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A LOWER BOUND ON THE NEUTRON ELECTRIC DIPOLE MOMENT IN THE WEINBERG ANSATZ FOR CP VIOLATION*

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ABSTRACT

We deduce a lower limit on the neutron electric dipole moment in the Weinberg ansatz for CP violation. The resulting number is comparable to the existing experimental upper limit; this ansatz will therefore be critically tested by the next round of experiments using ultra-cold neutrons.

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I. INTRODUCTION: A non-vanishing electric dipole moment for the neutron, d_N, would represent the first direct observation of a microscopic violation of time reversal invariance. In the SU(3)×SU(2)×U(1) Standard Model one predicts however, very tiny values: $d_N < 10^{-30} e$ cm. Instanton effects could produce much larger numbers; yet once a Peccei-Quinn symmetry is invoked to obtain a natural solution to the strong CP problem one again finds $d_N < 10^{-30} e$ cm.

Much larger predictions for d_N are obtained when using the Weinberg ansatz [1] for CP violation as noted by several authors [2]: the estimates range typically from 10^{-24} to $10^{-25} e$ cm. Experimentally the following bounds have been obtained:

$$d_N = \begin{cases} (-2.0 \pm 1.0) \times 10^{-25} e\ cm & \text{Ref. 4} \\ (-1.8 \pm 2.9) \times 10^{-25} e\ cm & \text{Ref. 3}\ . \end{cases}$$

It is expected that the experimental sensitivity will reach the $10^{-26} e$ cm level in the near future. Motivated by these exciting prospects we have re-examined as carefully as possible the prediction on d_N as it is obtained in the Weinberg ansatz. Our treatment is very similar to that of Ref. 12; yet we have analyzed the long distance effects in considerably more detail than has been done before and have specifically included top quark contributions as well as QCD radiative corrections. Our results based on all these considerations are presented in Fig. 1. In short: d_N indeed cannot be significantly smaller than $10^{-25} e$ cm; for most of the allowed range in the model parameters, d_N actually exceeds $10^{-25} e$ cm substantially. The next round of measurements, therefore, has to reveal a non-vanishing value for d_N if the Weinberg mechanism represents the major source of CP violation.

II. CONSTRAINTS ON THE MODEL PARAMETERS: There are three doublets of Higgs fields in addition to the three families of quarks and leptons and the gauge bosons. CP violation occurs spontaneously; thus the Kobayashi–Maskava (= KM) matrix is orthogonal and all CP asymmetries can be traced back to complex vacuum expectation values of Higgs fields that enter the Yukawa couplings after a redefinition of the Higgs fields:

$$\mathcal{L}_Y = \frac{g}{\sqrt{2}M_W} \sum_{i=1}^{3} \left(\alpha_i \bar{U}_R M_U K D_L + \beta_i \bar{U}_L M_D K D_R\right) H_i^+ + \text{ h.c. }, \tag{1}$$

with H_i denoting the (charged) Higgs fields and U and D the three families of quarks with diagonal mass matrixes M_U and M_D respectively: $U = (u, c, t)$; $D = (d, s, b)$; K is the KM matrix.

It is the relative phase between α and β that drives CP asymmetries; the range of allowed values for $Im\ \alpha^* \beta$ is derived from data on ϵ and ϵ'. One finds the general expressions [5]

$$\epsilon = \frac{1-D}{2\sqrt{2}}\, e^{i\pi/4} \left\{\epsilon_m + 2\xi + \frac{D}{1-D}\chi\right\}, \tag{2}$$

$$\frac{\epsilon'}{\epsilon} = -e^{i(\pi/4+\delta_2-\delta_o)} \ \frac{2\xi}{20(1-D)\left[\epsilon_m + 2\xi + \frac{D\chi}{1-D}\right]}, \tag{3}$$

with the following notation

$$\begin{gathered} M_{12} = (M_{12})_{SD} + (M_{12})_{LD} \equiv (M_{12})_{SD} + DM_{12} \\ \epsilon_m = \frac{Im(M_{12})_{SD}}{Re(M_{12})_{SD}}; \quad \xi = \frac{Im\ \langle 2\pi\,, I = O|H|K^o\rangle}{Re\langle 2\pi\,,\ I = O|H|K^o\rangle} \\ Im(M_{12})_{LD} = (-2\xi + \chi) Re(M_{12})_{LD}\ . \end{gathered} \tag{4}$$

$SD[LD]$ stands for short [long] distance dynamics.

