}$ where $(\bar{\pi}^i)\pi^i$ are the (anti-)chiral Nambu-Golstone superfields and recalling that the removal of $(\bar{g}-1)or(g-1)$ from within a normal product reduces the degree of subtraction by one, we see that the Yukawa interactions are minimally subtracted. However there remains the mass terms $mN_3[S\phi]$ and

$mN_3[\bar{\phi}\bar{S}]$ with degree of subtraction indicated by N_3 rather than the N_2 normal product. This over subtraction leads to the broken SUSY SU(N) Ward identity

$$\delta_N(\Lambda,\bar{\Lambda})\Gamma[A,B,g,\bar{g}] = -im\int dS(N_3[S\Lambda.\lambda\phi]\Gamma - N_2[S\Lambda.\lambda\phi]\Gamma)$$

$$+im\int d\bar{S}(N_3[\bar{\phi}\bar{\Lambda}.\lambda\bar{S}]\Gamma - N_2[\bar{\phi}\bar{\Lambda}.\lambda\bar{S}]\Gamma), \tag{9}$$

where the SU(N) Ward identity functional differential operator $\delta_N(\Lambda,\bar{\Lambda})$ is defined as

$$\delta_N(\Lambda,\bar{\Lambda}) = \int dV[\delta_N(\Lambda,\bar{\Lambda})A^i]\frac{\delta}{\delta A^i}$$

$$+\int dS[\delta_N(\Lambda,\bar{\Lambda})g]\frac{\delta}{\delta g} + \int d\bar{S}[\delta_N(\Lambda,\bar{\Lambda})\bar{g}]\frac{\delta}{\delta \bar{g}}. \tag{10}$$

The difference between the oversubtracted and minimally subtracted mass insertions can be evaluated using the SUSY Zimmermann identity and usual graphical techniques. Of the various terms appearing, most can be written as the gauge variation of dimension 3 local supersymmetric monomials composed of gauge fields only. Such terms can be absorbed by adding finite, renormalizable counterterms to the action or equivalently by specifying normalization conditions. There are, however, certain monomials which are functions of the gauge fields only that cannot be written as gauge variations of terms containing only gauge fields. These terms constitute the Adler-Bardeen anomaly and in a sense form a minimal set of breaking terms. The Adler-Bardeen anomaly is independent of the method of calculation and thus represents intrinsic quantum mechanical constraints on the field theory. These constraints are manifested in the form of violations to low energy theorems or obstructions to a consistant quantization procedure. Note, however, that both the minimal or non-minimal form for the anomaly yields the same low energy theorem violations[22] . Although they cannot be written as gauge variations of action terms composed of gauge fields only, the Adler-Bardeen anomaly can be written as gauge variations of an action depending on both the gauge fields and the Nambu-Goldstone fields. Such an action, however, is non-renormalizable when the fields are quantized. It is this effective action, Γ_{eff}, which we shall construct.

The SU(N)xU(1) Ward identities for this effective action are

$$\delta_N(\Lambda,\bar{\Lambda})\Gamma_{eff}[A,B,g,\bar{g}] = G_A[\Lambda,\bar{\Lambda},A] + G_B[\Lambda,\bar{\Lambda},A,B] \tag{11}$$

$$\delta_1(\Lambda_1,\bar{\Lambda}_1)\Gamma_{eff}[A,B,g,\bar{g}] = 0. \tag{12}$$

Here $G_A[\Lambda,\bar{\Lambda},A] = G_A[\Lambda,A] + \bar{G}_A[\bar{\Lambda},A]$ is the pure SUSY Yang-Mills gauge field contribution to the SUSY SU(N) Adler-Bardeen anomaly and $G_B[\Lambda,\bar{\Lambda},A,B] = G_B[\Lambda,A,B] + \bar{G}_B[\bar{\Lambda},A,B]$ is the mixed SUSY non-abelian and abelian gauge field contribution to the SUSY SU(N) Adler-Bardeen anomaly. The $(\bar{G}_A)G_A$ and $(\bar{G}_B)G_B$ are purely (anti-)chiral and can be written as

$$G_A[\Lambda,A] = \int dS\Lambda^i G_A^i(A)$$

$$\bar{G}_A[\bar{\Lambda},A] = \int d\bar{S}\bar{\Lambda}^i \bar{G}_A^i(A) \tag{13}$$

and

$$G_B[\Lambda,A,B] = \int dS\Lambda^i G_B^i(A,B)$$

$$\bar{G}_B[\bar{\Lambda},A,B] = \int d\bar{S}\bar{\Lambda}^i \bar{G}_B^i(A,B), \tag{14}$$

where

$$\bar{D}_{\dot{\alpha}} G_A^i(A) = 0 = D_\alpha \bar{G}_A^i(A)$$

$$\bar{D}_{\dot{\alpha}} G_B^i(A,B) = 0 = D_\alpha \bar{G}_B^i(A,B).$$

The Adler-Bardeen anomalies must satisfy the Wess-Zumino consistency conditions[7] (which are also known as the first gauge cohomology conditions[28]).

