A Conceptual Design of Final Focus Systems for Linear Colliders[*]

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1. Introduction

Linear colliders are a relatively recent development in the evolution of particle accelerators. This report discusses some of the approaches that have been considered for the design of Final Focus Systems to demagnify the beam exiting from a linac to the small size suitable for collisions at the interaction point. The system receiving the most attention is the one adopted for the SLAC Linear Collider. However, the theory and optical techniques discussed should be applicable to the design efforts for future machines.

2. The Problem

The principal problem in designing Final Focus Systems (FFS) for linear colliders is the elimination or minimization of the chromatic distortions introduced by the final lens system nearest to the interaction point (I.P.). If the beam exiting from the linac is monoenergetic and has a small enough emittance, then the FFS design for colliders is relatively simple. It could consist of a simple first–order optical system demagnifying the monoenergetic linac beam to the small size needed for collisions at the I.P. The ultimate limitation would then be the residual geometric (monoenergetic) aberrations of the optical system. The magnitude of these aberrations would be a function of the phase space emittance from the linac and the natural third–order geometric aberrations of quadrupole focusing systems. Unfortunately the beams from microwave linear accelerators are not monoenergetic. They have a finite momentum spread, and one is forced to cope with the chromatic distortions caused by this momentum spectrum.

There are several possible approaches to solving this problem:

1. A reduction in the momentum spread or the emittance of the beam exiting from the linac clearly eases the severity of the problem.

2. The magnitude of the chromatic distortion is a function of the distance, l^*, of the first lens from the I.P. The closer the lens system is to the I.P. the smaller the chromatic distortion. Hence strong, compact lenses are an asset to solving the problem.

3. Sextupoles can be introduced into the optical design to cancel the principal second–order chromatic aberrations, leaving the residual third–order chromatic and geometric aberrations as the fundamental limitation to achieving a small spot at the I.P.

4. Alternatively one might choose to "live with" the small residual second–order chromatic aberrations by choosing small aperture but very strong lenses as the final lenses in the FFS. Such systems were studied and considered for the initial SLC design but were ultimately rejected in favor of a "sextupole corrected" solution.

Unfortunately "sextupole corrected" solutions create other problems that must be minimized in order to find a satisfactory solution. Sextupoles require dipoles to be inserted into the chromatic correction system in order for the sextupoles to selectively couple to the chromatic aberrations that one is trying to eliminate. These dipoles, in turn, introduce emittance growth to the beam via the fluctuations in the synchrotron radiation energy losses. This complication demands a careful selection of the placement and strength of the dipoles so as to minimize this effect.

Sextupoles create an additional problem which ultimately limits the smallest spot size that can be achieved at the I.P. While sextupole families can be designed such that both the second–order geometric and chromatic aberrations vanish,[1] the cross–coupling of the sextupole families generates residual third– and higher–order aberrations. For the SLC case these residual aberrations limit the effective β^* at the I.P. to 8 mm. Without the sextupole corrections the SLC would have a β^* of 7 cms for the same l^*. Thus a factor of nine has been gained in this particular application.

3. Optics Definitions and Notation

For the theoretical discussion in this report, we use the six dimensional phase space parameters defined by the TRANSPORT[2] notation; namely

$$
X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix}
\tag{1}
$$

where $\delta = (p - p_0)/p_0$. The first– and second–order optics is represented by the R and T matrix elements as follows:

$$
x_i = \sum_{j=1}^{6} R_{ij} x_j + \sum_{j,k=1}^{6} T_{ijk} x_j x_k + \text{higher–order terms}
\tag{2}
$$

where the R matrix is commonly expressed in the form:

$$
R = \begin{pmatrix}
c_x(s) & s_x(s) & 0 & 0 & 0 & d_x(s) \\
c_x'(s) & s_x'(s) & 0 & 0 & 0 & d_x'(s) \\
0 & 0 & c_y(s) & s_y(s) & 0 & 0 \\
0 & 0 & c_y'(s) & s_y'(s) & 0 & 0 \\
R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\tag{3}
$$

The objective in the design of the final focus system for the SLC was to find a way of eliminating all of the second–order aberrations, T_{ijk}, and be left with only the appropriate linear terms plus the residual third– and higher–order aberrations. These residual terms then determined the ultimate β^* that could be achieved at the interaction point of the collider.

