SLAC – PUB – 4162
January 1987
[T/AS]

WIMP Distribution In and Evaporation From The Sun

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ABSTRACT

This paper analyzes the evaporation of WIMPs (weakly interacting massive particles) from the sun, both analytically and numerically. First an analytic approximation is made by defining an appropriate truncation of a Maxwell-Boltzmann distribution for the WIMPs and calculating the exact evaporation rate from this distribution due interaction with a truly thermal distribution of nuclei. Then, the actual (non-thermal) distribution of Dirac neutrino WIMPs in the sun is calculated numerically for WIMPs of mass 1 - 7 GeV. Evaporation from the actual and thermal distributions are compared. It is found that the evaporation mass for a solar life-time is 3.7 GeV and, for an 'annihilation life-time,' 2.9 GeV. These are about 8% lower than the most well-reasoned previous estimate.

Submitted to *Astrophysical Journal*

⋆ Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

1. Introduction

WIMPs (weakly interacting massive particles) may make up some or all of the 'dark matter' in the Milky Way. If they do, they will have been captured by the sun (Press and Spergel 1985) and other bodies (Faulkner and Gilliland 1985) perhaps generating observable consequences (Faulkner and Gilliland 1985; Spergel and Press 1985). Since these particles have elastic cross sections much smaller than those of the solar nuclei, they orbit almost freely inside the sun, interacting only occasionally. To accurately predict observable effects, one must know the space and velocity distribution of the WIMPs and the rate at which they evaporate from the sun. In the past, the WIMP distribution has usually been assumed to be thermal, with a characteristic temperature which is an average of the temperatures of the regions in the sun which it samples. Of course, it is known that the distribution cannot be truly thermal because the WIMPs are transporting heat, but it has been assumed (as it turns out, correctly) that this approximation introduces relatively small errors (Faulkner and Gilliland 1985; Spergel and Press 1985; Griest and Seckel 1987). However, the analyses of the evaporation from a thermal WIMP distribution have been less than precise and have, for the most part misestimated the evaporation rate by one or several orders of magnitude. The most well-reasoned previous analysis succeeded in finding the correct functional dependence on the various parameters by using a detailed balance argument for the case when the WIMP temperature is equal to the nuclei temperature (Griest and Seckel 1987). But the correction factor for the case of unequal temperatures was not evaluated correctly, and this led to an overestimation of the evaporation rate by a factor of 6.

In this paper I begin by solving the problem of evaporation from a thermal distribution of WIMPs with a velocity independent isotropic cross section exactly. I then develop a rigorous method of numerical analysis which can determine the actual (non-thermal) distribution with arbitrary accuracy. I also develop analytic techniques for understanding the differences between the thermal and actual rates. As it happens, this difference turns on rather subtle points.

The rate of evaporation from the true distribution is found to be about one third of the rate from the thermal one with the same average kinetic energy. These corrections to evaporation rate (a factor of about 18) lead to an 8% reduction in the evaporation mass (the WIMP mass at which the evaporation rate is the inverse of some specified evaporation time). Using an evaporation time equal to the lifetime of the sun (4.7 billion years), the evaporation mass is 3.7 GeV. Using the 'annihilation time' (computed for Dirac neutrinos), the evaporation mass is 2.9 GeV. (The annihilation time (Griest and Seckel 1987) is the lifetime of a WIMP orbiting in the presence of a thermal distribution of anti-WIMPs with an appropriate number density.) The large correction in evaporation rate leads to a relatively small correction in evaporation mass because the latter depends logarithmically on the former.

In section 2, I discuss qualitatively how one might expect the WIMP distribution to differ from a thermal one, and how one should parameterize this difference.

In section 3, I derive the general formula for evaporation from a Maxwell-Boltzmann distribution of WIMPs truncated at a given value of the kinetic energy, due to immersion in a truly thermal gas of nuclei with arbitrary temperature and mass per particle. The results of this calculation are used to illustrate the problems with assuming that the distribution is thermal.

In section 4, I describe the numerical simulation experiments I used to determine the actual WIMP distribution in the sun for various mass WIMPs. In section 5, the results of these experiments are given.

In section 6, I discuss the case of WIMPs with an enhanced interaction cross-section of order 10^{-36}cm^2 which have been proposed to explain the solar neutrino problem (Faulkner and Gilliland 1985; Spergel and Press 1985). The numerical methods used here are not strictly applicable to these WIMPs, but it is argued in this section that a good estimate of the evaporation rate can nevertheless be made by using them.

2. Qualitative Picture of WIMP Distribution

If the WIMP distribution were thermal with temperature T_W, then at every point in the sun, the WIMPs would have a distribution

$$f_{th}(v,r) = \frac{n_0}{V_1}\frac{4}{\pi^{\frac{1}{2}}}\left(\frac{M}{2T_W}\right)^{\frac{3}{2}} v^2 e^{-Mv^2/(2T_W)} e^{-M\Phi(r)/T_W}, \tag{2.1}$$

($k_B = 1$), where M is the mass of the WIMP, $\Phi(r)$ is the solar gravitational potential, n_0 is the total number of WIMPs, and

$$V_1 \equiv \int_0^{R_\odot} 4\pi r^2 e^{-M\Phi(r)/T_W}\, dr \tag{2.2}$$

is the effective volume of the sun. At every point in the sun the velocity distribution would be isotropic and the average kinetic energy would be equal to $(3/2)T_W$.

If the WIMPs interact only once every several orbits, one could also parameterize the WIMP distribution by energy and angular momentum rather than velocity and position. The thermal distribution would then be expressed in terms of these variables,

$$f_{th} = f_{th}(E,L). \tag{2.3}$$

How does one expect the true WIMP distribution to differ from this thermal one? For ~ 4 GeV WIMPs, the scale height of orbits is about 1/12 of the solar radius. Thus the bulk of the WIMPs will be in a region of relatively uniform temperature, so that they might at first be thought to have a nearly thermal distribution. However, all the WIMPs which are kicked into higher energies (the Boltzmann tail) will leave the central region of the sun and experience the colder outer regions. They will quickly scatter back down to lower energies. Thus the tail will be suppressed. In terms of equation (2.3) this may be expressed

$$\frac{f(E,L)}{f_{th}(E,L)} \sim 1 \quad (E \sim T_W) \tag{2.4}$$

$$\frac{f(E,L)}{f_{th}(E,L)} \to 0 \quad (E \gg T_W), \tag{2.5}$$

where $f(E,L)$ is the actual WIMP distribution. Further, one would expect the high-energy, high-angular-momentum part of the distribution to be highly suppressed relative to the high-energy, low-angular-momentum part. This is because most of the tail is created in the hot center of the sun and consequently the typical high energy WIMP is in a radial, low-angular-momentum orbit.

Another way to express these same qualitative features is to consider the average WIMP kinetic energy, $< E_k >$, and average square of the radial component of the velocity unit vector, $< (v_r/v)^2 >$, as functions of position in the sun. One would expect $< E_k >$ to fall with increasing radius and $< (v_r/v)^2 >$ to rise.

In section 5, it will be shown that for $\sim$ 4 GeV WIMPs these qualitative expectations are borne out and that, roughly, the WIMPs may be considered as being in a cut-off thermal distribution, with a cut-off energy of about 40% to 80% of the evaporation energy. (The value depends on whether one estimates the cut-off from the energy distribution integrated over all angular momenta, or just the low angular momenta. The high figure is more appropriate for the central region of the sun because the high energy WIMPs there have low angular momentum. Far from the center, the lower figure is more appropriate.)

3. Analytic Theory of Evaporation

Consider first a single WIMP of velocity w and mass M scattering off a thermal distribution of nuclei with density N, mass m, and temperature T. Assume there is a velocity independent cross section σ, with isotropic scattering. The rate at which w scatters to velocity v (derived in the appendix) is given by

$$R(w \to v)dv = \frac{2}{\pi^{\frac{1}{2}}}\frac{\mu_+^2}{\mu}\sigma N\frac{vdv}{w}[\chi(\pm\beta_-, \beta_+)e^{-\frac{M}{2T}(v^2-w^2)} + \chi(\pm\alpha_-, \alpha_+)], \qquad (3.1)$$

where

$$\chi(a,b) \equiv \int_a^b dye^{-y^2}, \qquad (3.2)$$

$$\alpha_\pm \equiv (m/2T)^{\frac{1}{2}}(\mu_+ v \pm \mu_- w), \qquad (3.3)$$

$$\beta_\pm \equiv (m/2T)^{\frac{1}{2}}(\mu_- v \pm \mu_+ w), \qquad (3.4)$$

$$\mu_\pm \equiv \frac{\mu \pm 1}{2} \qquad \mu \equiv \frac{M}{m}, \qquad (3.5)$$

and the upper (lower) sign in equation (3.1) refers to the case when $w < (>) v$.

To find the rate at which w 'escapes' to any velocity $v \geq v_e$, where v_e is an escape velocity greater than w, integrate equation (3.1) from the escape velocity to ∞:

$$R_{v_e}(w) \equiv \int_{v_e}^{\infty} R(w \to v)dv. \qquad (3.6)$$

The identities necessary for evaluating this integral are given in the appendix. The result is

$$R_{v_e}(w) = \frac{1}{2\pi^{\frac{1}{2}}}\frac{2T}{m}\frac{1}{\mu^2}\frac{\sigma N}{w}[\mu(\alpha_+ e^{-\alpha_-^2} - \alpha_- e^{-\alpha_+^2}) + (\mu - 2\mu\alpha_+\alpha_- - 2\mu_+\mu_-)\chi(\alpha_-, \alpha_+) + 2\mu_+^2\chi(\beta_-, \beta_+)e^{-\frac{M}{2T}(v^2-w^2)}], \qquad (3.7)$$

where $\alpha_\pm$ and $\beta_\pm$ are evaluated at $v = v_e$.