In the Weinberg ansatz it was found [6]that

$$\epsilon_m \ll 2\xi\ . \tag{5}$$

This leads to $\epsilon'/\epsilon \sim -0.05$ – a number clearly inconsistent with experimental bounds – unless chiral symmetry introduces a sufficiently strong suppression to yield [7,8]

$$|2\xi| \ll \left|\frac{D\chi}{1-D}\right| . \tag{6}$$

We assume this to happen – otherwise the model is already ruled out – and therefore read off [15]

$$|\epsilon| \approx \frac{1}{2\sqrt{2}}\, D\chi . \tag{7}$$

In this model ϵ is thus produced mainly by long distance dynamics; therefore they have to be studied very carefully.

$SU(3)$ symmetry together with current algebra implies the phases of $< 2\pi, I = O|H|K^\circ >$, $< \pi^\circ|H|K^\circ >$ and $< \eta_8|H|K^\circ >$ to be equal; in the Wu-Yang phase convention the $2\pi, \pi^\circ$ and η_8 contributions to $(M_{12})_{LD}$ are therefore purely real. $Im\ (M_{12})_{LD}$ must then be produced mainly by the $K^\circ \to \eta_\circ \to \bar{K}^\circ$ transition, $\eta_\circ$ being the $SU(3)$ singlet component in the nonet of pseudoscalar mesons:

$$\begin{aligned}\epsilon = \frac{Im\langle K^o|H|\eta_o\rangle\langle\eta_o|H|\bar{K}^o\rangle}{2\sqrt{2}\ Re\ M_{12}} &= \frac{8}{9}\,\rho\xi_o\frac{|\langle K^o|H|\pi_o\rangle|^2}{\sqrt{2}\Delta M^2} \times \\ \times \sum_{P=\eta,\eta'} &\frac{(1-4\rho)X_P^2 - (1+2\rho)Y_P^2 - \frac{1}{\sqrt{2}}(1+8\rho)X_P Y_P}{m_K^2 - m_P^2} .\end{aligned} \tag{8}$$

where[9]

$$\frac{\langle K^o|H|\eta_o\rangle}{\langle K^o|H|\eta_8\rangle} = -2\sqrt{2}\rho(1-i\xi_o),\ \Delta M^2 = 2m_K\Delta m_K .$$

We have used as representation for the pseudoscalar wavefunctions [18]:

$$|P\rangle = X_p|\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\rangle + Y_p|s\bar{s}\rangle + Z_p|G\rangle,\quad P = \eta,\eta' . \tag{9}$$

$|G>$ denotes an additional $SU(3)$ singlet component like a glueball.

Equation (8) contains three types of parameters – $\{X_p, Y_p\}$; ρ; ξ_o– which will be discussed in turn.

(i) A comprehensive analysis of decays involving η and η' in the initial or final state leads to the conservative bounds[13]

$$
\begin{aligned}
&0.6 \leq X_\eta \leq 0.85, \quad 0.55 \leq -Y_\eta \leq 0.95, \\
&0.3 \leq X_{\eta'} \leq 0.6, \quad 0.55 \leq Y_{\eta'} \leq 0.85 \ .
\end{aligned}
\tag{10}
$$

A very recent re-analysis of MARK III data yields [11]

$$
\begin{aligned}
&X_\eta = 0.81 \pm 0.04, \quad Y_\eta = -0.58 \pm 0.04, \\
&X_{\eta'} = 0.58 \pm 0.04, \quad Y_{\eta'} = -0.81 \pm 0.04 \ .
\end{aligned}
\tag{11}
$$

These numbers are quite consistent with an $\eta - \eta'$ mixing angle $\theta = -19 \ \pm \ 2^\circ$ as predicted by a $1/N$ treatment of QCD[16]. However, there is still room for a sizeable glueball component in the η' wavefunction.

