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CHARGE ASYMMETRY OF THE NUCLEAR INTERACTION AND NEUTRON–NEUTRON SCATTERING PARAMETERS*

G. F. DE TÉRAMOND†

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

and

Escuela de Física, Universidad de Costa Rica, San José, Costa Rica

and

B. GABIOUD

Institut de Physique Nucléaire, Université de Lausanne, Lausanne, Switzerland

ABSTRACT

Analyses of bound systems and low-energy scattering experiments give clear evidence of a departure from charge symmetry in the nuclear interaction. Furthermore, this effect could be enhanced in a nuclear medium thereby solving the Nolen–Schiffer anomaly observed in mirror nuclei. Since the extraction of the n-n scattering parameters is tied to the theoretical analysis of particular nuclear reactions, we have examined some theoretical treatments and approximations used in the study of the reaction $\pi^- d \to \gamma\, nn$ which is ideal for this purpose. In particular, we consider the neutron-neutron enhancement in higher n-n partial waves by introducing a simplified approach to the eigenamplitude and multipole expansions. We also give a description of the methods used in the calculation of the final state interactions in the $^1S_0$ n-n state to determine the low-energy n-n scattering parameters.

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1. INTRODUCTION

The introduction of isospin by Heisenberg[1] to describe the two charge states of the nucleon, proton and neutron, has proven to be a far-reaching contribution to the understanding of the structure of matter, and is still a problem of importance in nuclear and particle physics.[2] Isospin is the first internal symmetry that has been introduced which acts on the particle identity independent of space-time. The extension of the SU(2) symmetry to higher internal symmetries SU(3), SU(4), ... is extremely successful in the classification of the hadron spectrum and led to the recognition of more fundamental structures, quarks, from which all the hadrons are built.[3] Furthermore, the requirement of local gauge invariance under SU(2) rotations[4] played a central role in building the prototype of the modern renormalizable gauge theories which describe the fundamental interactions among the basic constituents of matter.

Isospin is broken by the electromagnetic and weak interactions which are flavor-dependent, i.e., they distinguish among the various types of quarks (u, d, s, c, ...). Yet, another source of isospin violation is the quark mass difference $m_u - m_d$. In the limit $m_u \to 0$ and $m_d \to 0$, the basic Lagrangian of quantum chromodynamics[5] (QCD), the candidate theory of the strong interactions, is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ invariant or, invariant under the isospin $\mathrm{SU}(2)_{L+R} \sim \mathrm{SU}(2)_V$ group if $m_u = m_d$. Although the origin of the masses of quarks and leptons is a deep unsolved problem, it is generally believed that the different masses are generated by the Higgs mechanism from different Yukawa couplings in the electroweak sector of the theory. In the standard $\mathrm{SU}(2) \times \mathrm{U}(1)$ model[6] there are no zeroth-order relations among quark masses, which are thus introduced as free parameters. Isospin is not a natural symmetry in this model and, unless additional symmetries[7] are introduced in the theory, m_u and m_d are unrelated. The observed conservation of isospin to within a few percent would reflect the smallness of the u- and d-quark masses with respect to the hadronic scale (accidental symmetry) rather than the degeneracy of m_u and m_d (exact SU(2) symmetry). Moreover, it is not likely that the isospin breaking effects would be generated radiatively within the framework of a broader scheme for the particle interactions.[8]

Early evidence of charge asymmetry in the nuclear interaction came from the precise measurement of binding energies in mirror nuclei,[9] which suggested a slightly stronger short-range n-n attraction to account for the observed differences.[10] The construction of $^1S_0$ charge asymmetric potentials from $\rho - \omega$ and $\pi^0 - \eta$ mixings[11,12] has been successful in describing the charge symmetry breaking, and accounts for a significant part of the observed effects[13,14] in the $^3\mathrm{H} - {}^3\mathrm{He}$ mirror nuclei. At the hadronic level, the particle mixing parameters can be obtained from experiment in a model-independent way.[15] The particle mixing is parametrized in terms of a tadpole which depends linearly on the quark mass difference. Consequently, any charge asymmetric effect should vanish in the isospin limit $e \to 0$, $m_u = m_d$.

The effect of the isospin corrections at the quark level has been examined by Chemtob and Yang[16] and by Hwang[17] using the resonating group approach in a quark cluster model. It is found that the isospin-violating contributions are enhanced in the quark description, relative to a mesonic description, due to the short-distance quark mass effects.[16] Furthermore, since the effect of quark exchange is sensitive to the nucleon radius, one could expect an enhancement of the charge symmetry breaking effects[17] in a nuclear medium due a change of size of the nucleon. This could resolve[17] the anomaly observed in the binding energy of mirror nuclei, the "Nolen–Schiffer" anomaly.[9]

The situation concerning the determination of charge-symmetry breakdown in low-energy N–N scattering experiments has been rather confusing. Since the properties of the neutron-neutron (n-n) interaction cannot be inferred from direct collision experiments, the extraction of the n-n scattering parameters is tied to the analysis of final-state interactions (FSI) where two or more nucleons are present in the final state. In the presence of three hadrons in the final state the theoretical calculation, which is generally based on the use of the Fadeev equations, becomes quite intrincate and the theoretical uncertainties difficult to evaluate. To avoid large errors in the extraction of the scattering parameters, the analysis is usually restricted to a limited kinematical region where n-n quasi-free scattering dominates. The often quoted value of the n-n $^1S_0$ scattering length $a_{nn} = -16.6 \pm 0.6$ fm [18] based on a straight average of a large number of measurements, is smaller in absolute value than the Coulomb corrected value of a_{pp} determined from low-energy p-p scattering: $a_{pp} = -17.1 \pm 0.2$ fm [19] thus suggesting a stronger p-p force.[20] The above value for a_{nn} is heavily weighted by experiments with three hadrons in the final state and does not include a theoretical uncertainty.[21] On the other hand, the utilization of the capture reaction

$$\pi^- d \rightarrow \gamma\, nn \tag{1.1}$$

to study the properties of the n-n interaction is of special interest, since the three particles in the final state are detectable and only the two neutrons interact strongly. Hence, the extraction of the low-energy n-n scattering parameters is free from the theoretical uncertainties inherent to other nuclear reactions and the study is not restricted to a particular kinematical region, thus making it possible to obtain also the effective range parameter r_{nn}.