4. Chromatic Aberrations

Assuming an upright ellipse, i.e., $\alpha_o = 0$, at the beginning of the final focus system, then β at the I.P. as a function of momentum may be expressed by the equations

$$\beta(\delta) = R_{11}^2(\delta)\beta_0 + \frac{R_{12}^2(\delta)}{\beta_0}$$

or

$$\beta(\delta)\beta_0 = R_{11}^2(\delta)\beta_0^2 + R_{12}^2(\delta) \tag{4}$$

where β_0 is the value of β at the beginning of the FFS, and $\delta = \Delta p/p_0$. R_{11} and R_{12} are matrix elements of the R transformation matrix measured from the beginning of the FFS to the I.P. $R_{11}(\delta)$ and $R_{12}(\delta)$ are a function of the chromatic aberrations in the system as follows:

$$R_{11}(\delta) = R_{11}(0) + T_{116}\delta + U_{1166}\delta^2 + ...$$
$$R_{12}(\delta) = R_{12}(0) + T_{126}\delta + U_{1266}\delta^2 + ... \tag{5}$$

where $R_{11}(0)$ and $R_{12}(0)$ are the values of the matrix elements for the central momentum P_0. Substituting Eqs. (5) into (4) and collecting terms, we obtain the result

$$\begin{aligned}
\beta(\delta)\beta_0 = & \left[R_{11}^2\beta_0^2 + R_{12}^2\right] \\
& + 2\left[R_{11}T_{116}\beta_0^2 + R_{12}T_{126}\right]\delta \\
& + \left[(T_{116}^2 + 2R_{11}U_{1166})\beta_0^2 + T_{126}^2 + 2R_{12}U_{1266})\right]\delta^2 \\
& + \text{higher order geometric and chromatic distortions} \quad .
\end{aligned} \tag{6}$$

The ideal objective would be to devise an "aberration free" optical system where all terms in Eq. (6) vanish except for the first term

$$\beta(\delta) = R_{11}^2\beta_0 = \beta^*(0)$$

where $R_{11} = M$ is the monoenergetic demagnification of the beam envelope at the I.P.

As a step in this direction, we now wish to derive some chromatic properties of telescopic systems which have been useful in solving our problem. We restrict the discussion to the x plane for simplicity, but the results and conclusions are applicable to both transverse planes, x and y.

PROPERTIES OF TELESCOPIC SYSTEMS

Shown below in Fig. 1 is a representative one–dimensional, telescopic system. For monoenergetic trajectories, telescopic systems have point–to–point $[R_{12}(0) = 0]$ and parallel–to–parallel $[R_{21}(0) = 0]$ imaging.
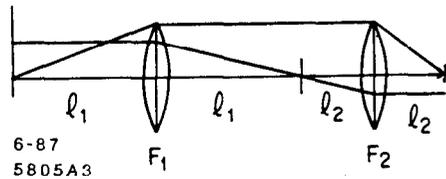

Fig. 1. A one–dimensional symmetric telescopic system.

In addition, telescopic systems with the symmetry shown in Fig. 1 have some interesting chromatic properties. To illustrate this, we define the following quantities

$$\epsilon = \frac{\Delta p}{p}; \qquad \frac{1}{F_1} = \frac{1-\epsilon}{l_1}; \qquad \frac{1}{F_2} = \frac{1-\epsilon}{l_2}; \qquad l = l_1 + l_2;$$

$$\text{and } M = \frac{l_2}{l_1}$$

where F_1 and F_2 are the focal lengths as a function of momentum of the two lenses comprising the system and M is the demagnification of the beam envelope for the central momentum p_0.

Note the relationship between ϵ and δ.

$$(1 - \epsilon) = \frac{1}{1+\delta} \quad \text{or} \quad \epsilon = \frac{\delta}{1+\delta}$$

ϵ has been used in this derivation so as to avoid a power series expansion in the matrix elements and thus simplify the result.