(ii) In the next step one obtains the parameter ρ by solving

$$
\begin{aligned}
A(K_L \to \gamma\gamma) = &\langle K_L | H | \pi^o \rangle A(\pi^o \to \gamma\gamma) \times \\
&\times \left\{ \frac{1}{m_K^2 - m_\pi^2} + \frac{1}{3} \sum_{P=\eta,\eta'} \frac{A(P \to \gamma\gamma)}{A(\pi^o \to \gamma\gamma)} \right. \\
&\left. \frac{1}{m_K^2 - m_P^2} \left(X_P - \sqrt{2}\, Y_P - 2\sqrt{2}\ \rho(\sqrt{2}\ X_P + Y_P) \right) \right\} .
\end{aligned}
\tag{12}
$$

The relative sign of the amplitudes $A(\eta \to \gamma\gamma)$ and $A(\eta' \to \gamma\gamma)$ is taken to be positive as predicted by the quark model. The quark model actually predicts $\rho = 1$. We have found this to hold to within a factor of two for the set of parameters that give the lower limit on d_N.

(iii) The remaining task consists of computing ξ_o in the Weinberg ansatz. One-loop diagrams involving Higgs exchange yield the CP odd transition operator[6,10]

$$\mathcal{L}_- = if\bar{d}\sigma^{\mu\nu}(1-\gamma_5)t^A sF^A_{\mu\nu} + \text{ h.c. }, \tag{13}$$

where $F^{\;A}_{\mu\nu}$ denotes the gluon field strength tensor and

$$\begin{aligned} f = &\frac{G_F}{\sqrt{2}}\,\frac{g_s}{32\pi^2}\,m_s m_c^2\,\frac{\alpha^*\beta}{m_H^2}\,\times \\ &\times\left[\eta_c K_{cs}K^*_{cd}\;G\left(\frac{m_c^2}{m_H^2}\right) + \frac{m_t^2}{m_C^2}\eta_t K_{ts}K^*_{td}G\left(\frac{m_t^2}{m_H^2}\right)\right], \end{aligned} \tag{14}$$

where $G(x) = -\left(\frac{1}{2} + \frac{1}{1-x} + \frac{1}{(1-x)^2}\;\log x\right)$; η_c and η_t denote QCD radiative corrections; a leading log treatment yields

$$\eta_Q \simeq \left[\frac{\alpha_s(m_Q^2)}{\alpha_s(\mu^2)}\right]^{\frac{-1}{6b}}\left[\frac{\alpha_s(m_Q^2)}{\alpha_s(\mu_H^2)}\right]^{\frac{8}{b}}, \quad Q = c,\ t;\ \ b = 11 - \frac{2}{3}\,n_F\ , \tag{15}$$

where μ denotes the normalization or infrared cut-off scale. Forming the matrix element will in principle lead to a compensating dependence on μ; in practice however, an uncertainty is thus introduced since the models used to evaluate the matrixelements do not exhibit the μ dependence explicitly. In this case the μ dependence is extremely mild due to the tiny exponent $1/6b$ and we use: $\eta_c \sim 3.2,\ \eta_t \sim 1.2$ for $m_t \sim 40\ GeV,\ M_H \sim 100-500\ GeV.$

Then one has

$$\langle K^o|\mathcal{L}_-|\eta_o\rangle \simeq -2\sqrt{\frac{2}{3}}\,\rho\langle K^o|\mathcal{L}_-|\pi^o\rangle = -2\sqrt{\frac{2}{3}}\,\rho f^* A_{K\pi}\ , \tag{16}$$

and therefore

$$\xi_o = \frac{Im f A_{K\pi}}{\langle K^o|H|\pi^o\rangle}\ . \tag{17}$$

Inserting (17) into (8) and solving for Imf we find

$$
\begin{aligned}
Imf &= \frac{9}{8\rho}\,\epsilon\frac{\sqrt{2}\,\Delta M^2}{A_{K\pi}|\langle K^o|H|\pi^o\rangle|}\,\frac{1}{F}\,, \\
F &= \sum_p \frac{(1-4\rho)X_p^2-(1+2\rho)Y_p^2-\frac{1}{\sqrt{2}}(1+8\rho)X_pY_p}{m_K^2-m_P^2}\,.
\end{aligned}
\tag{18}
$$

Equation (18) together with (14) allows us, finally, to determine $Im\alpha^*\beta$ for given values of M_H, M_t; for $A_{K\pi}$ we use the bag model result[10] $A_{K\pi}=0.4\ (\mathrm{GeV})^3$.