Next assume that the WIMPs are in a truncated thermal distribution with a cut-off velocity w_c:

$$f_W(w)dw = \frac{4}{\pi^{\frac{1}{2}}}\Big(\frac{M}{2T_W}\Big)^{3/2} N_W w^2 e^{-Mw^2/2T_W}\,\theta(w_c - w)dw. \tag{3.8}$$

Then the total evaporation is given by

$$R(w_c \mid v_e) \equiv \int_0^\infty f_W(w) R_{v_e}(w) dw. \tag{3.9}$$

The evaluation of this integral is outlined in the appendix. There are two cases. For $T = T_W$

$$\begin{aligned} R(w_c \mid v_e) = \frac{2}{\pi}(2T/M)^{\frac{1}{2}}\sigma N N_W [& e^{-E_e/T}\Big(-\beta_+\beta_- - \frac{1}{2\mu}\Big)\chi(\beta_-,\beta_+) \\ & + e^{-E_c/T}\Big(\alpha_+\alpha_- - \frac{1}{2\mu}\Big)\chi(\alpha_-,\alpha_+) \\ & - e^{-(E_c/T+\alpha_-^2)}\Big(\frac{m}{2T}\Big)^{\frac{1}{2}}\frac{v_e - w_c}{2} \\ & + e^{-(E_c/T+\alpha_+^2)}\Big(\frac{m}{2T}\Big)^{\frac{1}{2}}\frac{v_e + w_c}{2}], \end{aligned} \tag{3.10}$$

for $T \neq T_W$

$$\begin{aligned} & R(w_c \mid v_e) = \\ & \frac{2}{\pi}\Big(\frac{2T}{M}\Big)^{\frac{1}{2}}\Big(\frac{T}{T_W}\Big)^{\frac{3}{2}}\sigma N N_W \Big\{ e^{-\left(\frac{\mu_+}{\xi}\right)^2\frac{E_e}{T_W}}\Big[\frac{\mu\mu_-}{\nu\xi}\Big(\frac{\xi^2}{\nu} - \frac{\mu_+\mu_-}{\mu}\Big) + \frac{\mu_+^3}{\xi(\nu-\mu)}\Big]\chi(\gamma_-,\gamma_+) \\ & \qquad + e^{-E_c/T_W}\frac{\mu}{\nu}\Big[\alpha_+\alpha_- - \frac{1}{2\mu} + \mu_-^2\Big(\frac{1}{\mu} - \frac{1}{\nu}\Big)\Big]\chi(\alpha_-,\alpha_+) \\ & \qquad - e^{-E_c/T_W} e^{-(E_e-E_c)/T}\frac{\mu_+^2}{\nu-\mu}\chi(\beta_-,\beta_+) \\ & \qquad - e^{-(E_c/T_W+\alpha_-^2)}\frac{\mu}{2\nu}\alpha_+ + e^{-(E_c/T_W+\alpha_+^2)}\frac{\mu}{2\nu}\alpha_- \Big\}. \end{aligned} \tag{3.11}$$

In these equations,

$$\gamma_\pm \equiv (m/2T)^{\frac{1}{2}}(\rho v_e \pm \xi w_c), \tag{3.12}$$

$$E_e \equiv \frac{Mv_e^2}{2} \qquad E_c \equiv \frac{Mw_c^2}{2}, \tag{3.13}$$

$$\xi^2 \equiv \mu_-^2 + \nu \qquad \rho \equiv \frac{\mu_+\mu_-}{\xi}, \tag{3.14}$$

$$\nu \equiv \frac{T}{T_W}\mu. \tag{3.15}$$

Although these expressions are exact and completely general, they are also less than transparent. Fortunately, a great deal of information can be extracted by considering appropriate limits. Consider first the case of a thermal WIMP distribution (that is, one cut off at the escape velocity). When the temperatures are also equal equation (3.10) becomes

$$\frac{2}{\pi^{\frac{1}{2}}} N N_W \sigma \left(\frac{2T}{M}\right)^{\frac{1}{2}} e^{-E_e/T} \left(\frac{E_e}{T} - \frac{1}{2\mu}\right). \tag{3.16}$$

Except for very heavy nuclei, evaporation is virtually independent of the mass of the nuclei. Because of this, and because most of the evaporation occurs in regions where the WIMP temperature is roughly equal to the nuclei temperature, I will now restrict attention to the analytically simpler case of $\mu = 1$. For this case (assuming, as I always will that $E_e \gg T, T_W$, as is true in the sun) equations (3.10) and (3.11) become

$$A \frac{2}{\pi^{\frac{1}{2}}} N N_W \sigma \left(\frac{2T_W}{M}\right)^{\frac{1}{2}} e^{-E_e/T_W} \frac{E_e}{T_W} \tag{3.17}$$

where A is a correction factor

$$A = (1 - T_W/2E_e) \sim 1 \qquad (T = T_W), \tag{3.18}$$

$$A = \frac{T}{E_e}\frac{T}{\Delta T}\left[1 - \left(\frac{T_W}{T}\right)^{\frac{1}{2}} e^{-\frac{E_e}{T_W}\frac{\Delta T}{T}}\right] \sim \frac{T}{E_e}\frac{T}{\Delta T}\left[1 - e^{-\frac{E_e}{T_W}\frac{\Delta T}{T}}\right] \qquad (T \neq T_W), \tag{3.19}$$

and $\Delta T \equiv (T_W - T)$. Using equation (2.1), equation (3.17) may be written in the suggestive form

$$A[T(r)]N(r)\frac{E_e(r)}{E_0}\mathcal{R} \tag{3.20}$$

where

$$\mathcal{R} \equiv \frac{1}{V_1}\frac{2}{\pi^{\frac{1}{2}}}\sigma\left(\frac{2T_W}{M}\right)^{\frac{1}{2}}e^{-E_0/T_W}\frac{E_0}{T_W} \tag{3.21}$$

is independent of radius and E_0 is the escape energy at the center. That is, up to the correction factor, $A(E_e/E_0) \sim A$, all the nuclei in the sun contribute to evaporation equally. The nuclei high in the sun have fewer WIMPs to hit (because of Boltzmann suppression in equation (2.1)) but the escape energy is less, so there is a greater chance that any given collision will kick the WIMP out.

A may be approximated as follows. When $(|\Delta T|/T_W) < (T_W/E_e)$, A is of order unity. But as the nuclei temperature falls below this range, A rapidly drops toward $\sim (T_W/2E_e)$ at $T = .5T_W$. For $E_e/T_W \sim 30$, A is .47 at $\Delta T/T_W = .05$ and is .26 at $\Delta T/T_W = .1$. On the other hand, when the nuclei temperature is above the WIMP temperature, there is enhancement. At $\Delta T/T_W = -.05$, A is 2.25. However, there is significant enhancement over only an extremely small region of the solar core. It follows that the evaporation due to a thermal distribution of WIMPs may be reasonably approximated by setting $A = \theta(T - .95T_W)$ in equation (3.20),

$$\sim \theta(T(r) - .95T_W)N(r)\mathcal{R}. \tag{3.22}$$

The relation (3.20) was originally discovered by Griest and Seckel (1987). They argued (from detailed balance) that in the equal temperature case, the evaporation rate of WIMPs from the sun should be equal to the capture rate of the tail of a (hypothetical) ambient WIMP distribution. This in turn, they argued, should be virtually equal to the interaction rate of the WIMP tail with a gas of stationary nuclei. This part of the analysis was correct. They went on, however, to estimate the correction due to unequal temperatures by assuming that the ratio of the rates at different temperatures was equal to the ratio of nuclei phase space in a certain kinematic region, the region where a nucleus could promote a WIMP to escape velocity by donating all its kinetic energy to the latter. This approximation is not valid. The evaporation rate depends on nuclei phase space *coupled to* WIMP phase space. The nuclei dependence cannot be factored out. This physical fact is reflected in the mathematical form of the kinematic limits under various parameterizations of phase space. One may verify, by working through the appendix, that the kinematic limits of the problem assume a simple form when phase space is parameterized by certain linear combinations of the center of mass velocity, s, and the velocity of the WIMP in the center of mass frame, t; namely

$$t \pm s. \tag{3.23}$$

One may also verify, this time by hours of tedious algebra, that the kinematic limits assume an unbelievably ugly and complex form when phase space is parameterized by the velocities of the nuclei and WIMPs, and that in these coordinates the simplicity of the final result appears to stem from a miraculous series of cancellations. That is, if one insists on analyzing the problem from the standpoint of the nuclei velocity distribution, one is not free to make simple assumptions about the kinematic limits. With their approximation, Griest and Seckel calculated that $A(E_e/E_0)$ had a mean value of about .5 averaged over the mass of the sun. Using the θ-function approximation of equation (3.22), a WIMP temperature of 90% of the central solar temperature, and the standard solar model (Bachall *et. al.* 1982) (in which only 7% of the mass of the sun is above 85% of the central temperature) it can be seen that by taking the evaporation rate to be proportional to half the mass of the sun instead of the mass of the core, Griest and Seckel overestimated the evaporation rate by a factor of $\sim .5/.07 \sim 7$. A factor of 7 in rate translates into a correction of about 6% in evaporation mass because the rate depends exponentially on the mass.