A recent high-precision determination of a_{nn} from $\pi^- d \rightarrow \gamma\, nn$ [22,23] using a high statistics photon spectrum gives the value $a_{nn} = -18.5 \pm 0.4$ fm [24,25] consistent with the charge asymmetry in the $^3\mathrm{H} - ^3\mathrm{He}$ nuclei, but in contradiction with the results obtained from the study of nuclear reactions with three nucleons in the final state.

How to reconcile those contradicting results which differ by almost three standard deviations? A discrepancy which is clearly beyond experimental error. To find a way out of this dilemna, Šlaus, Akaishi and Takaka[26] suggest an ingenious mechanism to explain the difference between the radiative pion capture results and the results from the neutron-induced deuteron breakup reaction $nd \rightarrow pnn$, which in turn differ according to the distinct kinematical region studied: $a_{nn}(nd \rightarrow pnn,$ knock out$) = -20.7 \pm 2.0$ fm and $a_{nn}(nd \rightarrow pnn,$ pick-up$) = -16.7 \pm 0.5$ fm. The mechanism is based on a specific model for the three-body forces and operates differently for neutron pickup and proton knockon processes, hence removing the apparent discrepencies. It has also been pointed out[27] that a detailed study of the extended electromagnetic structure of the pnn state in the deuteron breakup reaction is relevant to the comparison with the scattering parameters obtained from (1.1).

Coon and Scadron have examined the charge asymmetric and charge dependent effects in the N–N interaction including two pion exchange contributions.[28] The calculated effect on the charge asymmetry in the scatttering length $| a_{nn} | - | a_{pp} | = 1.2$ fm from $\rho - \omega, \pi - \eta - \eta'$, 2π and $\gamma\pi^0$ boson exchange, is in good agreement with the experimental value of a_{nn} obtained from (1.1) and the $^3H-^3He$ mass difference. The short distance quark mass effects calculated in Refs. 16 and 17 gives a contribution of the same sign and even still larger. However, a recent calculation including QCD charge dependent corrections[29] finds a small effect of the quark mass difference on the asymmetry of the low-energy scattering parameters, due to a partial cancellation of the quark kinetic energy and the color magnetic interaction.

In view of the new developments discussed above, spurred by the interest in understanding the properties of the nuclear interaction in terms of the basic degrees of freedom, we will reexamine some of the theoretical calculations and assumptions used to analyze the energy spectra of reaction (1.1). This is of importance to make a precise determination of the n-n scattering parameters from the strong enhancement in the energy spectrum of (1.1) due to the n-n interaction in the final state, as originally proposed by Watson and Stuart[30] to test the charge symmetry of the nuclear forces.

We should mention here that subsequent theoretical developments of the reaction (1.1)[31–36] have shown that the approximations used in Ref. 30 are indeed remarkably good, and the extraction of a_{nn} using the different theories gives essentially the same results within 0.4 fm. We can understand this result as follows: the value of a_{nn} depends mainly on the asymptotic behavior of the n-n wave function, and is largely independent of the short-range description of the nuclear force. The scattering length is determined at low energies of the n-n system, where the normalized spectrum is insensitive to the secondary effects calculated here and elsewhere. For this reason, we should consider the recent measurement of a_{nn} from (1.1)[22,23] as a serious indication of a departure from charge symmetry.

Indeed, a completely different experimental technique has been used recently[37] to detect the neutrons in coincidence with the photon. The analysis of the time-of-flight neutron spectrum is much less dependent on theoretical assumptions, since the value of the scattering length is determined in this experiment from the low-energy n-n spectrum where the theoretical uncertainty is negligible. The neutron spectrum gives $a_{nn} = -18.7 \pm 0.6$ fm.[37] This new determination of a_{nn} rules out a possible systematic error in the previous experiment[22,23] based on the photon spectrum analysis.

In this paper we discuss the methods utilized previously[23] in the description of the FSI in the $^1S_0$ n-n scattering state, and develop a convenient formalism to include the FSI in the higher partial waves. The latter calculation has not been carried out in detail before due to the rather complex spin structure present in this reaction, which is manifest in the large number of independent invariant amplitudes.

Let us resume briefly the contents of this article. After discussing the eigen-amplitude and multipole expansion in Section 2, we describe the FSI in the $^1S_0$ channel in Section 3 and in higher waves in Section 4. The discussion of the results and some concluding remarks are given in Section 5. Some useful formulas are given in the Appendix.

2. MULTIPOLE EXPANSION OF EIGENAMPLITUDES

Since the neutron-neutron interaction acts differently for each transition from a given orbital momentum, the scattering amplitude has to be projected in partial waves by a multipole expansion to describe the n-n FSI in each scattering state of (1.1). In practice, this is a difficult task due to the presence of three particles in the final state and the rich spin structure present in this reaction. Each multipole has to be coupled with the spin of the deuteron and expanded into singlet or triplet n-n amplitudes of given total angular momentum. To avoid unnecessary complications in our discussion we shall follow here a rather simple approach in constructing the eigenamplitudes, by decomposing the orbital angular momentum states into their tensor representation. The nucleon spin is taken into account by contracting the spin variables with the components of the orbital angular momentum tensor, according to the transformation properties corresponding to the total angular momentum of the n-n system. The multipole amplitudes are eigenvalues of angular momentum and parity rather than linear momentum.