The transformation matrix $R(\epsilon)$ for the telescopic system shown in Fig. 1 is then

$$R(\epsilon) = \begin{bmatrix} \epsilon & -l_2(1+\epsilon) \\ -\frac{1-\epsilon}{l_2} & \epsilon \end{bmatrix} \cdot \begin{bmatrix} \epsilon & l_1(1+\epsilon) \\ -\frac{1-\epsilon}{l_1} & \epsilon \end{bmatrix}.$$

or finally

$$R(\epsilon) = \begin{bmatrix} -M + (M+1)\epsilon^2 & l\epsilon + l\epsilon^2 \\ -\frac{l}{l_1 l_2}(\epsilon - \epsilon^2) & -\frac{1}{M} + (1 + \frac{1}{M})\epsilon^2 \end{bmatrix}. \tag{7}$$

Comparing Eq. (7) with Eq. (5), we observe that, in addition to $R_{12}(0) = 0$, $T_{116} = 0$ for this telescopic system. To appreciate the importance of this, we substitute the result into Eq. (6) and find

$$\begin{aligned}
\beta(\delta)\beta_0 &= R_{11}^2 \beta_0^2 \\
&+ 2\,[0]\,\delta \\
&+ \left[2R_{11} U_{1166} \beta_0^2 + T_{126}^2 \right] \delta^2 \\
&+ \text{higher-order terms} .
\end{aligned} \tag{8}$$

So for telescopic systems having the symmetry shown in Fig. 1, we find that the first derivative of β with respect to δ vanishes and we are left with the δ^2 term as the first term of importance.

If now we equate

$$2R_{11} U_{1166} \beta_0^2 = -T_{126}^2 \tag{9}$$

then the δ^2 term also vanishes and the telescope becomes achromatic to second order in $(\frac{\Delta p}{p_0})$. Note that no sextupoles have been used to obtain this result! While this is an interesting theoretical result one finds that for most physically realizable situations

$$T_{126}^2 \gg 2R_{11} U_{1166} \beta_o^2 .$$

At this point one has to decide whether to tolerate the second–order chromatic distortion generated by the quadrupoles via the T_{126} aberration or, alternatively, eliminate T_{126} by introducing sextupoles and defer the optical distortions to higher–order terms.

For the moment we consider the first alternative and evaluate the magnitude of the T_{126} aberration for an all–quadrupole telescopic FFS. Equation 8 may then be truncated and rewritten in the following form:

$$\beta(\delta) \simeq R_{11}^2(0)\beta_0 + \frac{[T_{126}\delta]^2}{\beta_0}$$

or

$$\beta(\delta) = \beta^*(0) + \frac{[R_{11}T_{126}]^2\delta^2}{\beta^*(0)} \tag{10}$$

where $\beta^*(0) = R_{11}^2(0)\beta_0$ is the monoenergetic β at the interaction point.

We now wish to calculate the average value $\bar{\beta}(\delta)$ and find its minimum. The result clearly depends upon the distribution function representing the momentum spectrum of the linac. We shall consider two possible cases: a gaussian and a rectangular distribution. For the gaussian distribution, the average value of $\beta(\delta)$ is

$$\bar{\beta}(\delta) = \beta^*(0) + \frac{[R_{11}T_{126}]^2\sigma_\delta^2}{\beta^*(0)} \tag{11}$$

where σ_δ is the sigma of the gaussian momentum distribution function.

For a rectangular momentum distribution the result is –

$$\bar{\beta}(\delta) = \beta^*(0) + \frac{[R_{11}T_{126}]^2}{\beta^*(0)} \cdot \frac{\delta_{max}^2}{3} \tag{12}$$

where δ_{max} is the maximum value of δ in a rectangular (constant value) distribution function representing the momentum spectrum of the linac.

If now we wish to find the minimum value of $\bar{\beta}(\delta)$, using $\beta^*(0)$ as the variable, the result is obtained by equating the two terms in each equation. The result is

$$\bar{\beta}(\delta)_{min} = 2\beta^*(0) = 2[R_{11}T_{126}]\sigma_\delta \tag{13}$$

for the gaussian distribution and

$$\bar{\beta}(\delta)_{min} = 2\beta^*(0) = 2[R_{11}T_{126}] \cdot \frac{\delta_{max}}{\sqrt{3}} \tag{14}$$

for the rectangular distribution.

The rectangular distribution is more representative of the SLC linac in its present mode of operation, so for the remainder of this report we shall assume Eq. (14) to be valid.