These are obtained by applying the SU(N)xU(1) algebra satisfied by the Ward identity differential operators

$$[\delta_N(\Lambda,\bar{\Lambda}),\delta_N(\Lambda',\bar{\Lambda}')] = \delta_N(\Lambda\times\Lambda',\bar{\Lambda}\times\bar{\Lambda}') \tag{15}$$

$$[\delta_N(\Lambda,\bar{\Lambda}),\delta_1(\Lambda_1,\bar{\Lambda}_1)] = 0 \tag{16}$$

$$[\delta_1(\Lambda_1,\bar{\Lambda}_1),\delta_1(\Lambda_1',\bar{\Lambda}_1')] = 0 \tag{17}$$

to the action $\Gamma_{eff}[A,B,g,\bar{g}]$ yielding

$$\delta_N(\Lambda,\bar{\Lambda})G_A[\Lambda',\bar{\Lambda}',A] - \delta_N(\Lambda',\bar{\Lambda}')G_A[\Lambda,\bar{\Lambda},A]$$

$$= G_A[\Lambda\times\Lambda',\bar{\Lambda}\times\bar{\Lambda}',A] \tag{18}$$

$$\delta_1(\Lambda_1,\bar{\Lambda}_1)G_A[\Lambda,\bar{\Lambda},A] = 0 \tag{19}$$

and

$$\delta_N(\Lambda,\bar{\Lambda})G_B[\Lambda',\bar{\Lambda}',A,B] - \delta_N(\Lambda',\bar{\Lambda}')G_B[\Lambda,\bar{\Lambda},A,B]$$

$$= G_B[\Lambda\times\Lambda',\bar{\Lambda}\times\bar{\Lambda}',A,B] \tag{20}$$

$$\delta_1(\Lambda_1,\bar{\Lambda}_1)G_B[\Lambda,\bar{\Lambda},A,B] = 0. \tag{21}$$

In obtaining these results we have used the U(1) invariance of $\Gamma_{eff}[A,B,g,\bar{g}]$: $\delta_1(\Lambda_1,\bar{\Lambda}_1)\Gamma_{eff}[A,B,g,\bar{g}] = 0$ and the independence of G_A and G_B.

We now turn to the solution of the Ward identities of Eq. (11-12) for the effective action. This action can be written as the sum of two terms

$$\Gamma_{eff}[A,B,g,\bar{g}] = \Gamma_{WZ}[A,g,\bar{g}] + \Gamma_{mixed}[A,B,g,\bar{g}], \tag{22}$$

where each term is separately U(1) invariant while the SU(N) variation of Γ_{WZ} (Γ_{mixed}) yields $G_A(G_B)$. The structure of the mixed gauge field contribution to the effective action, $\Gamma_{mixed}[A,B,g,\bar{g}]$, is similar to the pure non-abelian gauge field term, $\Gamma_{WZ}[A,g,\bar{g}]$, which is the supersymmetric extension of the usual Wess-Zumino action and has the form[8]

$$\Gamma_{WZ}[A,g,\bar{g}] = -\int_0^1 dt e^{t\delta_N^{(A)}(i\pi,-i\bar{\pi})} G_A[i\pi,-i\bar{\pi},A], \tag{23}$$

where

$$g = e^{-\pi}, \quad \bar{g} = e^{-\bar{\pi}}$$

and

$$\pi = -i\pi^i t^i, \quad \bar{\pi} = i\bar{\pi}^i t^i.$$

Here $\delta_N^{(A)}(\Lambda,\bar{\Lambda})$ indicates the SU(N) variation of the Yang-Mills fields, $A = A.t$, only. Introducing the gauge transformed Yang-Mills field $A_{g(t)}$ as

$$e^{2A_{g(t)}} = e^{t\delta_N^{(A)}(i\pi,-i\bar{\pi})} e^{2A} = e^{\delta_N^{(A)}(it\pi,-it\bar{\pi})} e^{2A}$$