However, as discussed in section 2, one expects that the actual distribution has a truncated tail and that the truncation point gets lower with increasing radius. Equations (3.10) and (3.11) throw light on the question of how this affects evaporation. Consider first the case $\mu = 1$, $E_e > E_c \gg T, T_W$ which reasonably approximates helium-WIMP interactions in the sun for WIMPs near the evaporation mass. Equations (3.10) and (3.11) become

$$\frac{2}{\pi^{\frac{1}{2}}} N N_W \sigma \left(\frac{2T_W}{M}\right)^{\frac{1}{2}} e^{-E_e/T_W} \left(\frac{E_c}{T_W} - \frac{1}{2}\right) \quad (T = T_W), \tag{3.24}$$

$$\frac{2}{\pi^{\frac{1}{2}}} N N_W \sigma \left(\frac{2T}{M}\right)^{\frac{1}{2}} \left(\frac{T}{T_W}\right)^{\frac{3}{2}} e^{-E_e/T} \frac{T_W}{\Delta T} \left[e^{\frac{E_c}{T}\frac{\Delta T}{T_W}} - \left(\frac{T_W}{T}\right)^{\frac{1}{2}}\right] \quad (T \neq T_W). \tag{3.25}$$

These formulas may be combined as before in the form of equation (3.17) where now

$$A \sim E_c/E_e \quad (T = T_W), \tag{3.26}$$

$$A \sim \frac{T}{E_e}\frac{T}{\Delta T}\left[\exp\left(-\frac{E_e - E_c}{T_W}\frac{\Delta T}{T}\right) - \exp\left(-\frac{E_e}{T_W}\frac{\Delta T}{T}\right)\right] \quad (T \neq T_W). \tag{3.27}$$

When $T \sim T_W$, A exhibits the same temperature dependence as before but is now, in addition, linear in the cut-off energy. This means that near the center of the sun where the bulk of evaporation occurs, the non-thermal character of the distribution has relatively little effect. However, away from the center, where there is both a lower truncation point and a lower temperature, new effects come into play. Now, for $(\Delta T/T_W) > (T_W/E_e)$, A is given roughly by

$$A \sim \exp\left(-\frac{E_e - E_c}{T}\frac{\Delta T}{T_W}\right). \tag{3.28}$$

Using $E_e/T \sim 30$, $E_c \sim .5E_e$, this means that at nuclei temperatures more than 7% below the WIMP temperature, evaporation is exponentially suppressed. [When $\mu \neq 1$ equation (3.11) becomes considerably more complicated but numerical studies indicate that the similar qualitative conclusions hold.]

Next consider the case when $\mu \neq 1$ but $T = T_W \ll E_c < E_e$. Recall from equation (3.16) that for $E_c = E_e$, A was virtually independent of the mass of the nucleus. However, for a *truncated* WIMP distribution this is no longer the case; there is "impedance matching." Evaporation due to nuclei which are not matched to the WIMP mass is suppressed. This may be seen by graphing equation (3.10) against $\log_{10}\mu$ for $E_e/T = 30$ with various values of E_c/E_e (see Figure 1). This means that evaporation due to hydrogen relative to helium is somewhat suppressed in the core and very suppressed further out. It also means that heavier elements [which despite their large cross sections play only a minor role in the WIMP mass range of interest even in the thermal approximation (Griest and Seckel 1987)] can be ignored altogether.

The main effect of these two modifications is to further justify the assumption that no evaporation takes place outside the core region. For a thermal distribution, the relatively mild suppression of A in the outer regions of the sun is somewhat compensated by the large total number of nuclei in these regions. But for the truncated distributions, evaporation is exponentially suppressed there and is further suppressed because of the relatively high abundance of hydrogen.

4. Numerical Model

In order to find the actual WIMP distribution, I numerically solve the Boltzmann collision equation. First I outline a method for doing this that is valid for WIMPs of arbitrary cross-section. Later I will make the additional simplifying assumption (valid for ordinary Dirac neutrino WIMPs) that the number of collisions per orbit is small compared to one.

I begin by assuming that the WIMPs are interacting with a *fixed* solar medium. In principle, one should take into account the fact that the WIMPs alter the thermal conductivity of the sun and hence the solar model, and use a model which depends on WIMP properties. This could be done, for example, by solving for the WIMP distribution for a given solar model, finding out how this

WIMP distribution changed the solar model and iterating. For purposes of this calculation, however, I will simply assume that the given solar model is consistent with the presence of the WIMPs. In the case of ordinary Dirac neutrino WIMPs, this assumption is fully justified by the fact that these WIMPs produce little change in the solar model. In my numerical calculations, I simply used the standard solar model (Bachall *et. al.* 1982).

Next, I assume that the WIMPs do not interact with each other, but only with the nuclei in the sun. This makes the Boltzmann equation linear in the WIMP distribution. Finally, I assume radial symmetry. These assumptions imply that the WIMP distribution can be described by three continuous parameters, which might be taken to be v, v_r, and r, the velocity, radial component of velocity, and height. However, these parameters are not convenient. The transport terms in the collision equation assume a complicated form.

A better set of continuous parameters are E and L, the energy and angular momentum of the orbit, and R, the radial coordinate at the point where the WIMP entered the orbit. These must be supplemented by the discrete parameter $\Lambda = \pm$ which specifies whether the WIMP was going up or down when it entered its orbit. For each E and L one may find the probability of scattering at each point along the orbit. When, in addition, one knows the entry point, one may calculate the probability $p(E, L, R, \Lambda, r, \lambda)$ that a WIMP with parameters (E, R, L, Λ) is at a given height, r, and going in a given direction λ. At each such point, construct a local scattering matrix $S_r(E_i, L_i, \lambda_i; E_f, L_f, \lambda_f)$ by Monte Carlo methods. The full scattering matrix, the rate at which a WIMP in state $(E_i, L_i, R_i, \Lambda_i)$ scatters to a state $(E_f, L_f, R_f, \Lambda_f)$ can be obtained:

$$S(E_i, L_i, R_i, \Lambda_i; E_f, L_f, R_f, \Lambda_f) = \sum_{\lambda} p(E_i, L_i, R_i, \Lambda_i, r_f, \lambda) S_{r_f}(E_i, L_i, \lambda; E_f, L_f, \Lambda_f). \tag{4.1}$$

This procedure may seem rather cumbersome, but it greatly simplifies the Boltzmann collision equation because *it eliminates the transport terms.* In this basis of states, WIMPs leave a given state only if they collide. The Boltzmann equation reads simply

$$\begin{aligned}\frac{df(E,L,R,\Lambda)}{dt} = &-f(E,L,R,\Lambda)\sum_{E_f,L_f,R_f,\Lambda_f} S(E,L,R,\Lambda;E_f,L_f,R_f,\Lambda_f)\\ &+\sum_{E_i,L_i,R_i,\Lambda_i} f(E_i,L_i,R_i,\Lambda_i)S(E_i,L_i,R_i,\Lambda_i;E,L,R,\Lambda).\end{aligned} \tag{4.2}$$

The above equation is completely general. It can be used regardless of how frequently or infrequently the WIMPs collide. As a practical matter, it is not useful in the limit of frequent collisions because the height grid size must be small compared to the mean free path. However, in the sun this restriction is not critical: Even $10^{-35}\mathrm{cm}^2$ per baryon cross sections yield mean free paths $\sim .01R_\odot$

For Dirac neutrino WIMPs, moreover, this parameterization leads to a further simplification. Since the WIMPs interact only about once per 50 orbits, $p(E,L,R,\Lambda,r,\lambda)$ is essentially independent of R and Λ. The states may be described with only two continuous variables, E and L. To model the WIMPs in the sun, I discretized these states into 64 equal energies, and within each energy level, into 5 equal angular momentum brackets. (A variety of tests showed that the discreteness of the grid introduced errors in the evaporation rate of about 10%, which corresponds to less than 0.5% in the evaporation mass. This is less than errors caused by uncertainties in the solar model.)

I constructed a scattering matrix, $S(E_i,L_i;E_f,L_f)$ by Monte Carlo methods described below. I then numerically integrated the Boltzmann collision equation

$$\frac{df(E,L)}{dt} = -f(E,L)\sum_{E_f,L_f} S(E,L;E_f,L_f) + \sum_{E_i,L_i} f(E_i,L_i)S(E_i,L_i;E,L) \tag{4.3}$$

until the root mean square of

$$\frac{\sum(fS) - f\sum S}{\sum(fS) + f\sum S} \tag{4.4}$$

was less than one half percent. (Errors introduced by this cut-off were negligible compared to those mentioned above.) For every state I calculated the fraction of the time the WIMP spent in each of 26 regions of the standard solar model (Bachall *et. al.* 1982). In each region I calculated the evaporation rate due to hydrogen and helium using equation (3.7), the average kinetic energy, and the average of the square of the radial component of the velocity unit vector. This data could then be summarized by state, by energy level, or by solar region.