5. The Relationship between Path Length and Chromatic Aberrations

Using the optics formalism developed in SLAC Report-75,[2] it can be shown that

$$R_{11}T_{126} = \int_*^L (s_x')^2 ds \tag{15}$$

where $R_{11}T_{126}$ is measured from the beginning of the FFS to the interaction point(*), but s_x' is the R_{22} matrix element measured from the I.P.(*) back through to the beginning of the final focus system.

Evaluating the integral in Eq. (15), we find

$$\int_*^L (s'_x)^2 ds = \int_*^L \left[\frac{x'(s)}{x'(0)}\right]^2 ds \simeq 2 \int_*^L \frac{[1 - \cos x'(s)]}{[x'(0)]^2} ds \qquad (16)$$

$$= 2\frac{\Delta l}{[x'(0)]^2} \quad ,$$

where $x'(s)$ is the angle an arbitrary trajectory makes with respect to the central trajectory at a position s. $x'(0)$ is its angle at the interaction point (*) and $x(0)=0$. Δl is the path–length difference between this arbitrary trajectory and the central trajectory.

Using TRANSPORT notation, we define T_{522} as follows:

$$\frac{\Delta l}{[x'(0)]^2} = T_{522}$$

then

$$\int_*^L (s'_x)^2 ds = 2T_{522} \qquad (17)$$

T_{522} is the normalized second–order path length difference measured from the interaction point (*) back through the system. Similar results may be derived for the y plane optics.

Note that the integral is proportional to the square of the angle a trajectory makes at any position s in the system. Therefore the main contribution to the integral comes from the first lens system nearest to the I.P. where the angles are largest.

This relationship between the transverse chromatic aberrations and the path length of the monoenergetic trajectories through a system can be shown to be a general result of Hamiltonian Mechanics. It is related to the fact that E and t are conjugate variables in the Hamiltonian description of the system.

EXAMPLES:

Let's consider this integral for some representative lens systems. Consider first a simple thin lens module of length $L_s = 2F$ as shown in Fig. 2.
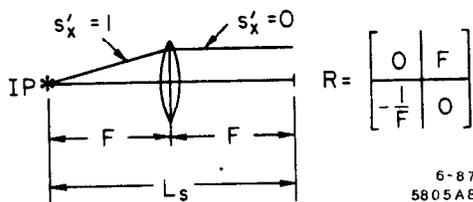

Fig. 2. A thin lens module.

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The integral for this module may be calculated by observing that $s'_x = 1$, by definition, from the interaction point (*) to the input face of the lens. The distance to the lens is F. $s'_x = 0$ after passing through the lens. Thus,

$$\int_*^L (s'_x)^2 ds = 2T_{522} = F = \frac{L_s}{2} \quad .$$ (18)

Thus we learn that short focal length lenses are desirable if one wishes to minimize the chromatic distortion in a final focus system.

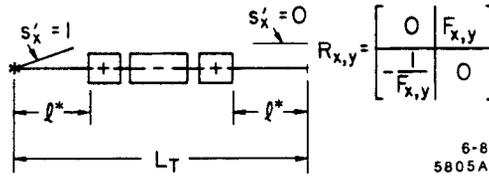Consider now a symmetric triplet lens system as shown in Fig. 3.

Fig. 3. A symmetric triplet lens module.

If we impose the condition that the chromatic distortion be equal in both transverse planes (x and y), i.e., that

$$T_{522} = T_{544},$$

then in a well designed triplet module the following empirical scaling law has been found to apply:

$$l^* \left[\frac{B_0}{aB\rho} \right]^{\frac{1}{2}} \geq \frac{4}{3}$$ (19)

where l^* is the distance to the input face of the first lens in the triplet, $\frac{B_0}{a} = \frac{\partial B}{\partial x}$ is the field gradient in each lens (assumed to be the same for all elements of the triplet) and $B\rho$ is the magnetic rigidity of the beam. For an optimized symmetric triplet system, it has also been empirically determined that

$$R_{11}T_{126} = \int_*^L (s'_x)^2 ds = 2T_{522} \approx L_T$$

and

$$L_T \approx 4l^* \quad .$$ (20)

Evaluation of the path length integral in Eq. (20) shows that approximately one-fourth of the total chromatic distortion is generated by l^* and the remaining three-fourths is generated by the path–length excursions within the triplet lens structure itself.