III. PREDICTION ON THE NEUTRON ELECTRIC DIPOLE MOMENT: In the non-relativistic approximation d_N is simply expressed in terms of d_d and d_u, the electric dipole moments of down and up quarks:

$$
d_N = \frac{1}{3}\,(4d_d - d_u)\ . \tag{19}
$$

The one-loop diagrams lead to (since $d_u \ll d_d$)

$$
\begin{aligned}
d_N &= \frac{2\sqrt{2}G_F e}{18\pi^2}\,\frac{m_c^2 m_d}{m_H^2}\,Im(\alpha\beta^*)\,\times \\
&\times\left[\bar\eta_c|K_{dc}|^2\,g\left(\frac{m_c^2}{m_H^2}\right)+\bar\eta_t\frac{m_t^2}{m_c^2}\,|K_{td}|^2\,g\left(\frac{m_t^2}{m_H^2}\right)\right],
\end{aligned}
\tag{20}
$$

with $g(x)=\frac{1}{(1-x)^2}\left[\frac{5}{4}\,x-\ \frac{1-\frac{3}{2}\,x}{1-x}\ \log x-\frac{3}{4}\right]$; $\bar\eta_c$ and $\bar\eta_t$ are the radiative QCD corrections. In the leading log approximation one finds

$$
\bar\eta_Q \simeq \left[\frac{\alpha_s(m_Q^2)}{\alpha_s(\mu^2)}\right]^{\frac{4}{35}}\left[\frac{\alpha_s(m_Q^2)}{\alpha_s(m_H^2)}\right]^{\frac{8}{5}}, \tag{21}
$$

and we therefore use $\bar\eta_c \sim 2.5$–3, $\bar\eta_t \sim 0.7-0.9$ in the same spirit as expressed after Eq. (15).

It turns out that the minimum of d_N is obtained by minimizing the t quark contribution. This occurs when the inequalities

$$m_t \geq 23 \text{ GeV}, \ \ K_{td} \geq 0.001 \ , \tag{22}$$

are saturated. The former follows from PETRA data, the latter from the unitarity of the KM matrix (assuming there are only three families).

The resulting lower bound for d_n has only a weak dependence on M_H: the variation is at most 20% in the range 23 GeV $\leq M_H \leq$ 500 GeV. In our evaluation we have set $M_H = 500$ GeV.

IV. UNCERTAINTIES: In many computations of this type, one encounters large cancellations between π° and η, η' contributions, which amplify uncertainties introduced by, for example, $SU(3)$ breaking and chiral symmetry breaking. In Eq. 8, we are spared from this possibility since only η and η' contribute. In Eq. 12, we have taken the symmetry breaking effect into account by introducing

$$\frac{\langle K^o|H|\eta_o\rangle}{\langle K^o|H|\pi^o\rangle} = \frac{1}{\sqrt{3}} \, (1 + .17) \ ,$$

which was computed in Ref. 17. In principle, the same correction factor should be incorporated in Eqs. 16 and 18. Here, the uncertainty comes in as an overall multiplicative correction and can be treated together with the uncertainty in $A_{K\pi}$. Therefore, we are confident that a *further* reduction of the lower bound by a factor of three reflects these uncertainties sufficiently. This has been done in Fig. 1 which shows our findings.

V. SUMMARY: As stated in the beginning the experimental sensitivity for d_N is expected to reach the $10^{-26}e$ cm level soon. These measurements will have to reveal a non-vanishing value for d_N if the Weinberg ansatz describes the major source of CP violation. Otherwise this model would clearly be ruled out as a significant contributor to ϵ. Two further notes in passing:

(*i*) The $\bar{\theta}$ parameter is calculable in this model. It vanishes naturally on the tree level; yet on the one-loop level one finds[14] $\bar{\theta}(1-\text{loop}) \sim O(10^{-3})$ which is much too large thereby creating a pronounced need for a Peccei-Quinn symmetry.

(*ii*) The presence of scalar couplings produces a transverse polarization of muons in $K^+ \to \mu^+ \nu \pi$ decays. Yet we find $\text{Pol}(\mu) \sim O(10^{-4})$. It appears hopeless to observe such a tiny effect however important it would be.