To construct the scattering matrix, I numerically integrated each orbit and, at each of 50 points, allowed the WIMPs to randomly scatter off hydrogen and helium at their density and temperature as given by Bachall *et. al.* (1982). I assumed the WIMPs were Dirac neutrinos with cross sections (Griest and Seckel 1987)

$$\sigma_i = 2.1 \cdot 10^{-39} \left[\frac{m_i M}{m_i + M} Q_i \right]^2 cm^2 \tag{4.5}$$

where $Q_i^2 = (N - (1 - 4\sin^2\theta_w)Z)^2$ for helium and $Q_H^2 = 3g_A^2 \sim 3(1.25)^2$. $[(1 - 4\sin^2\theta_w) = .124$; masses are in GeV.] Because it was necessary to get equally good statistics on events which happened with roughly unit probability and events which happened with probability 10^{-16} or less, I used the following technique: I divided the range of possible nuclei velocities with which the WIMP of velocity w could interact into, say, 16 segments by velocities $(0, u_1, u_2 ... u_{15}, \infty)$ such that 90% of the interactions happened with velocity less than u_1, 99% with less than u_2, 99.9% less than u_3, etc. Then one collision was allowed in each segment and its probability appropriately weighted. The number of segments was chosen so that, given the local temperature and density, all events at least 0.1% as probable as WIMP evaporation would be sampled.

From the final WIMP distribution, I calculated the average kinetic energy and defined a WIMP temperature $T_W = (2/3) < E_k >$. I then populated the sun with a 'thermal' distribution of WIMPs at this temperature and compared these two distributions and their resulting evaporation rates. (The term 'thermal distribution of WIMPs' will always be used to designate a thermal distribution truncated at the evaporation energy. The term 'truncated distribution of WIMPs' will always mean truncated *below* the evaporation energy.) It was found that while the

T_W varied about 1%, the evaporation rate varied by only 3% (which corresponds to a variation in evaporation mass of only 0.1%). At first one might think that the numerical fluctuation in WIMP temperature and that in the evaporation mass should be about the same. But from equation (3.26) one can see that roughly equal amounts of evaporation should come from every energy bin, whereas the average kinetic energy is determined mainly by the lowest T_W/E_0 fraction of the bins. Since my statistics were about equally good in every bin, one would expect roughly $\left(E_0/T_W\right)^{\frac{1}{2}} \sim 6$ times more fluctuation in the WIMP temperature than in the evaporation mass.

[The method described above has some interesting, sometimes indirect, resemblances to methods developed for the analysis of globular clusters (Shapiro 1985).]

The computations just reported were done for the precise solar model given by Bachall *et. al.* (1982). However, the standard solar model does not fix the core temperature exactly; a small range of temperatures is consistent with the input data, provided that the central pressure is also adjusted. This range is of order 1% (Bachall and Ulrich 1987). Since, for small changes, WIMP evaporation is proportional to core temperature, the numerical errors in this calculation are of the same order as those introduced by uncertainties in the solar model. If one considers a larger variation in the solar model, one should scale the WIMP evaporation mass proportional to the change in the core temperature and inversely proportional to changes in the gravitational potential difference between the center and infinity. This method of adjustment can be verified by considering equation (3.17). [The central temperature I used, based on the standard solar model (Bachall *et. al.* 1982), was 15.5 million degrees kelvin. My *calculated* value for the gravitational potential difference, using data from this same model, was $5.077 GM_\odot/R_\odot$.]

5. Results

Figures 2, 3 and 4 demonstrate the non-thermal character of the WIMP distribution for 3.7 GeV WIMPs. Figure 2 shows the average kinetic energy (normalized to the WIMP temperature) and average square of the radial component of the velocity unit vector as functions of solar radius. The curves would be horizontal (at 1 and 1/3) if the distributions were thermal. Also shown on this figure is the local temperature of the sun normalized to the WIMP temperature. Figure 3 shows the ratio of the real WIMP distribution to a thermal one by energy. The lowest curve includes all the angular momenta; the highest one includes only angular momenta which are less than 20% of maximum for that energy (states which account for about 1/3 of the evaporation). The middle curve shows angular momenta below 40%. These curves indicate that the WIMP distribution might reasonably be approximated as truncated at 40% of escape energy if all WIMPs are included, or at 80% if only those passing through the core region are included. Figure 4 shows the $\log_{10}$ of the evaporation rate per unit solar mass as a function of solar radius for thermal and actual distributions. Even on a log graph evaporation cuts off extremely rapidly for the thermal distribution and still more rapidly for the actual distribution. This confirms the analysis of section 3 that almost all evaporation takes place in the core. If the sun had a uniform temperature, these curves would be virtually straight lines.

In view of the fact that the local WIMP average kinetic energy falls with radius (albeit more slowly than the temperature) it may be asked why, in section 3, I chose to analyze the deviations from a uniform temperature as a cut-off Maxwell-Boltzmann distribution at a single temperature, as opposed to, say, a true thermal distribution at a radially dependent temperature. The answer is to be found in the fact (mentioned in a completely different context in section 4) that evaporation is extremely sensitive to the structure of the tail of the WIMP distribution, whereas the local value of the average kinetic energy is sensitive to the low-energy 'hump.' The hump is in turn relatively sensitive to local conditions, because, especially near the center, low energy WIMPs tend to stay where they are. The tail, on the other hand, tends to be highly non-local, because virtually the entire tail at every point in the sun is created in the core. Thus the local average kinetic energy is not a good indicator of WIMP evaporation. [By contrast, the sliding average kinetic energy may well have a depressing effect on heat transport (Nauenberg 1986).]

Figures 5, 6, and 7 illustrate the behavior of the WIMP distribution and the corresponding thermal distribution over a range of WIMP masses from 1 to 7 GeV. Figure 5 shows the WIMP temperature normalized to the central temperature of the sun. As the WIMPs get lighter they tend to sample the higher (colder) regions of the sun and this depresses their temperature. Figure 6 shows evaporation due to the actual WIMP distributions compared to that of corresponding thermal distributions. For high mass WIMPs, as the temperature asymptotically approaches the central solar temperature, the $\log_{10}$ of the evaporation rate for the thermal distribution approaches a straight line with slope $-\Delta\Phi/(T_0 \ln 10) = -3.5/\mathrm{GeV}$. However, for low mass WIMPs, the lower WIMP temperature tends to suppress the rate relative to this line. For the actual distribution, the rate is suppressed by about a factor of 3-5 relative to the thermal rate for high mass WIMPs, and is about the same for low mass WIMPs. It turns out that the low mass WIMP distribution (while not thermal) more closely resembles a thermal distribution than a truncated one. However, the higher mass WIMPs do have a truncated distribution (see Figure 3) and so (according to the analysis of section 3) should have their evaporation somewhat suppressed. Figure 7 shows the relative contribution of hydrogen to evaporation. For higher mass WIMPs hydrogen plays a reduced role in the actual distribution compared to the thermal one. This is due to the "impedance matching" analyzed in section 3. For low mass WIMPs there is no hydrogen enhancement relative to helium because, as mentioned above, the actual distribution more closely resembles a thermal one.

The evaporation rate for 3.68 GeV WIMPs is (4.7 billion years)$^{-1}$. Thus the evaporation mass for a solar lifetime is 3.68 ± 0.04GeV. (The error does not include possible inaccuracies in the solar model.) If there are equal numbers of WIMPs and anti-WIMPs which annihilate through the weak interaction then Griest and Seckel (1987; Griest 1986) have calculated an annihilation time for 3.0 GeV WIMPs of 15 million years. This is also the inverse evaporation rate for 2.94 GeV WIMPs. Thus the evaporation mass for an annihilation time is 2.94 ± 0.03GeV. (The error accounts for neither the substantial errors in the annihilation time calculation nor those in the solar model). According to Figure 6, the annihilation signal from solar WIMPs which are lighter than 3 GeV will be suppressed by a factor of 10^2 for each .3 GeV compared to the signal above this value.

Finally, I wish to again emphasize that the evaporation mass is proportional to $\Phi(\infty) - \Phi(0)$, inversely proportional to the core temperature and logarithmic in the helium cross section. Thus, any small change in the first two of these parameters, or a comparatively large change in the last, can be compensated easily without redoing the calculation.

6. 10^{-36} cm^2 WIMPs

Faulkner and Gilliland (1985) and independently, Spergel and Press (1985) have proposed WIMPs with about 200 to 800 times greater than weak interaction cross sections to explain the solar neutrino problem. The approximation made in going from equation (4.2) to equation (4.3) are not valid for these WIMPs because they interact about four to 16 times per orbit. However, at least in the lower range the *distribution* of these WIMPs should not be very different from the WIMPs considered above. If the WIMPs interacted many times per orbit, then their local distribution would be thermal at the local temperature. For the Faulkner-Gilliland WIMPs (200 times weak cross section) there will be a moderate deformation of the distribution in this direction relative to the WIMPs in my model. However, since the deformation will be moderate and since most of the evaporation occurs in regions of the sun where the WIMP and the solar temperatures are about equal anyway, it seems reasonable to ignore this difference. This assumption is further justified by the fact that the presence of these WIMPs alters the solar model so that the whole core has a nearly uniform temperature.

Making the assumption that my model would produce the correct distribution, I ran my program using Faulkner's solar model (Faulkner and Gilliland 1985; Faulkner *et. al.* 1986; Faulkner 1987) and cross sections which were 190 times the ones used above. In view of the lack of a microscopic theory to account for these WIMPs, I simply scaled up the Dirac neutrino cross sections so that hydrogen and helium each had about 10^{-36}cm^2 per baryon cross sections. Finally, I introduced an additional suppression factor of .2 because this is the fraction of WIMPs reaching evaporation velocities in the core which will actually make it to the surface before they rescatter. Using these approximations I get an evaporation mass (for a solar lifetime) of 3.8 GeV. Assuming that my combined assumptions have introduced errors in the evaporation rate of a factor of 3 (the difference between the thermal non-thermal rates), the error in mass may be guessed to be about .15 GeV.