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If the triplet and singlet lens systems occupy the same physical length, i.e., $L_T = L_S$, then we conclude that the total chromatic distortion in the triplet system is approximately twice that found in the singlet system!

Substituting the results of Eq. (20) for the symmetric triplet into Eq. (14) for a rectangular momentum distribution, we find for an all quadrupole FFS (no sextupoles) the result

$$\bar{\beta}(\delta)_{min} \geq 8l^* \frac{\delta_{max}}{\sqrt{3}} \quad . \tag{21}$$

For a gaussian momentum distribution of the linac, the result is

$$\bar{\beta}(\delta)_{min} \geq 8l^* \sigma_\delta \quad . \tag{22}$$

The inequality signs in Eqs. (21) and (22) are included because the path length integral has been evaluated only for the triplet lens nearest to the interaction region. Additional contributions to the integral will come from the remaining portions of the telescopic system. The total $\bar{\beta}$ will be of the order (1+M) times the above values, where M is the demagnification of the telescopic system. Since M is normally very small, Eqs. (21) and (22) are usually good approximations to the final answer.

BEAM–BEAM DISRUPTION EFFECTS

If the disrupted beams must pass back through the apertures of the FFS optical elements, then the maximum disruption angle, θ_D, becomes an important factor in the design.

The maximum radius of the beam envelope within the quadrupole triplet depends upon θ_D. For an optimized triplet, the result is found empirically from TRANSPORT to be

$$a_{beam} \cong 3l^* \theta_D \quad . \tag{23}$$

For a gaussian beam profile, in the transverse coordinates x and y, θ_D has the form[3]

$$\theta_D = \frac{Nr_e}{\gamma\sigma} \cdot K_p \tag{24}$$

where N is the number of particles in a bunch, $r_e = \frac{e^2}{mc^2}$ is the classical electron radius, $\gamma = \frac{E}{mc^2}$ and $\sigma = \sqrt{\beta^*\epsilon}$, where $\beta^* = \bar{\beta}(\delta)_{min}$. ϵ is the beam emittance and K_p is a design "safety factor" to allow for the beamstrahlung pinch effect. K_p should be determined from computer simulations. In the SLC design a value of $K_p = 3$ has been used.

For two intersecting beams having gaussian cross sections, the luminosity is defined as

$$\mathcal{L} = \frac{N^2 f}{4\pi\sigma^2} \cdot 10^{-4} cm^{-2} sec^{-1} \tag{25}$$

where f is the repetition rate and σ is the gaussian sigma beam size at the interaction point given in meters.

Equations (19), (23), (24) and (25) may be combined for the "optimized triplet" to yield the inequality

$$B_0 l^* \geq 9 \cdot 10^{-14} K_p \left(\frac{\mathcal{L}}{f}\right)^{\frac{1}{2}} \text{kGauss m} \ . \tag{26}$$

This equation applies for a round beam spot at the interaction point. It tells us that for a given luminosity there is a minimum product of B_0 and l^* required. This result is independent of whether sextupoles are used for the chromatic correction process. As will be seen later for a given luminosity, sextupoles allow a larger l^* than would otherwise be possible with an uncorrected system.

Combining Eqs. (21) and (25), the attainable luminosity for the triplet without sextupole corrections is

$$\mathcal{L} \leq (\frac{\sqrt{3}}{32\pi})\left(\frac{N^2 f}{\epsilon l^* \delta_{max}}\right) \cdot 10^{-4} cm^{-2} sec^{-1} \tag{27}$$

for a beam with a rectangular momentum spectrum. If satisfactory luminosity cannot be achieved with just a simple symmetric triplet, then the introduction of sextupoles to eliminate the quadrupole chromatic distortions, manifested by the T_{126} and T_{346} aberrations, is a logical recourse. This is what was done for the present SLC design.

6. The Final Focus System for the SLAC Linear Collider

The basic parameters for the initial SLC design were the following:

- Beam Energy $\geq$ 60 GeV.
  (Nominal Design Value = 50 GeV.)

- Linac Emittance, $\epsilon = 3 \cdot 10^{-10}$ meters.

- Linac Momentum Spread $\delta_{max} = \frac{1}{200}$.
  (The distribution is assumed to be rectangular.)