We shall follow in this paper the notation and conventions of Refs. 35 and 36, henceforth referred to as I and II, respectively. The electric and magnetic multipole transitions are denoted by $E\lambda({}^{2S+1}L_J)$ and $M\lambda({}^{2S+1}L_J)$, where λ is the total angular momentum of the photon, $\lambda \geq 1$, and J, L and S are the total angular momentum, the orbital momentum and the total spin of the two neutrons in the final state. The dipole, quadrupole and octopole states for the allowed transitions from the S state of the pionic-deuterium atom are listed for reference in Table 1.

A state of orbital angular momentum L will be described by an irreducible tensor representation of the rotation group, with components $T_{i_1,i_2,\ldots,i_L}$ symmetric and traceless in each pair of indices

$$
\begin{aligned}
L=0, &\qquad T = 1 \quad , \\
L=1, &\qquad T_i = \hat{p}_i \quad , \\
L=2, &\qquad T_{ij} = \frac{1}{2}(3\hat{p}_i\hat{p}_j - \delta_{ij}) \quad , \\
L=3, &\qquad T_{ijk} = \frac{1}{2}(5\hat{p}_i\hat{p}_j\hat{p}_k - \hat{p}_i\delta_{jk} - \hat{p}_j\delta_{ik} - \hat{p}_k\delta_{ij}) \quad , \\
L=4, &\qquad T_{ijk\ell} = \frac{1}{8}(35\hat{p}_i\hat{p}_j\hat{p}_k\hat{p}_\ell - 5(\hat{p}_i\hat{p}_j\delta_{k\ell} + \hat{p}_i\hat{p}_k\delta_{j\ell} \\
&\qquad\qquad + \hat{p}_i\hat{p}_\ell\delta_{jk} + \hat{p}_j\hat{p}_k\delta_{i\ell} + \hat{p}_j\hat{p}_\ell\delta_{ik} + \hat{p}_k\hat{p}_\ell\delta_{ij}) \\
&\qquad\qquad + \delta_{ij}\delta_{k\ell} + \delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk}) \quad \ldots \quad ,
\end{aligned}
\tag{2.1}
$$

where $\vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$ is the relative n-n momentum. The tensor components of the representation are normalized in each state L according to

$$
P_L(x) = T_{i_1 i_2 \cdots i_L}\hat{n}_{i_1}\hat{n}_{i_2}\ldots\hat{n}_{i_L} \quad , \quad x = \hat{p}\cdot\hat{n} \quad , \tag{2.2}
$$

where $\hat{n}$ is a unit vector along some arbitrary direction, and $P_L(x)$ is a Legendre polynomial.

We have indicated in Table 2 the tensor decomposition of total angular momentum eigenamplitudes of the n-n system for the lower transitions. The singlet states are invariant under rotations, whereas the triplet states transform as a vector and are thus linear in the spin variable σ_i. The coupling of the orbital eigentensor T with the spin functions is determined by the total angular momentum J of a given n-n state. For example, the $^3P_0$ state with $J = 0$, is described by the scalar product of $\vec{\sigma}$ and $\vec{p}$ which is invariant under rotations. The $^3P_1$

state with $J = 1$, is obtained from the vector product of $\vec{\sigma}$ and $\vec{p}$, and the ${}^3P_2$ state corresponding to $J = 2$ transforms as a traceless symmetric tensor. The ${}^3F_2$ state is obtained by contracting the spin component σ_k with the tensor T_{ijk}, which represents the orbital state F. Finally, the amplitude components of Table 2 are contracted with the deuteron polarization vector $\vec{\eta}$ and with a tensor representing a given electric or magnetic multipole transition, with components written in terms of the photon direction $\hat{k}$ and the photon polarization $\vec{\epsilon}$:

$$
\begin{aligned}
&E1 \quad \epsilon_i = E_i \quad , \\
&M1 \quad (\hat{k} \times \vec{\epsilon})_i = M_i \quad , \\
&E2 \quad \epsilon_i \hat{k}_j + \epsilon_j \hat{k}_i = E_{ij} \quad , \\
&M2 \quad (\hat{k} \times \vec{\epsilon})_i \hat{k}_j + (\hat{k} \times \vec{\epsilon})_j k_i = M_{ij} \quad \ldots \quad ,
\end{aligned}
\tag{2.3}
$$

We shall neglect correction terms which are smaller by a factor p^2/m^2, k^2/m^2 and kp/m, which are of the same order of magnitude as the relativistic contributions. To this approximation, we shall keep the electric dipole amplitudes $E1({}^1S_0)$, $E1({}^1D_2)$, the magnetic dipole $M1({}^3P_0)$, $M1({}^3P_1)$, $M1({}^3P_2)$, the electric quadrupole $E2({}^3P_1)$ and $E2({}^3P_2)$, and neglect the higher multipole amplitudes given in Table 1. For completeness, we also include the magnetic dipole amplitude $M1({}^3F_2)$, although its contribution is very small.

In terms of the 2×2 matrices given in Table 2 and the multipole components (2.3), the electric and magnetic dipole and electric quadrupole transition amplitudes are given by

$$
\Im_{E1} = \chi^+ \left\{ E1({}^1S_o)\vec{\epsilon} \cdot \vec{\eta} + E1({}^1D_2)\epsilon_i \eta_j T_{ij} \right\} \chi^c \quad , \tag{2.4}
$$

$$
\begin{aligned}
\Im_{M1} = \chi^{+} \Big\{ & M1(^3P_0) i(\vec{\sigma}\cdot\hat{p})\vec{\eta}\cdot(\hat{k}\times\vec{\epsilon}\,) \\
& + M1(^3P_1) i(\vec{\sigma}\times\hat{p})\cdot\left[\vec{\eta}\times(\hat{k}\times\vec{\epsilon}\,)\right] \\
& + M1(^3P_2) i\epsilon_{ijk} U_{km}\eta_m \hat{k}_i \epsilon_j \\
& + M1(^3F_2) i\epsilon_{ijk} V_{km}\eta_m \hat{k}_i \epsilon_j \Big\} \chi^c \quad ,
\end{aligned}
\tag{2.5}
$$