Nauenberg (1986) has shown that maximum heat transport (which is what is needed to solve the solar neutrino problem) occurs at about $7 \times 10^{-36}\text{cm}^2$ per baryon. This cross section implies about 30 collisions per orbit. To properly calculate evaporation for these WIMPs would require solving equation (4.2) rather than equation (4.3).

ACKNOWLEDGEMENTS

I would like to thank Kim Griest, Michael Nauenberg, and Michael Peskin for their many helpful discussions.

APPENDIX Thermal Scattering

Here I evaluate $R(w \to v)$, the rate at which a WIMP with isotropic, velocity-independent cross-section scatters off a thermal distribution of nuclei, and display the identities which make the derivation of (3.10) and (3.11) tractable. I have gone into some detail because I believe the substitutions and identities introduced here are of general use in solving a broad class of double Maxwell-Boltzmann distribution scattering problems. I use the notation introduced in section 3.

The differential rate at which w scatters off a nucleus with velocity u and lab-frame angle θ is equal to the product of the cross section, the Maxwell-Boltzmann number density, and the relative velocity,

$$\sigma(u^2 + w^2 + 2uwz_i)^{\frac{1}{2}} \frac{2}{\pi^{\frac{1}{2}}} N\kappa^3 u^2 e^{-\kappa^2 u^2} \theta(1- \mid z_i \mid) du dz_i, \qquad \text{(A1)}$$

where $\kappa^2 = m/2T$ and $z_i = \cos\theta$. Switching to coordinates s and t which specify the velocity of the center of mass and the velocity of the WIMP in the center of mass frame,

$$(1+\mu)s = \mid \vec{u} + \mu\vec{w} \mid \qquad (1+\mu)t = \mid \vec{w} - \vec{u} \mid, \qquad \text{(A2)}$$

$$\frac{\partial(s,t)}{\partial(u,z_i)} = \frac{u^2 w}{8\mu_+^3 st}, \qquad \text{(A3)}$$

this becomes

$$\frac{32\mu_+^4}{\pi^{\frac{1}{2}}} \kappa^3 N\sigma \frac{t^2 s}{w} e^{-\kappa^2 u^2} \theta(w- \mid s-t \mid)\theta(s+t-w) ds dt, \qquad \text{(A4)}$$

where u is now regarded as a function of s and t:

$$u^2 = 2\mu\mu_+ t^2 + 2\mu_+ s^2 - \mu w^2. \qquad \text{(A5)}$$

The rate at which w scatters to v is given by integrating equation (A4) against

$$dv \frac{1}{2} \int_{-1}^{1} dz_f \delta(v - (s^2 + t^2 - 2stz_f)^{\frac{1}{2}}) = \frac{1}{2} \frac{v dv}{st} \theta(v- \mid s-t \mid)\theta(s+t-v) \qquad \text{(A6)}$$

over all s and t. Here z_f is the cosine of the recoil angle in the center of mass frame. Thus

$$R(w \to v)dv = \int_0^\infty ds \int_0^\infty dt \frac{16\mu_+^4}{\pi^{\frac{1}{2}}} \kappa^3 N\sigma \frac{vdv}{w} te^{-\kappa^2 u^2}$$

$$\theta(w - \mid s-t \mid)\theta(s+t-w)\theta(v - \mid s-t \mid)\theta(s+t-v). \tag{A7}$$

Again changing variables to

$$x = t + s \qquad y = t - s, \tag{A8}$$

noting that

$$\mu_+ + \mu_- = \mu_+^2 - \mu_-^2 = \mu, \tag{A9}$$

$$u^2 = (\mu_+ x + \mu_- y)^2 + \mu(y^2 - w^2) = (\mu_+ y + \mu_- x)^2 + \mu(x^2 - w^2), \tag{A10}$$

and choosing the case $w > v$, this becomes

$$R(w \to v) = \frac{4\mu_+^4}{\pi^{\frac{1}{2}}} N\sigma \frac{v}{w} \int_0^\infty dx \int_{-\infty}^\infty dy \kappa^3 (x+y) e^{-\kappa^2 u^2} \theta(v - \mid y \mid)\theta(x - w) \tag{A11}$$

$$= \frac{4\mu_+^4}{\pi^{\frac{1}{2}}\mu} N\sigma \frac{v}{w} \int_w^\infty dx \int_{-v}^v dy \kappa^3 \left[(\mu_+ x + \mu_- y) e^{-\kappa^2[(\mu_+ x + \mu_- y)^2 + \mu(y^2 - w^2)]} \right.$$
$$\left. + (\mu_+ y + \mu_- x) e^{-\kappa^2[(\mu_+ y + \mu_- x)^2 + \mu(x^2 - w^2)]}\right] \tag{A12}$$

$$= \frac{2\mu_+^3}{\pi^{\frac{1}{2}}\mu} N\sigma\kappa \frac{v}{w} \left[\int_{-v}^v dy e^{-\kappa^2[(\mu_+ w + \mu_- y)^2 + \mu(y^2 - w^2)]} \right.$$
$$\left. + \int_w^\infty dx \left(-e^{-\kappa^2(\mu_+ v + \mu_- x)^2} + e^{-\kappa^2(\mu_+ v - \mu_- x)^2}\right) e^{-\kappa^2 \mu (x^2 - w^2)} \right] \tag{A13}$$

$$= \frac{2\mu_+^3}{\pi^{\frac{1}{2}}\mu} N\sigma\kappa \frac{v}{w} \Bigg[\int_{-v}^{v} dy e^{-\kappa^2(\mu_+ y + \mu_- w)^2}$$

$$+ \int_{w}^{\infty} dx (-e^{-\kappa^2(\mu_+ x + \mu_- v)^2} + e^{-\kappa^2(\mu_+ x - \mu_- v)^2}) e^{-\kappa^2 \mu (v^2 - w^2)} \Bigg] \quad \text{(A14)}$$

$$= \frac{2}{\pi^{\frac{1}{2}}} \frac{\mu_+^2}{\mu} N\sigma \frac{v}{w} [\chi(-\alpha_-, \alpha_+) + \chi(-\beta_-, \beta_+) e^{-\frac{M}{2T}(v^2 - w^2)}]. \quad \text{(A15)}$$

The case of $w < v$ can either be done the same way or deduced from detailed balance arguments. Next, one may easily establish the following identities:

$$\alpha_\pm^2 = \beta_\pm^2 + \mu\kappa^2(v^2 - w^2) = \gamma_\pm^2 + \varsigma - \nu\kappa^2 w^2, \quad \text{(A16)}$$

$$\frac{\mu_-}{\mu_+} \frac{d}{dv} \chi(\pm\alpha_-, \alpha_+) = e^{-\mu\kappa^2(v^2 - w^2)} \frac{d}{dv} \chi(\pm\beta_-, \beta_+), \quad \text{(A17)}$$

$$e^{-\nu\kappa^2 w^2} \frac{d}{dw} \chi(\pm\alpha_-, \alpha_+) = \frac{\mu_-}{\mu_+} e^{-\mu\kappa^2(v^2 - w^2)} e^{-\nu\kappa^2 w^2} \frac{d}{dw} \chi(\pm\beta_-, \beta_+)$$

$$= \frac{\mu_-}{\xi} e^{-\varsigma} \frac{d}{dw} \chi(\pm\gamma_-, \gamma_+), \quad \text{(A18)}$$

$$2\kappa^2\mu_+^2 \int dv v \chi(\pm\alpha_-, \alpha_+) = \frac{1}{2}\alpha_- e^{-\alpha_+^2} \mp \frac{1}{2}\alpha_+ e^{-\alpha_-^2} + (\alpha_+\alpha_- - \frac{1}{2})\chi(\pm\alpha_-, \alpha_+), \quad \text{(A19)}$$

$$2\kappa^2\mu_+^2 \int dw w \chi(\pm\beta_-, \beta_+) = -\frac{1}{2}\beta_- e^{-\beta_+^2} \pm \frac{1}{2}\beta_+ e^{-\beta_-^2} + (-\beta_+\beta_- - \frac{1}{2})\chi(\pm\beta_-, \beta_+), \quad \text{(A20)}$$

where

$$\varsigma = \frac{\nu\kappa^2\mu_+^2 v^2}{\xi^2}. \quad \text{(A21)}$$

Using equations (A17), (A18), and (A19) it is trivial to derive equation (3.7).

By writing the evaporation integral as

$$R(w_c \mid v_e) = \frac{2}{\pi}\Big(\frac{T}{T_W}\Big)^{\frac{3}{2}}\Big(\frac{2T}{M}\Big)^{\frac{1}{2}}\sigma N N_W$$
$$\Bigg\{\frac{-1}{2\nu}\int\limits_{w=0}^{w_c}\big[\mu\big(\alpha_+ e^{-\alpha_-^2} - \alpha_- e^{-\alpha_+^2}\big) + (\mu - 2\mu\alpha_+\alpha_- - 2\mu_+\mu_-)\chi(\alpha_-,\alpha_+)\big]\, de^{-\nu\kappa^2 w^2}$$
$$+ \frac{-1}{2(\nu-\mu)}\int\limits_{w=0}^{w_c} 2\mu_+^2 e^{-\mu\kappa^2 v^2}\chi(\beta_-,\beta_+)\, de^{-(\nu-\mu)\kappa^2 w^2}\Bigg\}, \tag{A22}$$

integrating by parts, noting that in

$$\frac{1}{\kappa\mu_-}\frac{d}{dw}\big[\mu\big(\pm\,\alpha_+ e^{-\alpha_-^2} - \alpha_- e^{-\alpha_+^2}\big) + (\mu - 2\mu\alpha_+\alpha_- - 2\mu_+\mu_-)\chi(\pm\alpha_-,\alpha_+)\big]$$
$$= 2(\mu - \mu_+\mu_-)\big(\pm\, e^{-\alpha_-^2} + e^{-\alpha_+^2}\big) + 4\mu\mu_-\kappa w\chi(\pm\alpha_-,\alpha_+), \tag{A23}$$

the complicated terms cancel, and using equations (A16) and (A18), one may obtain equation (3.11) with only half a dozen lines of algebra. One may similarly obtain equation (3.10) by using equations (A16), (A18), and (A20).