- Required l^* to accommodate the particle detectors

<div align="center">

For the MarkII l^* = 2.8 meters

For the SLD l^* = 2.2 meters

</div>

- $\sigma^*_{x,y} = \sqrt{\beta^* \epsilon} \leq 2 \ \mu m$ round spot (including optical distortions).
- Assumed maximum disruption angle $\theta_D \leq 2$ mrad (for magnet aperture purposes).
- The maximum number of particles/beam $N = 5 \cdot 10^{10}$.
- Beam repetition rate = 180 pps (later reduced to 120 pps).

The task was to design a final focus system meeting the above specifications which was also compatible with physical constraints dictated by the SLAC site. To achieve these specifications, it was decided to adopt a "sextupole corrected" solution.

Several possible FFS systems were studied during the preliminary design studies.[4] Eventually the design converged to the basic optical system shown in the block diagram below. Details of the completed design have been reported at the 1987 Washington Accelerator Conference.[5]

<div align="center">9</div>

In the remainder of this report, we discuss the conceptual design considerations leading up to the adopted SLC design.

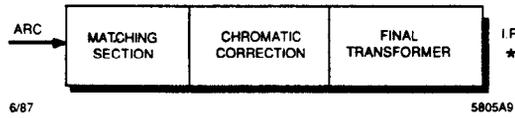## 7. Block Diagram of SLC Final Focus System

Fig. 4. A block diagram of the SLC final focus system.

- All segments of the block diagram are telescopic transformers.

- The final lens system is a symmetric quadrupole triplet.

- The chromatic correction section is a modified second-order achromat.[1] Sextupoles and dipoles are needed in this region to eliminate the second-order chromatic aberrations in the x and y transverse planes which originate in the final lens system nearest to the interaction region.

- The matching sections are needed to match the β and η functions at the exit of the SLC arcs.
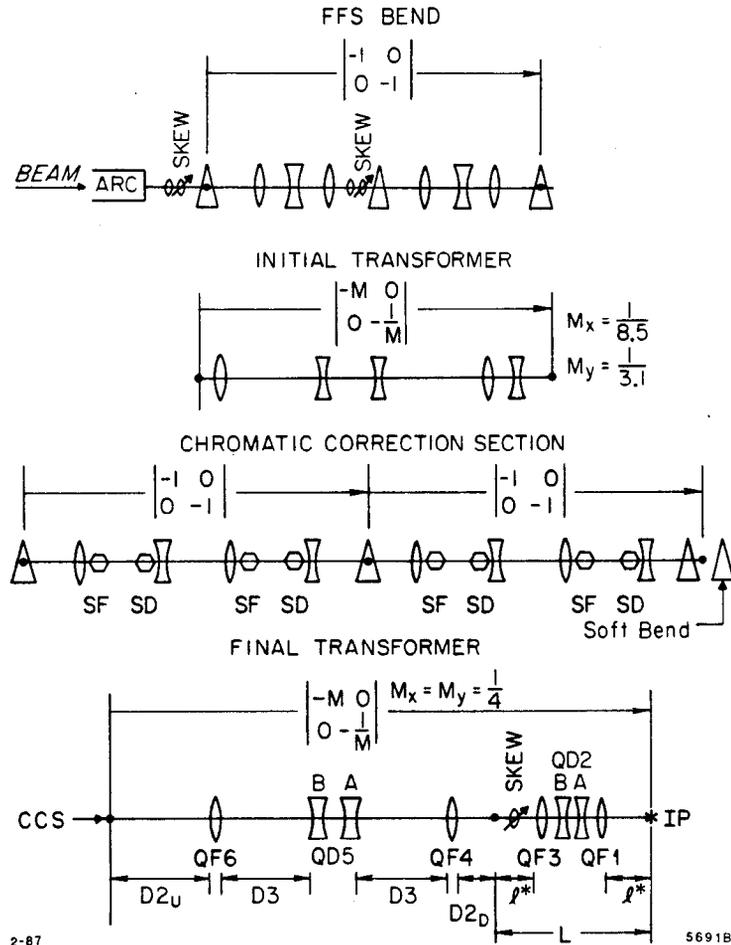

Fig. 5. Detailed schematic diagram of the SLC final focus system.