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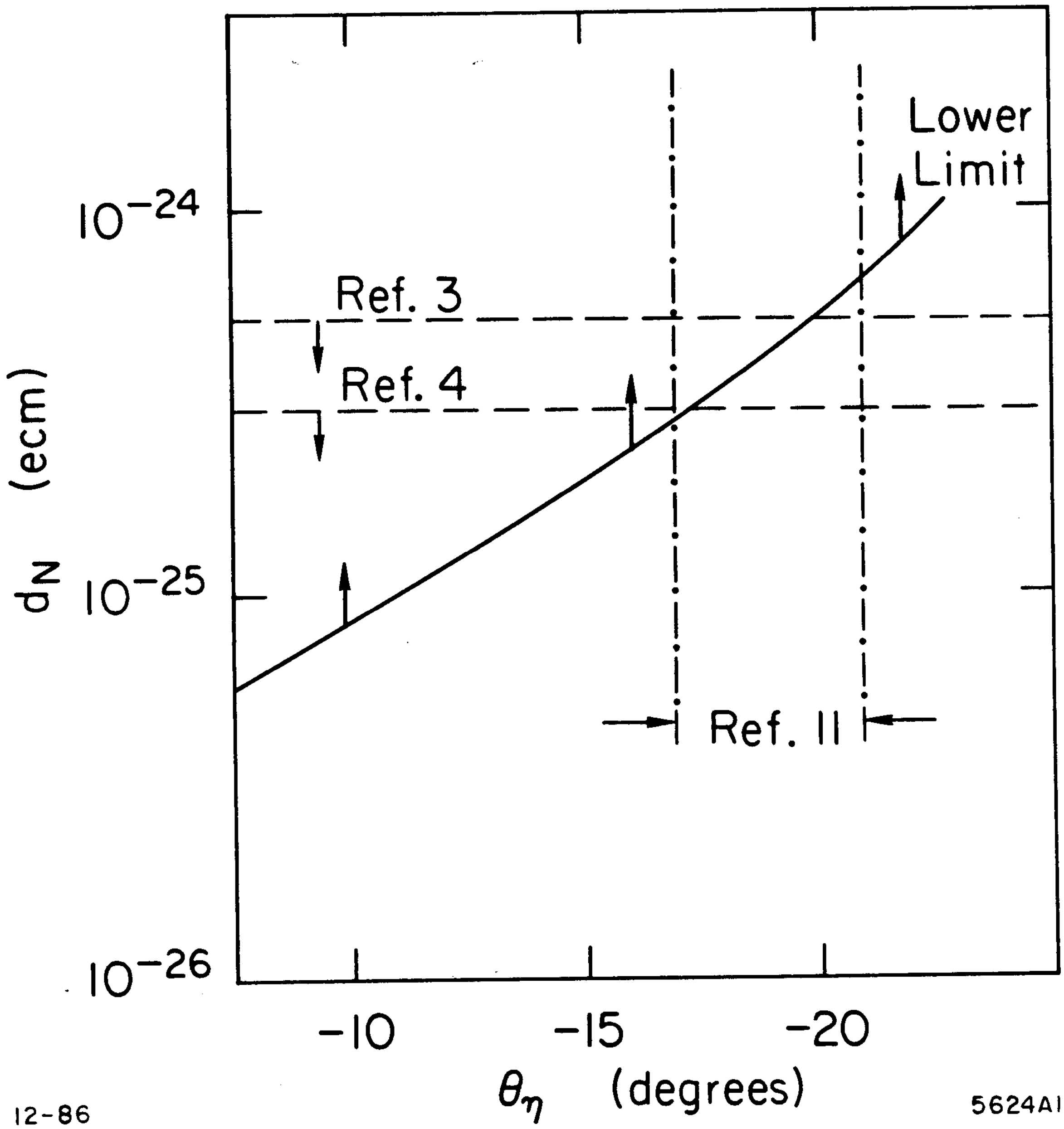
FIGURE CAPTION

Fig. 1. Lower limit on the neutron electric dipole moment.



Lower
Limit
Ref. 3
Ref. 4
Ref. 11
10^{-24}
10^{-25}
10^{-26}
d_N (ecm)
-10
-15
-20
θ_η (degrees)
12-86
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Fig. 1