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FIGURE CAPTIONS

1. IMPEDANCE MATCHING: Evaporation rate as a function of $\log_{10}(\mu)$ for $T = T_W = E_e/30$ and $E_c/E_e = .5, .6, .7, .8, .9, 1.0$. Curves are normalized to rate for $\mu = E_c/E_e = 1$.

2. NON-THERMAL RADIAL DISTRIBUTION: ($2/3\times$) average WIMP kinetic energy (solid) and average square of the radial component of WIMP direction (dots) as functions of solar radius. Also shown is solar temperature (dashes). Temperatures are normalized to WIMP temperature. WIMP mass is 3.7 GeV.

3. NON-THERMAL ENERGY DISTRIBUTION: Ratio of true WIMP distribution to a thermal distribution at the WIMP temperature, as a function of energy; for all angular momenta (dashes), for the angular momenta less than 20% of maximum (solid), and for less than 40% of maximum (dots). WIMP energies are normalized to escape energy. WIMP mass is 3.7 GeV.

4. EVAPORATION SUPPRESSION: The $\log_{10}$ of the evaporation rate per unit solar mass (in inverse seconds) as a function of solar radius for 3.7 GeV WIMPs. Actual distribution (solid) and thermal distribution at WIMP temperature (dots) are shown.

5. WIMP temperature as a function of WIMP mass (in GeV). Temperatures are normalized to the central solar temperature.

6. The $\log_{10}$ of the total evaporation rate (in inverse seconds) as a function of WIMP mass (in GeV) is shown for the actual distributions (solid) and thermal distributions at the WIMP temperature (dots). The ratios of the thermal to actual rates are also shown (dashes).

7. Fraction of total evaporation which is due to hydrogen as a function of WIMP mass (in GeV) is shown for actual distributions (solid) and thermal distributions at the WIMP temperature (dots).

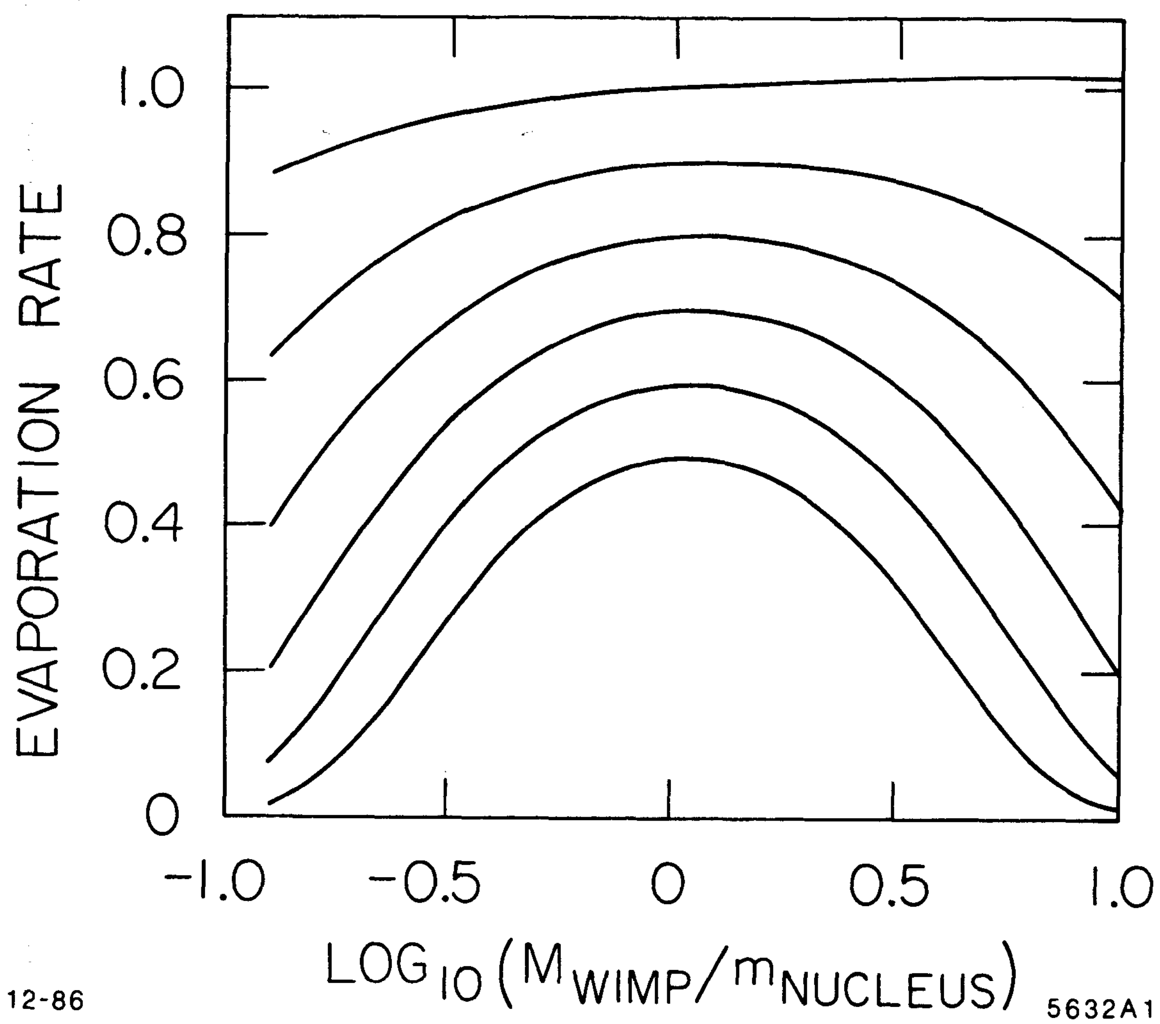
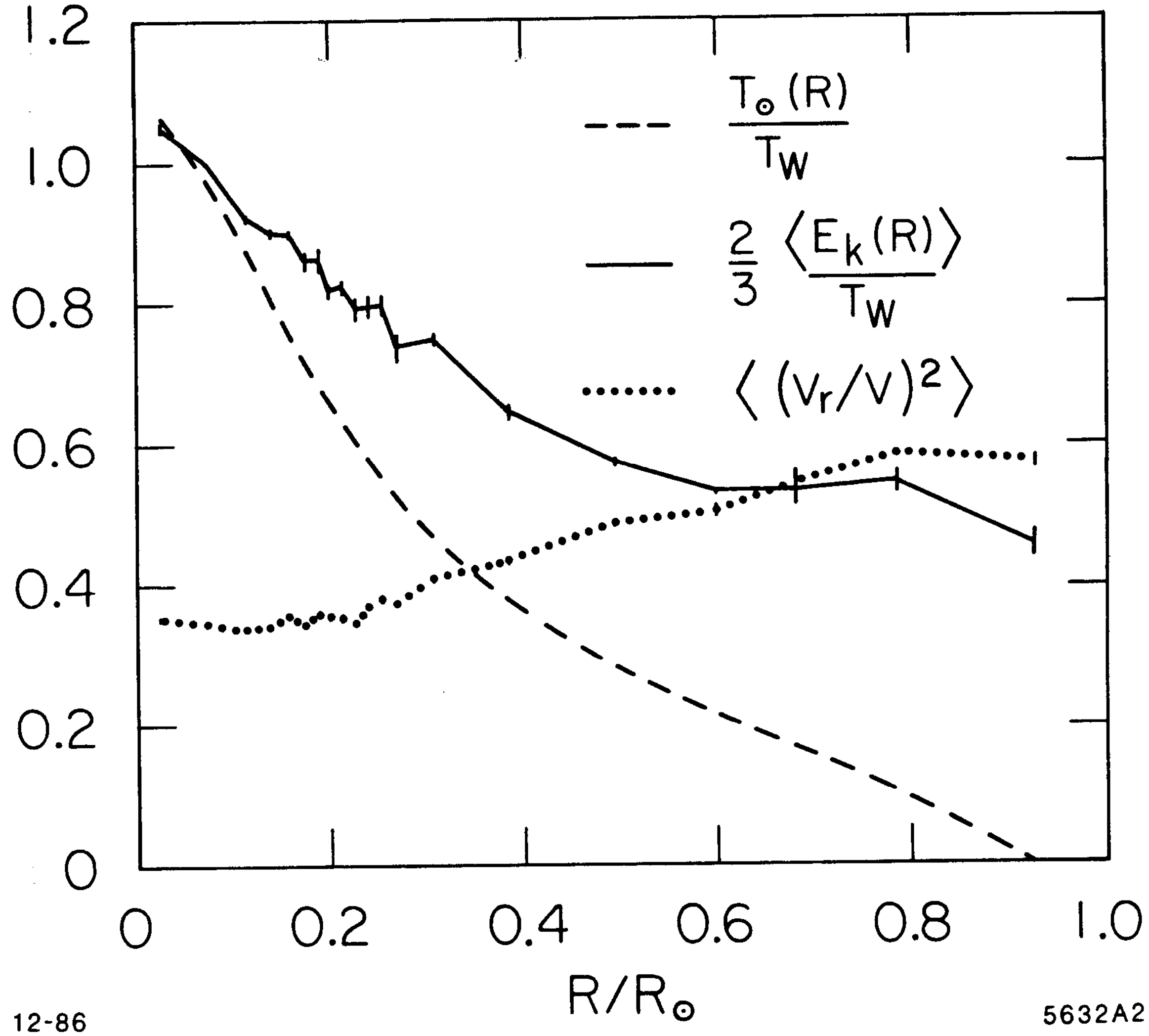
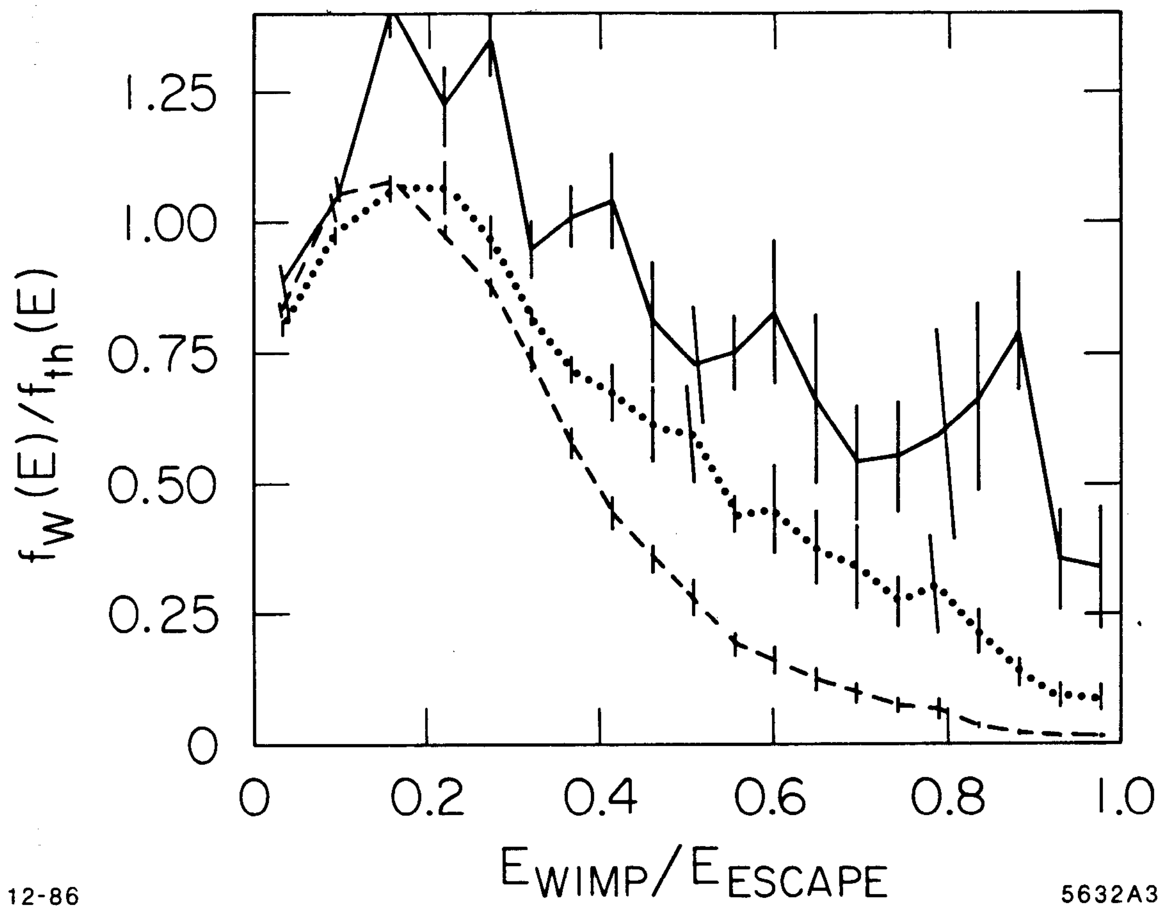