$$
\begin{aligned}
\Im_{E2} = \chi^{+} \{ & E2(^3P_1) i\epsilon_{ijk}\sigma_j \hat{p}_k E_{im}\eta_m \\
& + E2(^3P_2) i\epsilon_{ijk} U_{jm} E_{km}\eta_i \} \chi^c \quad ,
\end{aligned}
\tag{2.6}
$$

with $\chi^c = i\sigma_2\chi^*$. The transition matrix can also be expanded in terms of 12 rotational invariant independent forms $\lambda^{(i)}, i = 1,\ldots,12$, constructed from the indepedent vectors and the Pauli spin matrices as follows[35]

$$
\Im = \chi^{+} \sum_{i=1}^{12} \lambda^{(i)} f_i(p^2, z) \chi^c \quad , \tag{2.7}
$$

where $z = \hat{p}\cdot\hat{k}$, and the scalar amplitudes f_i are functions of the kinematic variables. The $\lambda^{(i)}$ are listed in Table 1 of I. The number of invariants correspond to the number of independent helicity amplitudes for the capture of the pion from rest. We can express the scalar amplitudes f_i in terms of the multipole amplitudes as follows:

$$
f_1 = E1(^1S_o) - \frac{1}{2}E1(^1D_2) \quad , \tag{2.8a}
$$

$$
f_2 = M1(^3P_o) - M1(^3P_2) - \frac{1}{2}M1(^3F_2) - 3E2(^3P_2) \quad , \tag{2.8b}
$$

$$
f_3 = M1(^3P_1) + \frac{3}{2}M1(^3P_2) - \frac{1}{2}M1(^3F_2) - E2(^3P_1) - \frac{9}{2}E2(^3P_2) \ , \tag{2.8c}
$$

$$
f_4 = -M1(^3P_1) + \frac{3}{2}M1(^3P_2) - \frac{1}{2}M1(^3F_2) - E2(^3P_1) + \frac{9}{2}E2(^3P_2) \ , \tag{2.8d}
$$

$$
f_5 = \frac{3}{2}E1(^1D_2) \quad , \tag{2.8e}
$$

$$f_6 = \frac{5}{2} M1(^3F_2) \quad , \tag{2.8f}$$

$$f_7 = 6z E2(^3P_2) \quad , \tag{2.8g}$$

$$f_8 = 2E2(^3P_1) + 3E2(^3P_2) \quad , \tag{2.8h}$$

$$f_9 = f_{10} = f_{11} = f_{12} = 0 \quad , \tag{2.8i}$$

It is not difficult to construct the invariant amplitudes in higher waves or extend the procedure introduced here to other particle reactions.

3. FINAL-STATE INTERACTIONS IN THE $^1S_0$ STATE

The large scattering length of the almost bound $^1S_0$ N–N state dominates the low-energy scattering which is largely independent of the specific form of the nuclear potential. To first approximation, it is expected that the energy spectrum of (1.1) will show little sensitivity to the model-dependent features of the theoretical analysis, since the force between the two neutrons at low energies have a large effect on the photon spectrum. Nonetheless, to improve the bounds of theoretical uncertainty to a limit where the determination of charge symmetry breaking is meaningful, a careful analysis is needed even if the scattering length is known to be very sensitive to any charge-dependent effect. Indeed, using the first order perturbation formula for the scattering length differences,[2] $\delta a = a_{nn} - a_{pp}$,

$$\frac{\delta a}{a} = am \int dr\; u_0^2(r)\delta V(r) \quad , \tag{3.1}$$

where $u_0(r)$ is the $^1S_0$ asymptotic wave function normalized by $u_0(r) = \sin\,(kr+\delta)/\sin\,\delta$, it follows that

$$\frac{\delta a}{a} \sim \frac{a}{b}\frac{\delta V}{V} \quad , \tag{3.2}$$

with b the radius of the potential. Since the coefficient a/b in (3.2) is a factor of order 10, the charge dependent effects are largely magnified. Typically a 1% change in the potential produces a change of 3 fm in the scattering length. The contribution from nonelectromagnetic terms are of the order of $(m_u - m_d)/\Lambda$, with Λ the QCD scale (taken here as the nucleon mass). Since this effect is at the 0.5% level, we could expect a 1.5 fm different for δa. Consequently, we should maintain our theoretical uncertainties well below: 0.15–0.20 fm. In this section we discuss the different approaches used in the description of the FSI in the $^1S_0$ state of the neutron pair and present a method for evaluating some integrals appearing in the calculations.

Let us discuss first the problems encountered in the treatment of the FSI based on a specific nuclear potential.[30,31,34] Leaving apart the computational complexity, the main obstacle encountered in this approach is a problem of sensitivity of the theoretical analysis to the scattering parameters. The standard N–N potentials are constructed to reproduce the proton-proton data, and thus correspond to a fixed set of values for the scattering length and effective range. A model based on such potential, has only asymptotic sensitivity; i.e., only sensitivity to a variation of the low-energy parameters appearing in the n-n phase shift:

$$p \cot \delta_o(p) = -1/a_{nn} + \frac{1}{2}\, r_{nn} p^2 + \dots \quad . \tag{3.3}$$

As we mentioned above, such lack of flexibility for a given nuclear potential is of little importance in the extraction of the scattering length a_{nn}, which depends largely on the properties of the wave function outside the range of the nuclear forces, but should be taken into account in a serious attempt to determine the effective range r_{nn}. The effective range parameter represents the zero-energy variation of the actual wave function with respect to the asymptotic wave function within the range of the nuclear interaction, and is thus dependent on the model used to describe the nuclear forces. To give a specific example, a model based on a wave function obtained from a Reid soft-core potential (RSC),[38] has a sensitivity to r_{nn} lowered by a third,[23] as compared with the methods discussed below.