A schematic diagram of the optics is given in Fig. 5 showing the individual optical lenses required as taken from SLAC-PUB-4219. The functions of these individual subsystems are as follows:

THE FINAL FOCUS BEND (FFB)

- Removes dispersion which at the ends of the Arcs is given by $\eta_x = 0.0474$m and $\eta_y = \eta'_x = \eta'_y = 0$.

- Transfers the Arc beams (with $\beta_x = 8.70$ m, $\beta_y = 1.14$ m and $\alpha_{x,y} = 0$) to the next subsystem with a transfer matrix equal to the negative identity matrix.

- Provides a large, achromatic bend needed to satisfy global geometry constraints.

- Provides quad/skew-quad pairs separated in phase by $\sim \pi/2$ to remove the accumulated anomalous dispersion caused by perturbations in the Arcs.

THE INITIAL TRANSFORMER (IT)

- Initiates demagnification and provides a round beam, with $\beta_{x,y} = 0.12$ m and $\alpha_{x,y} = 0$, to the next subsystem.

- Provides trims on the two last quadrupoles to adjust $\beta^*_{x,y}$ at the I.P.

- Provides a region where the e^+, e^- path-length difference was adjusted.

THE CHROMATIC CORRECTION SYSTEM (CCS)

- Corrects the dominant second-order chromatic aberrations, T_{126} and T_{346}, at the I.P. without introducing other second-order geometric or chromatic aberrations.

- Provides s bend geometry to reduce synchrotron radiation (SR) background in the particle detector.

- Provides space for a low field *soft bend* magnet to further reduce SR background.

THE FINAL TRANSFORMER (FT)

- Completes the demagnification process. The magnification in this transformer is the same in the x and y planes, so that at the I.P. the ideal beam is round, with an optimum $\beta^*_{x,y} = 0.008$ m and $\alpha_{x,y} = 0$.

- Allows independent adjustment of the axial position of the beam waists to match the position of coincidence at the I.P.

- Provides a skew quad for suppression of cross-plane coupling between x, y' and x', y at the I.P.

8. Characteristics of Second-Order Aberrations

The following optics principles were used in the SLC design so as to avoid or minimize the accumulation of second-order aberrations.

- Separated-function quadrupoles do not introduce second-order geometric aberrations.

- Identical sextupole pairs separated by a minus unity (-I) transformation matrix can selectively couple to chromatic aberrations without introducing second-order geometric aberrations.

- Identical dipole pairs separated by a minus unity (-I) transformation matrix do not introduce second–order geometric aberrations.

The above principles are the basis of the second–order achromat[1] which can more generally be stated by the theorems listed below.

9. The Second–Order Achromat

THEOREM A

If one combines four or more identical cells consisting of dipole, quadrupole, and sextupole components with the parameters chosen so that the total first–order transfer matrix is unity (+I) in both transverse planes, then all geometric (on momentum) second–order aberrations will vanish.

THEOREM B

If the two sextupole families are adjusted so as to make one second–order chromatic aberration vanish in each transverse plane, then all second–order aberrations (geometric, chromatic, and path length) will vanish except for those path–length terms which depend only upon the momentum of the particles. The non–vanishing terms are R_{56} and T_{566}.

If both theorem A and theorem B are satisfied, then we call the resulting system a Second–Order Achromat. An optical lattice having these properties is shown in Fig. 6.
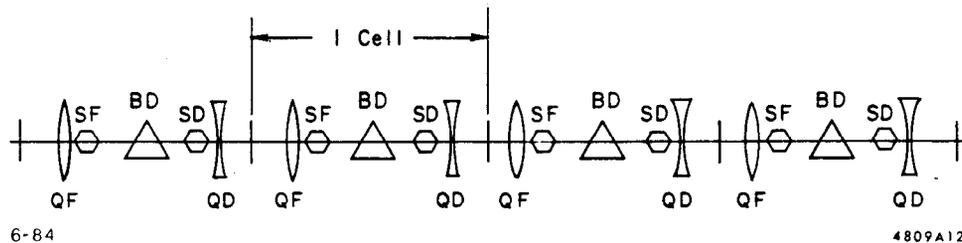

Fig. 6. Optical lattice of a typical four–cell second–order achromat.

The optical model adopted for the chromatic correction section of the SLC, as shown in Fig. 5, is a modified second–order achromat. The essential difference is that in the chromatic correction section the dipoles are positioned only in places of low η so as to minimize emittance growth caused by fluctuations in the synchrotron radiation energy loss.