EVAPORATION RATE
1.0
0.8
0.6
0.4
0.2
0
−1.0
−0.5
0
0.5
1.0
LOG10 (MWIMP/mNUCLEUS)
12-86
5632A1

Fig. 1



1.2
1.0
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1.0
R/R⊙
$T_\odot(R)/T_W$
$\frac{2}{3}\langle E_k(R)/T_W \rangle$
$\langle (V_r/V)^2 \rangle$
12-86
5632A2

Fig. 2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1.00
0.75
0.50
0.25
0
$f_W(E)/f_{th}(E)$
0
0.2
0.4
0.6
0.8
1.0
E_{WIMP}/E_{ESCAPE}
12-86
5632A3

Fig. 3

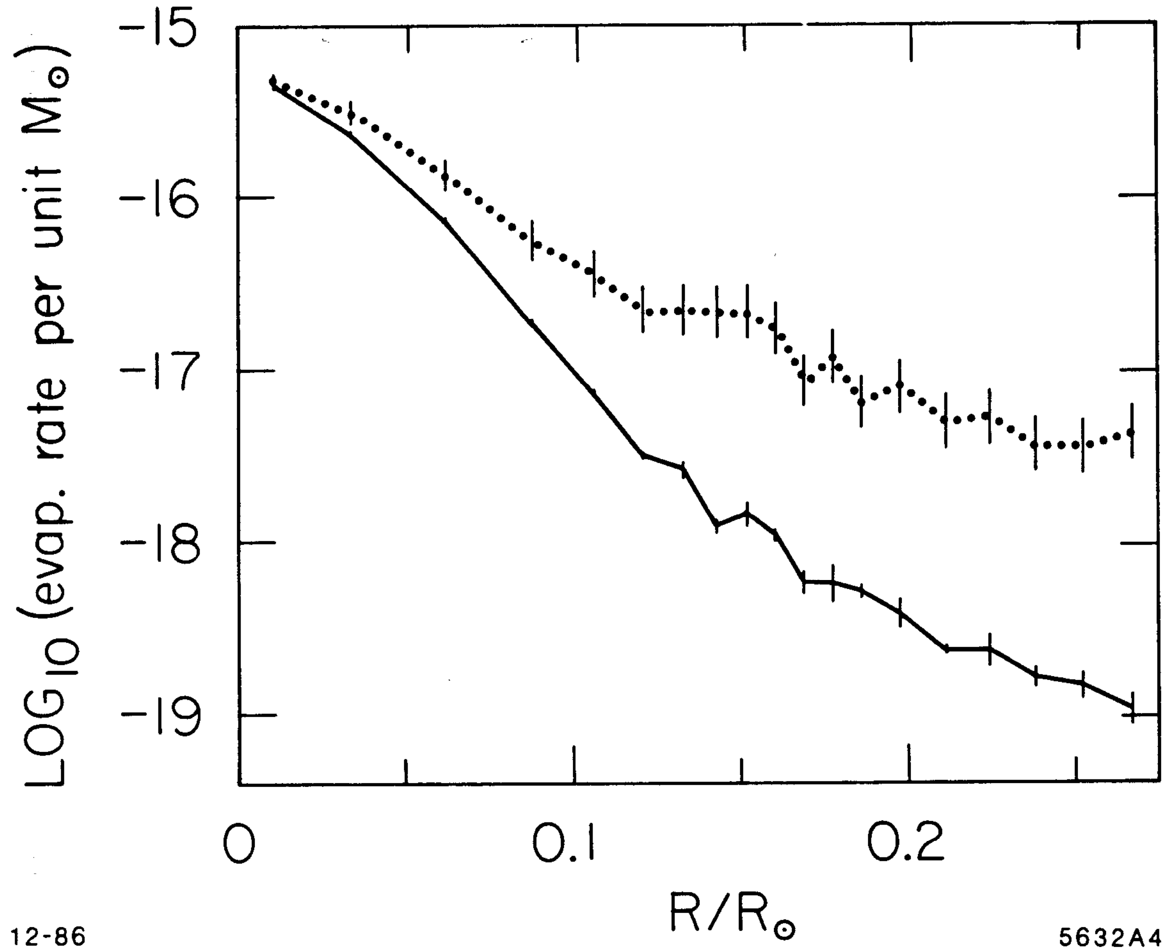
-15
-16
-17
-18
-19
LOG$_{10}$ (evap. rate per unit $M_\odot$)
0
0.1
0.2
R/$R_\odot$
12-86
5632A4