An alternative model-independent description of the FSI in the reaction (1.1) is based on dispersion relations.[32,35,36] The dispersion relations were solved to first order in Ref. 32, and an exact solution in closed from was found in I (Ref. 35) by studying the analytic properties of the Omnès–Muskhelishvili equation in the complex energy plane. The analytic solution was extended in II (Ref. 36) to describe the pion rescattering effects including the energy dependence of the pion-nucleon amplitude.

In the usual treatment of final-state interactions based on dispersion relations,[32,35] only the singularities from the Born term are considered in the analytical structure of the transition amplitude. Elastic unitary gives the enhancement to the Born amplitude in the form of an Omnès–Muskhelishvili solution to the dispersion problem. This corresponds to the neglect of all the singularities not included in the amplitude constructed from the model-independent part of the n-n wave function, sin $(pr)/pr$. To neglect the remaining singularities it is argued that, due to the short range of the final-state interaction, they are far away from the physical region in the energy plane.[39] This approximation is justified in a calculation aimed at the determination of the scattering length,[35] which is independent of finite range effects, but is insufficient to determine the effective range parameter, since the model-dependent part of the wave function is very sensitive to the finite range of the interaction. The singularities arising from the model-dependent part of the wave function cannot be ignored in the latter case. In fact, this approximation which amounts to neglect left-hand cuts from crossed channels in the dispersion integrals is responsible for an overestimate of 50% in the effect of the finite range of the interaction.[23] This result clearly contradicts the common wisdom, which ignores the effects from left-hand cuts at low energies. Similar results were obtained some time ago by Truhlík[40] related to the spectrum normalization of the negative muon capture in deuterium.

How shall we describe the nuclear interaction? A suitable approach to this problem is to follow the inverse scattering method of Gelfand–Levitan[41] to generate the n-n wave function from a given set of phase shifts, leaving a_{nn} and r_{nn} as free parameters. Other parameters are varied within a reasonable range, and the effect on a_{nn} and r_{nn} is evaluated. A most convenient form of parameterization of the phase shift, which presents great advantage for simplifying the actual calculations, is either in terms of Bargmann potentials[42] for which the Jost function $f(p)$ is a rational function

$$f(p) = \prod_{k=1}^{N} (p - i\beta_k)/(p - i\alpha_k) \quad , \tag{3.4}$$

with the phase determined by

$$e^{2i\delta_0(p)} = f(p)/f(-p) \quad , \tag{3.5}$$

or by separable potentials of the Yamaguchi type[43] given by

$$\langle q \mid V \mid p \rangle = -g(q)\, g(p) \quad , \tag{3.6}$$

with

$$g(p) = \sum_{k=1}^{N} A_k/(p^2 + a_k^2) \quad . \tag{3.7}$$

The phase shift is determined for the separable potential by

$$p \cot \delta_0(p) = (1 + G(p))/g^2(p) \quad , \tag{3.8}$$

with

$$G(p) = \sum_{k=1}^{N} \frac{A_k^2}{2a_k} \frac{a_k^2 - p^2}{(p^2 + a_k^2)^2} + 2\sum_{j=1}^{k-1} \frac{A_k A_j}{(a_k^2 - a_j^2)} \left(\frac{a_j}{p^2 + a_j^2} - \frac{a_k}{p^2 + a_k^2} \right) \quad . \tag{3.9}$$

For $N = 1$ we have

$$\alpha_1 = \left[(1 + 2r_{nn}/a_{nn})^{1/2} + 1 \right] / r_{nn} \quad , \tag{3.10}$$

$$a_1 = \left[(1 + 16r_{nn}/9a_{nn})^{1/2} + 1 \right] / r_{nn} \quad . \tag{3.11}$$

For $N = 2$ the Bargmann or Yamaguchi potentials depends on four adjustable parameters which are related to a_{nn}, r_{nn}, the energy where the phase goes through zero, and the coefficient of the asymptotic expansion of the phase shift[44], $\delta_o(p) \to A/p$ when $p \to \infty$. The transition amplitude corresponding to the Yamaguchi potentials have the same analytical structure in the complex energy plane as the closed-form solutions given in I and II, and the formulas in those references can be used with a simple rescaling of the range parameters. For example, for $N = 1$ we make the replacement $\alpha_1 \to a_1$. It is a relatively simple exercise to extend the analytical formalism expound in I and II to include more elaborate separable potentials, as the recently proposed separable representation[45] of the Paris nucleon-nucleon potential.[46]

Finally, we would like to have a convenient method for dealing with any n-n wave function, obtained from an arbitrary potential. This will allow us, in particular, to compare the result obtained from simple parameterizations with the shape of the spectrum obtained from realistic potentials[38,46] for determined values of the scattering parameters. To this purpose, we obtain a closed-form solution for the asymptotic n-n wave function, and integrate the difference with the exact wave function within the range of the nuclear interaction where the scattering wave function is modified.