In this modified achromat, the quadrupoles and sextupoles still satisfy repetitive symmetry such that all second–order geometric aberrations cancel. The dipoles are positioned so as to minimize emittance growth from synchrotron radiation quantum fluctuations and the two sextupole families are adjusted so as to cancel the T_{126} and T_{346} second order chromatic aberrations for the entire final focus system.

10. Residual Third–Order Aberrations

Having introduced the above symmetries into the design, all of the second–order aberrations will vanish or at least become sufficiently small that they are of no practical consequence. We are then left with the residual third–order geometric and chromatic aberrations as the limiting factors in the achievable luminosity.

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In TRANSPORT notation we find that the important residual third–order aberrations in the x plane are the following:

$$U_{1266} = (x|x'\delta^2), \quad U_{1446} = (x|y'^2\delta)$$

and

$$U_{1222} = (x|x'^3), \quad U_{1244} = (x|x'y'^2) \quad .$$

Similar residual aberrations occur in the y plane optics.

The design is optimized by observing that U_{1266} is the dominant residual aberration in the x plane. The other three aberrations can be shown to be functions of the magnitude of the dipole strengths used in the CCS. The stronger the dipole, the weaker these aberrations become. However this has to be balanced against the emittance growth caused by the dipoles. The design is adjusted so as to make the contribution from these three aberrations approximately equal to that from the U_{1266} term. The combination is then folded with the monoenergetic beam size so as to achieve the minimum $\bar{\beta}(\delta)$. An approximate equation has been derived representing this minimization process; the result is the following:

$$\bar{\beta}(\delta)_{min} \cong 2\sqrt{2}(R_{11}U_{1266})l^* \frac{\delta_{max}^2}{3} \tag{28}$$

where $\bar{\beta}(\delta)_{min} = 2\beta^*(0)$ is the average beta function at the interaction point. $(R_{11}U_{1266})$ is evaluated for the entire FFS measured from the input to the interaction point when $l^* = 1$, i.e., U_{1266} is proportional to l^*. Using TRANSPORT to evaluate U_{1266}, the net result for the final SLC design is

$$\bar{\beta}(\delta)_{min} \geq 100 \; l^* \delta_{max}^2 \tag{29}$$

where as before δ_{max} is the maximum $\frac{\delta p}{p}$ for a rectangular linac momentum distribution and l^* is the distance to the interaction point from the exit face of the last quadrupole in the symmetric triplet lens system nearest to the I.P. (measured in m).

Combining this result with Eq. (25) for the luminosity, we obtain

$$\mathcal{L}_{sex} \leq \left(\frac{N^2 f}{4\pi\epsilon}\right)\left(\frac{1}{l^*\delta_{max}^2}\right) \cdot 10^{-6} cm^{-2} sec^{-1} \quad . \tag{30}$$

These equations are approximate but seem to be valid for $\delta_{max} \geq \frac{1}{500}$. For δ_{max} less than $\frac{1}{500}$, the residual geometric aberrations begin to dominate and the assumptions break down, at least for the SLC design.

It is interesting to compare this equation for luminosity with that for the triplet alone. We find that

$$\frac{\mathcal{L}_{sex}}{\mathcal{L}_{triplet}} \approx \left(\frac{8}{\sqrt{3}}\right)\left(\frac{1}{100\delta_{max}}\right)$$

so for the SLC specification of $\delta_{max} = \frac{1}{200}$, the gain in luminosity is $\frac{16}{\sqrt{3}}$ as a result of using the sextupole corrected solution.

11. Summary

As a summary, a "cookbook" solution for designing SLC type systems is given below as a guide to the reader.

- Use telescopic transformers for the basic optics modules.

- Design the transformer nearest the interaction region so as to minimize the path lengths between the monoenergetic trajectories.

- Use a modified form of the second–order achromat for the design of the chromatic correction section.

- Choose the location of dipoles in the chromatic correction section so as to minimize emittance growth.

- Choose the dipole strength so as to minimize the overall effect of third–order aberrations and emittance growth.

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ERRATUM

A CONCEPTUAL DESIGN OF FINAL FOCUS SYSTEMS FOR LINEAR COLLIDER*

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