Fig. 4

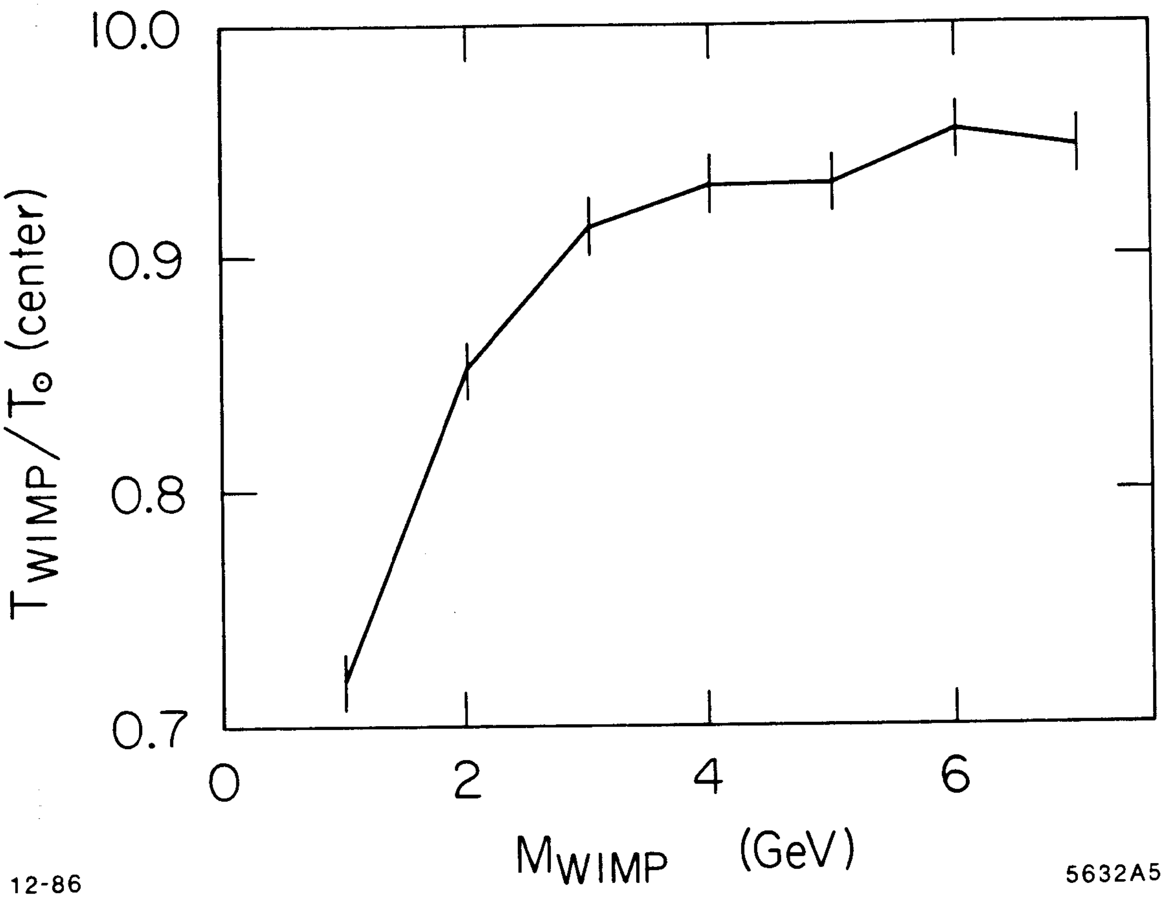

Fig. 5

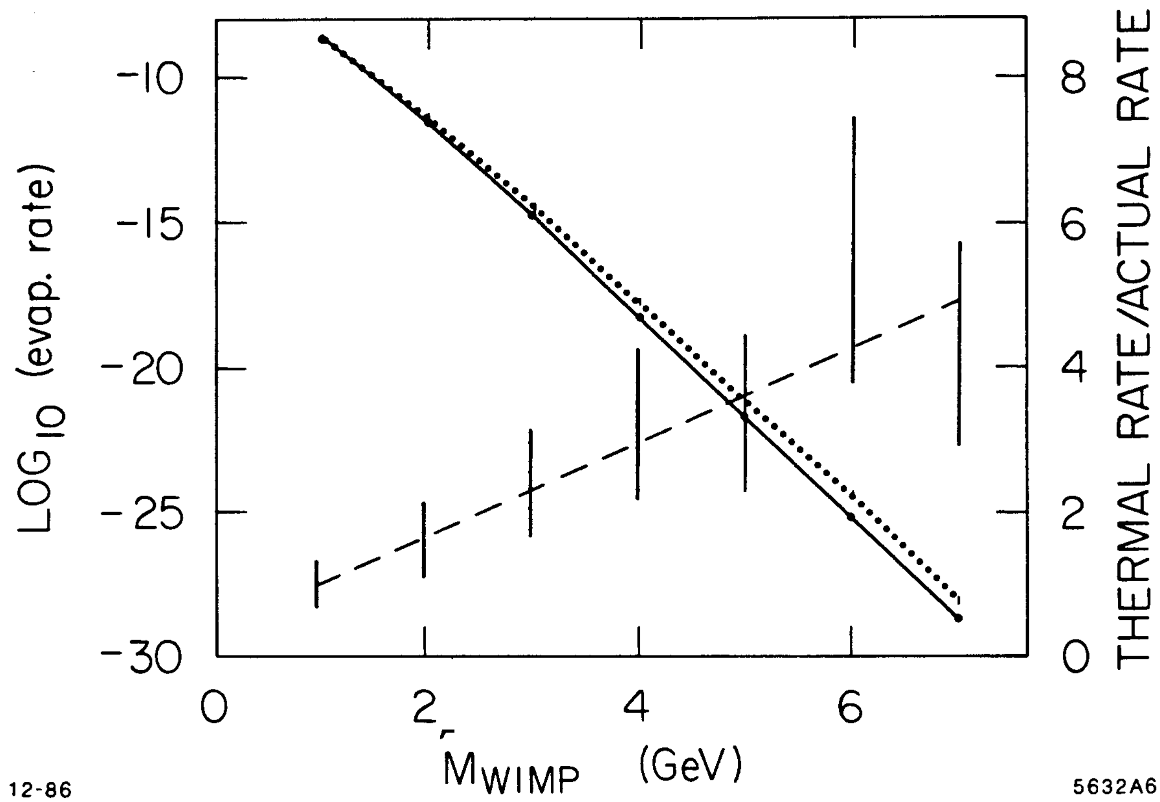
-10
-15
-20
-25
-30
LOG10 (evap. rate)
0
2
4
6
MWIMP (GeV)
8
6
4
2
0
THERMAL RATE/ACTUAL RATE
12-86
5632A6

Fig. 6

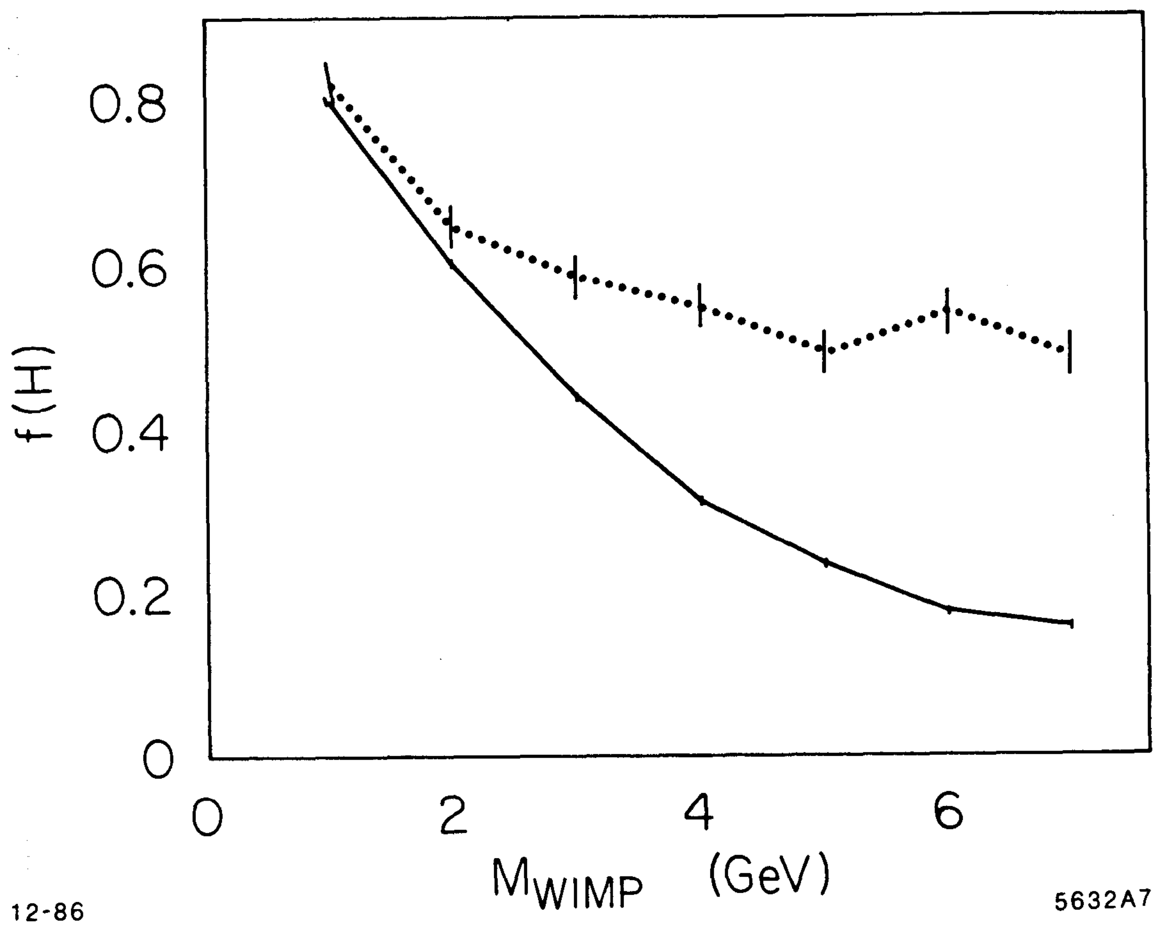

Fig. 7