Writing the transition amplitude as

$$M_S = \int \psi_S^{*(-)}(p,r)\ r f(r) d^3r \quad , \tag{3.12}$$

where $\psi_S^{*(-)}(p,r)$ denotes the S–wave of the outgoing neutrons with relative momentum p

$$\psi_S^{*(-)}(p,r) = e^{i\delta_0(p)} w_S(p,r)/pr \quad , \tag{3.13}$$

and $f(r)$ all the other factors which are given explicitly in I. Asymtotically

$$w_S(p,r) \underset{r\to\infty}{\longrightarrow} w_S^0(p,r) = \sin(pr+\delta_0) \quad . \tag{3.14}$$

We can express (3.12) as follows

$$M_S = \frac{e^{i\delta_0(p)}}{p}\left[\int d^3r\ \Delta w_S(p,r) f(r) + \int d^3r\ w_S^0(p,r) f(r)\right] \quad , \tag{3.15}$$

where $\Delta w_S(p,r)$ is the difference of the actual wave function with the asymptotic wave function at relative momentum p

$$\Delta w_S(p,r) = w_S(p,r) - w_S^0(p,r) \quad , \tag{3.16}$$

and the second integral above is the transition amplitude in the zero range approximation, that we shall label by M_S^0. A closed form solution for M_S^0 can be obtained if the deuteron wave function is written in the convenient form given by Gourdin, *et al.*[47] Furthermore, a quite accurate parametrization has been obtained from the Paris potential, which reproduces very well the deuteron static properties and form factors.[48] Using the expressions of I for the deuteron wave function and the transition operator for the radiative pion capture by the proton, we can express all the integrals which appear in M_S^0 in terms of spherical Bessel functions

$$I_\ell(\gamma, p, q) = \int_0^\infty e^{ipr} j_\ell(qr) h_\ell(i\gamma r)\gamma r dr \quad , \tag{3.17}$$

where γ is a deuteron range parameter. To evaluate this integral, we use the Gegenbauer addition theorem of Bessel functions

$$e^{i\omega}/i\omega = \sum_{n=0}^{\infty}(2n+1) j_n(z) h_n(Z) P_n(\cos\theta) \quad , \tag{3.18}$$

where

$$\omega = (Z^2 + z^2 - 2zZ\cos\theta)^{1/2} \quad , \quad \text{and } \mid z \mid < \mid Z \mid \quad . \tag{3.19}$$

Using the orthogonality of the Legendre polynomials and integrating over r we obtain

$$I_\ell(\gamma, p + i\epsilon, q) = \frac{\gamma}{2} \int_{-1}^{1} \frac{P_\ell(x)}{t(x)\left[t(x) + p + i\epsilon\right]} dx \quad , \tag{3.20}$$

with $t(x) = (q^2 + \gamma^2 - 2i\gamma x)^{1/2}$. The integral has been evaluated at $p + i\epsilon$ to ensure convergence at infinite r, but this procedure corresponds precisely to fix the variable p at physical values of the energy. In terms of the variable t

$$I_\ell(\gamma, p, q) = \frac{1}{2iq} \int_{-q-i\gamma}^{q-i\gamma} dt \, \frac{P_\ell(q^2 - \gamma^2 - t^2/2i\gamma q)}{p - t} \quad . \tag{3.21}$$

It follows from the above equation that there is a branch cut from $-q - i\gamma$

to $q - i\gamma$ in the complex t-plane, with a discontinuity given by

$$\text{disc } I_\ell(\gamma, p, q) = -\frac{\pi}{q} P_\ell(q^2 - \gamma^2 - p^2/2i\gamma q) \quad , \tag{3.22}$$

and thus

$$I_\ell(\gamma, p, q) = \frac{1}{2iq} P_\ell\left(\frac{q^2 - \gamma^2 - p^2}{2i\gamma q}\right) \ell n \frac{\gamma - i(p+q)}{\gamma - i(p-q)} + W_{\ell-1}(\gamma, p, q) \quad , \tag{3.23}$$

where $W_{\ell-1}(\gamma, p, q)$ is a polymonial of degree $\ell - 1$ in the variable $(p^2 + \gamma^2 - q^2)/\gamma q$, and is determined by the condition that the function $I_\ell(\gamma, p, q)$ is not singular near $q = 0$. In practice $W_{\ell-1}$ is obtained after taking real and imaginary parts of (3.23) for a given ℓ. For an S–wave transition we need to consider only $\ell = 0$ and 2, and we obtain

$$\begin{aligned} M_S^0 = \frac{16\pi}{kp} \sin\delta_0 e^{i\delta_0} \sum_\lambda C_\lambda \Bigg\{ & \left[C\left(\gamma_\lambda, p, \frac{1}{2}k\right) + \cot\,\delta_0 S\left(\gamma_\lambda, p, \frac{1}{2}k\right)\right] \tau_\lambda(p^2, k) \\ & + \frac{3\rho}{4\sqrt{2}} \frac{\gamma_\lambda}{k}\left(\frac{p^2 + \gamma_\lambda^2 - \frac{1}{4}k^2}{\gamma^2}\right)\Bigg\} \quad , \end{aligned} \tag{3.24}$$

which corresponds to the result obtained in I in the zero range limit. In the above expression, the sum is over the number of poles γ_λ of the deuteron wave function with residues C_λ, ρ is the asymptotic D– to S–ratio, and

$$\tau_\lambda(p^2, k) = 1 + \frac{\rho}{\sqrt{2}} \frac{\gamma_\lambda^2}{\gamma^2} + \frac{3\rho}{2\sqrt{2}} \frac{p^2}{\gamma^2}(1 - x_\lambda^2) \quad , \tag{3.25}$$

with

$$x_\lambda = (p^2 + k^2 + \gamma_\lambda^2)/pk \quad . \tag{3.26}$$

The functions $S(\gamma, p, q)$ and $C(\gamma, p, q)$ are defined by

$$S(\gamma, p, q) = \frac{1}{4} \, \ell n \, \frac{\gamma^2 + (p+q)^2}{\gamma^2 + (p-q)^2} \quad , \tag{3.27}$$

and

$$C(\gamma, p, q) = \frac{1}{2} \left[\tan^{-1} \left(\frac{2\gamma q}{\gamma^2 + p^2 - q^2)} \right) + \epsilon\pi \right] \quad , \tag{3.28}$$

with

$$\epsilon = 0 \text{ if } \gamma^2 + p^2 - q^2 \geq 0 \quad ,$$

$$\epsilon = 1 \text{ if } \gamma^2 + p^2 - q^2 < 0 \quad .$$

The calculation of the transition amplitude for an arbitrary nuclear potential is reduced to the numerical evaluation of the first integral in (3.15) in terms of $\Delta w_S(p, r)$ within the range of the nuclear forces. We show in Fig. 1 the difference function $\Delta w_S(p, r)$ for various interesting examples discussed here. For practical purposes the integral vanishes beyond 3-4 fermis. We have indicated in Fig. 2 the effect in the shape of the normalized energy spectrum of different approximations used in the description of the final-state interactions in the $^1S_0$ state of the outgoing neutrons. The effect is almost indistinguishable at the level of the drawing for the various models considered. However, a significant departure is observed for the zero-range approximation and the dispersion relation solution.