$$= e^{-t\bar{\pi}} e^{2A} e^{t\pi} = \bar{g}(t)^{-1} e^{2A} g(t)^{-1}, \tag{24}$$

with $g(0) = 1 = \bar{g}(0), g(1) = g, \bar{g}(1) = \bar{g}$, and using

$$e^{t\delta_N^{(A)}(i\pi,-i\bar{\pi})} G_A[i\pi,-i\bar{\pi},A] = G_A[i\pi,-i\bar{\pi},A_{g(t)}], \tag{25}$$

the Wess-Zumino action can be cast into the form

$$\Gamma_{WZ}[A,g,\bar{g}] = -\int_0^1 dt G_A[i\pi,-i\bar{\pi},A_{g(t)}]. \tag{26}$$

By construction, we note that Γ_{WZ} is U(1) invariant, while its SU(N) variation gives the pure Yang-Mills field contribution to the non-abelian anomaly

$$\delta_N(\Lambda, \bar{\Lambda})\Gamma_{WZ}[A, g, \bar{g}] = G_A[\Lambda, \bar{\Lambda}, A]. \tag{27}$$

In a similar fashion, we can construct the effective action whose variation reproduces the mixed field contribution to this anomaly. Noting that

$$\begin{aligned} &\left[\delta_N(\Lambda, \bar{\Lambda})\right]^n \Gamma_{mixed}[A, B, g, \bar{g}] \\ &= \left[\delta_N(\Lambda, \bar{\Lambda})\right]^{n-1} G_B[\Lambda, \bar{\Lambda}, A, B] \\ &= \left[\delta_N^{(A)}(\Lambda, \bar{\Lambda})\right]^{n-1} G_B[\Lambda, \bar{\Lambda}, A, B], \end{aligned} \tag{28}$$

it follows that

$$\begin{aligned} &e^{\delta_N(\Lambda,\bar{\Lambda})}\Gamma_{mixed}[A, B, g, \bar{g}] \\ &= \Gamma_{mixed}[A, B, g, \bar{g}] + \left[\frac{e^{\delta_N^{(A)}(\Lambda,\bar{\Lambda})} - 1}{\delta_N^{(A)}(\Lambda, \bar{\Lambda})}\right] G_B[\Lambda, \bar{\Lambda}, A, B] \\ &= \Gamma_{mixed}[A, B, g, \bar{g}] + \int_0^1 dt e^{t\delta_N^{(A)}(\Lambda,\bar{\Lambda})} G_B[\Lambda, \bar{\Lambda}, A, B]. \end{aligned} \tag{29}$$

We recognize the left hand side of this equation as the gauge transform of $\Gamma_{mixed}[A, B, g, \bar{g}]$. However, we can always find gauge parameters which rotate the Goldstone fields to the origin. Specifically, by choosing $\Lambda^i = i\pi^i, \bar{\Lambda}^i = -i\bar{\pi}^i$, the gauge transformed Nambu-Goldstone fields are

$$e^{\delta_N(i\pi,-i\bar{\pi})}g = ge^{i(i\pi).\lambda} = e^{\pi.\lambda}e^{-\pi.\lambda} = 1$$

$$e^{\delta_N(i\pi,-i\bar{\pi})}\bar{g} = e^{-i(-i\bar{\pi}).\lambda}\bar{g} = e^{-\bar{\pi}.\lambda}e^{\bar{\pi}.\lambda} = 1 \tag{30}$$

so that equation (29) takes the form

$$\Gamma_{mixed}[A,B,1,1] = \Gamma_{mixed}[A,B,g,\bar{g}]$$

$$+\int_0^1 dte^{t\delta_N^{(A)}(i\pi,-i\bar{\pi})}G_B[i\pi,-i\bar{\pi},A,B]. \tag{31}$$

Moreover, since no U(1) invariant action functional of the gauge fields alone can satisfy the SU(N) anomalous Ward identity, it follows that $\Gamma_{mixed}[A,B,1,1]$ must vanish. Finally, using the same reasoning as that leading to Eq. (25), it follows that

$$G_B[i\pi,-i\bar{\pi},A_{g(t)},B] = e^{t\delta_N^{(A)}(i\pi,-i\bar{\pi})}G_B[i\pi,-i\bar{\pi},A,B] \tag{32}$$

and hence that

$$\Gamma_{mixed}[A,B,g,\bar{g}] = -\int_0^1 dtG_B[i\pi,-i\bar{\pi},A_{g(t)},B]. \tag{33}$$