We have not performed a systematic evaluation of the nonlocal effects on the low-energy scattering parameters, due to our present ignorance of the importance of the presence of short-range nonlocality in the nuclear forces. We know, however, from the work of Gibbs, Gibson, and Stephenson[34] that if the nonlocal behavior near the origin is similar for the initial np and final nn systems, the effects on the spectrum are cancelled nearly completely.

4. FINAL–STATE INTERACTIONS IN HIGHER WAVES

To include the effect of the n-n FSI in higher waves, we perform a partial wave expansion of the n-n scattering wave function. Pauli exclusion principle is taken into account by decomposing the transition amplitude into the rotational invariants given in Table 1 of I corresponding to the 12 amplitudes f_i, which are in turn expanded into multipole eigenamplitudes using (2.8). At low relative n-n momentum, only the P–waves are of some importance. The dominant contribution corresponds to the $^3P_0$ state of the neutron pair. However, the probability amplitude to find the neutrons in the final state of $\pi^- d \to \gamma nn$ in a $^3P_0$ state is zero in the impulse approximation. This is clear from Eq. (2.8), since the $^3P_0$ amplitude contributes only to f_2 and, as shown previously in I, this amplitude is identically zero. Since we are computing a small correction, we shall use the distorted plane wave approximation to describe the higher n-n outgoing waves, and compute the higher wave enhancement keeping only the S–wave of the deuteron in the initial state ($\rho = 0.0265$). Going back to I, and looking at Eq. (2.11) of that reference, we see that only f_1 and f_7 contribute to this approximation. Also, neglecting D and higher partial wave enhancement in the final state, we obtain from (2.8) the following result:

$$f_1 = E1(^1S_0) \quad , \quad f_7 = 6zE2(^3P_2) \quad , \tag{4.1}$$

and only the $^3P_2$ enhancement survives. The effect from the $^3P_1$ wave would have been three times bigger, and from the $^3P_0$ five times more important than the $^3P_2$ enhancement. Let us now perform the explicit calculation. For a P–wave:

$$\psi_P^{*(-)}(p,r) = 6ie^{i\delta_1(p)}\omega_P(p,r)\hat{p}\cdot\hat{r}/pr \quad , \tag{4.2}$$

with

$$\omega_P(p,r) = \sin\left(pr - \frac{\pi}{2} + \delta_1\right) \quad . \tag{4.3}$$

Making use of recurrence relations for the spherical Bessel functions, all the integrals appearing in the calculation are amenable to the form

$$J_\ell(\gamma,p,q) = \int\limits_0^\infty e^{ipr} j_\ell(qr) h_0(i\gamma r)\gamma r dr \quad . \tag{4.4}$$

This integral is solved by using the expression

$$j_\ell(z) = \frac{1}{2i^\ell} \int\limits_{-1}^{1} P_\ell(\mu) e^{iz\mu} d\mu \quad , \tag{4.5}$$

and integrating over r. The result is

$$J_\ell(\gamma,p,q) = -\frac{1}{(-i)^{\ell+1}} \frac{1}{q} Q_\ell\left(\frac{p+i\gamma}{q}\right) \quad , \tag{4.6}$$

where the $Q_\ell(z)$ are Legendre functions of the second kind. The following result is obtained for the P–wave amplitude:

$$\begin{aligned} M_P = {} & \frac{48\pi}{pk} e^{i\delta}\left({}^3P_2\right) \cos\delta\left({}^3P_2\right) \cos\theta \\ & \times \sum_\lambda C_\lambda \left[S_P\left(\gamma_\lambda, p, \frac{1}{2}k\right) + \tan\delta\left({}^3P_2\right) C_P\left(\gamma_\lambda, p, \frac{1}{2}k\right)\right] \quad , \end{aligned} \tag{4.7}$$

where the functions $S_P(\gamma,p,q)$ and $C_P(\gamma,p,q)$ are defined by

$$S_P(\gamma,p,q) = (p/q)S(\gamma,p,q) + (\gamma/q)C(\gamma,p,q) \quad , \tag{4.8}$$

and

$$C_P(\gamma,p,q) = (p/q)C(\gamma,p,q) - (\gamma/q)S(\gamma,p,q) \quad . \tag{4.9}$$

Only the ${}^3P_2$ enhancement is relevant under the approximations discussed above. The amplitude corresponds to an electric quadrupole transition $E2({}^3P_2)$.

The amplitudes f_i are obtained by removing a given n-n partial wave from the symmetric or antisymmetric combination of plane waves for the spin singlet or triplet states, and adding the scattering eigenfunction $\psi^{(-)}(p,r)$ in the corresponding partial wave for specified boundary conditions. To fix our signs and normalization conventions, we indicate the modification on the scalar amplitudes f_i [Eq. (2.11) of I] from $S-$ and $P-$wave enhancement in the zero range approximation, and for a deuteron wave function $\phi(r) = e^{-\gamma r/r}$. The result is