Combining this term with $\Gamma_{WZ}[A,g,\bar{g}]$, we secure the effective action satisfying the anomalous Ward identities of Eq. (11-12) as

$$\Gamma_{eff}[A,B,g,\bar{g}] = \Gamma_{WZ}[A,g,\bar{g}] + \Gamma_{mixed}[A,B,g,\bar{g}]$$

$$= -\int_0^1 dt\left[G_A[i\pi,-i\bar{\pi},A_{g(t)}] + G_B[i\pi,-i\bar{\pi},A_{g(t)},B]\right]. \tag{34}$$

The form of the consistent Adler-Bardeen SU(N) anomaly is much more difficult to construct in the SUSY case than in the ordinary case. This is due to the zero dimensionality of the gauge vector superfield and the constraints of SUSY as well as chirality. The determination of the pure Yang-Mills field contribution to the anomaly has been discussed by several authors using various methods. We will not discuss the pure non-abelian gauge field contribution to the anomaly any further in this paper, but will instead turn our attention to the determination of the mixed field contribution which has not been previously constructed.

The imposition of U(1) invariance and chirality immediately restricts the form of the mixed anomaly to

$$G_B[\Lambda, A, B] = i \int dS \frac{\bar{D}\bar{D}}{-4} D^{\alpha} B Tr \left[\Lambda \frac{\bar{D}\bar{D}}{-4} R_{\alpha}(A) \right],$$

$$\bar{G}_B[\bar{\Lambda}, A, B] = -i \int d\bar{S} \frac{DD}{-4} \bar{D}_{\dot{\alpha}} B Tr \left[\bar{\Lambda} \frac{DD}{-4} \bar{R}^{\dot{\alpha}}(A) \right], \tag{35}$$

where $R_{\alpha}(A)$ is a mass dimension 1/2 function of the SU(N) gauge vector superfield and its covariant derivatives. Since A is dimensionless, we can in general write

$$R_{\alpha}(A) = \sum_{m,n=0}^{\infty} r_{mn} A^m D_{\alpha} A A^n$$

$$\bar{R}^{\dot{\alpha}}(A) = \sum_{m,n=0}^{\infty} r_{mn} A^m \bar{D}^{\dot{\alpha}} A A^n, \tag{36}$$

where the coefficients r_{mn} are fixed by the consistency condition Eq. (20). The coefficient r_{00} provides the overall normalization and is determined for example, by usual graphical techniques to be $r_{00} = 1/16\pi^2$. The remaining coefficients are then recursively determined by using Eq. (20) and demanding that the anomaly satisfy the Wess-Zumino consistency conditions. In terms of the component fields, the anomaly reduces to

$$G_B[\Lambda,\bar{\Lambda},A,B] = -\frac{1}{4\pi^2}\int d^4x\epsilon^{\mu\nu\rho\sigma}\partial_\mu B_\nu Tr\left[\omega\partial_\rho A_\sigma\right]$$

$$+SUSY\ partners, \tag{37}$$

where $\omega = 1/2(\Lambda+\bar{\Lambda})|_{\theta=\bar{\theta}=0}$ parameterizes the ordinary SU(N) gauge transformations. This form for the ordinary mixed anomaly is precisely the same as that previously determined. Note that the covariant form of the mixed anomaly[24] is also readily obtained as

$$G_B[\Lambda,A,B]|_{covariant} = i\int dS\frac{\bar{D}\bar{D}}{-4}D^\alpha BTr\left[\Lambda W_\alpha\right]$$

$$\bar{G}_B[\bar{\Lambda},A,B]|_{covariant} = -i\int d\bar{S}\frac{DD}{-4}\bar{D}_{\dot{\alpha}}BTr\left[\bar{\Lambda}\bar{W}^{\dot{\alpha}}\right], \tag{38}$$

with $(\bar{W}^{\dot{\alpha}})W_\alpha$ the (anti-)chiral field strength spinor.

In conclusion, given the SU(N)xU(1) anomalous Ward identity functional differential equations (11-12) for the effective action Γ, the solution has the form of the generalized Wess-Zumino action given in equation (34). Γ_{WZ} is the original Wess-Zummino action and it is determined solely by the SU(N) gauge field contribution to the SU(N) anomalous Ward identity, $G_A[\Lambda,\bar{\Lambda}]$. Γ_{mixed}, on the other hand, is determined by the mixed SU(N) and U(1) gauge field contribution to the SU(N) anomalous Ward identity, $G_B[\Lambda,\bar{\Lambda},A,B]$. The SU(N) anomalies, G_A and G_B, are given in terms of supersymmetric power series in the SUSY gauge fields and their derivatives and are restricted, as usual, by SUSY power counting and discrete symmetry requirements. The coefficients of the various terms in each series can be evaluated by many methods such as customary perturbation theory or SUSY path integral and heat kernal techniques. Alternatively, one may calculate the lowest order terms by one of these methods and then extract the remaining higher order terms recursively by implementation of the Wess-Zumino consistency conditions, Eqs. (18-20). For the pure Yang-Mills field anomaly G_A the solution of this constraint can also be found through the structure of the SUSY SU(N) BRS transformation decent equations[19]. For the case of the mixed anomaly G_B, however, the BRS descent equations associated with the U(1) factor in the product group are trivial. Consequently, a direct application of this approach fails to solve the consistency conditions for G_B. As such, one is led to the iterative solution outlined in the discussion surrounding Eqs. (35-36).

REFERENCES

1) S. Adler, *Phys. Rev.* **177** (1969) 2426;
J. Bell and R. Jackiw, *Nuovo Cimento* **60A** (1969) 47;
S.L. Adler and W.A. Bardeen, *Phys. Rev.* **182** (1969) 1517;
W.A. Bardeen, *Phys. Rev* **184** (1969) 1848.

2) The mixed anomaly also plays a pivotal role in constructing the effective action resulting when a fermion doublet is made very massive. See, for example, E. D'Hoker and E. Farhi, *Nucl. Phys.* **B248** (1984) 59; B248(84)77.

3) T.E. Clark and S.T. Love, *Phys. Lett.* **158B** (1985) 234.

4) L. Alvarez-Gaume' and E. Witten, *Nucl. Phys.* **B234** (1983) 269.

5) H.T. Nieh, *Phys. Rev. Lett.* **53** (1984) 2219;
S. Yajima and T. Kimura, *Phys. Lett.* **B173** (1986) 154.

6) T.E. Clark and S.T. Love, *Phys. Rev.* **D33** (1986) 1199.

7) J. Wess and B. Zumino, *Phys. Lett.* **37B** (1971) 95.

8) T.E. Clark and S.T. Love, *Phys. Lett.* **138B** (1984) 289.

9) N.K. Nielsen, *Nucl. Phys.* **B244** (1984) 499; B274(86)448.

10) O. Piguet and K. Sibold, *Nucl. Phys.* **B247** (1984) 484.

11) E. Guadagnini, K. Konishi and M. Mintchev, *Phys. Lett.* **157B** (1985) 37; Non-Abelian Chiral Anomalies in Supersymmetric Gauge Theories, preprint IFUP-TH-10/85, Pisa.

12) R. Garreis, M. Scholl and J. Wess, *Z. Phys.* **C28** (1985) 623.

13) K. Harada and K. Shizuya, *Phys. Lett.* **162B** (1983) 322.

14) M. Pernici and F. Riva, *Nucl. Phys.* **B267** (1986) 61.

15) A.W. Fischer, *Phys. Lett.* **159B** (1985) 42; The Relation Between the Minimal and Non-Minimal Forms of the Supersymmetric Chiral Anomaly, preprint King's College, London, 1986.

16) V.K. Krivoshchekov, P.B. Medvedev and L.O. Chekov, Explicit Form of Non-Abelian Consistent Chiral Susy Anomaly, preprint ITEP-169, Moscow.

17) S. Ferrara, L. Girardello, O. Piguet and R. Stora, *Phys. Lett.* **157B** (1985) 179.

18) I.N. McArthur and H. Osborn, *Nucl. Phys.* **B268** (1986) 573.

19) R. Grimm, BRS Transformations in Supersymmetric Yang-Mills Theories and the Algebraic Construction of the Chiral Non-Abelian Supersymmetric Anomaly, preprint LAPP-Th-176, 1986.

20) J. Wess and J. Bagger, Supersymmetry and Supergravity (Princeton University Press, 1983).

21) W. Zimmermann, Lectures on elementary particles and quantum field theory, 1970 Brandeis Lectures (MIT Press, Cambridge, 1970);
T.E. Clark, O. Piguet and K. Sibold, *Ann. of Phys.* **109** (1977) 418.

22) T.E. Clark, O. Piguet and K. Sibold, *Phys. Lett.* **169B** (1986) 85.

23) C. Becchi, A. Rouet and R. Stora, *Ann. of Phys.* **98** (1976) 287.

24) B. Zumino and W.A. Bardeen, *Nucl. Phys.* **B244** (1984) 421.