$$
\begin{aligned}
f_1 &= 4\pi\left\{\frac{1}{q_-^2+\gamma^2}+\frac{1}{q_+^2+\gamma^2}-\frac{4}{pk}\,S\left(\gamma,p,\frac{1}{2}k\right)\right.\\
&\quad\left.+\frac{4}{pk}\,\sin\delta_0 e^{i\delta_0}\left[C\left(\gamma,p,\frac{1}{2}k\right)+\cot\delta_0\,S\left(\gamma,p,\frac{1}{2}k\right)\right]\right\}\quad,\\
f_7 &= 4\pi\left\{\frac{1}{q_-^2+\gamma^2}-\frac{1}{q_+^2+\gamma^2}-\frac{12}{pk}\cos\theta\,S_P\left(\gamma,p,\frac{1}{2}k\right)\right.\\
&\quad\left.+\frac{12}{pk}\cos\theta\,\cos\delta_1 e^{i\delta_1}\left[S_P\left(\gamma,p,\frac{1}{2}k\right)+\tan\delta_1\,C_P\left(\gamma,p,\frac{1}{2}k\right)\right]\right\}\quad,
\end{aligned}
\tag{4.10}
$$

with $\vec{q}_\pm = \vec{p} \pm \frac{1}{2}\vec{k}$ and $\delta_1 = \delta({}^3P_2)$. The explicit formulas for the modifications of f_1 and f_7 from the pion rescattering contributions is given in II.

5. DISCUSSION AND OUTLOOK

We have presented evidence for charge symmetry breaking from the study of mirror nuclei and low-energy scattering experiments with nucleons. In our opinion, a consistent picture is emerging which suggests that the strong interaction is slightly more attractive for the n-n interaction than for the p-p system. Some theoretical implications were discussed as well as the inconsistency with the "world average" value[18] for the n-n scattering length, a_{nn}, which implies a stronger p-p force. It is not clear at the present time, however, if this significant difference would be explained by the three-body forces for the pick-up and knock-on reactions by the mechanism proposed by Šlaus, Akaishi and Tanaka,[26] since no conclusive evidence of three-body forces on scattering processes has been found yet.[49] Furthermore, the distinction between three-body forces and two-body off-shell effects is still an open problem.[50] Further theoretical and experimental effort is needed to ascertain the validity of the proposed mechanism.[51] The "world average" value is a straight average over some 40 measurements, most of them with three strongly interacting particles in the final state and with large uncertainties. There is no reason to give too much relevance to this value.

In the absence of colliding neutron beam experiments, the most unambiguous results from low-energy scattering reactions comes from the study of $\pi^- d \to \gamma nn$. We have made a careful analysis of the methods used in the calculation of the final-state interactions in this reaction. We have also developed a simplified approach for dealing with the n-n interaction in higher partial waves which, otherwise, is a cumbersome task due to the number of independent helicity amplitudes. Our study shows that the relevant contribution from the triplet P–waves comes only from the $^3P_2$ state, being itself a small effect. The contribution from the $^3P_0$

state is zero in the impulse approximation, and the effect from the $^3P_1$ state is absent if we do not include the D state of the deuteron in the higher partial wave enhancement, this having a negligible effect. It is now clear why the previous calculations, without including a proper P–wave enhancement, gave a very good agreement with the photon spectrum of (1.1) at relatively high n-n momentum: the dominant n-n P–waves are suppressed in the radiative pion capture from rest in deuterium. Although unimportant for the extraction of a_{nn}, the inclusion of higher n-n partial waves is relevant for the extraction of r_{nn}. We have developed a simple and coherent theory with all the important elements, valid in the energy range relevant to the Lausanne–Munich–Zurich experiment[22,23] (for relative n-n momenta up to 100 MeV/c). The present analysis of final-state interactions for the outgoing neutrons in the capture reaction (1.1) confirms our previous results[23] for the n-n scattering length and effective range: $a_{nn} = -18.5 \pm 0.4$ fm and $r_{nn} = 2.80 \pm 0.11$ fm.

The corrections to the production mechanism of $\pi^- d \to \gamma nn$ were calculated in the framework of a covariant theory,[52] without recourse to the impulse approximation, by studying all the relevant Feynman diagrams and treating the deuteron as a composite object in quantum field theory. The deuteron structure was introduced by means of neutron-proton-deuteron vertex functions depending on the momentum transfer, and the effect of meson-exchange currents was evaluated by making use of the gauge invariance of the theory.[52] The results from this elaborate theory do not modify our present conclusions regarding the charge symmetry breaking of the nuclear forces.

It is important to understand qualitatively the origin of the charge symmetry breakdown in the N–N interaction, where QCD and the quark substructure of the nucleon could play a significant role.[16,17,29] In particular, the quark mass difference and the exchange process of the quark degrees of freedom would be expected to give an important contribution to the charge asymmetry. The latter effect is reminiscent of the exchange force which arises in the study of the hydrogen molecule due to the symmetry properties of the wave function of two identical electrons. Similarly, the exchange properties of the wave function of the p-p or n-n system, due to the Pauli exclusion principle for quarks, gives rise to an exchange interaction which explains the short-range repulsion between nucleons.[53] The symmetry properties of the wave function are different for the p-p or n-n systems due to the distinct identity of the quark content. The role of the nucleon substructure in determining the intermediate-range attraction of the nuclear interaction is still an open problem,[54] and further investigation on the effect of the quark exchange in the charge symmetry breaking is needed to further elucidate this problem.

A most interesting effect would originate from the modification of the nucleon properties in a nuclear medium, were a size increase in the nucleon could be expected.[53] Since the quark exchange effects depend on the overlap of the nucleon wave function, an increase in the nucleon size would enhance the effects from the charge asymmetry of the nuclear interactions. For a 10% increase in the nucleon size, as suggested by the EMC (European Muon Collaboration) effect,[55] a difference of -1.2 fm in the scattering length difference $\delta a = a_{nn} - a_{pp}$ will increase to -1.6 fm in nuclear matter.[17] The discrepancy of binding energy differences of mirror nuclei, or Nolen–Schiffer anomaly, exists throughout the periodic table and increases significantly with the mass number A.[56] Although it is still premature to determine if the mechansim proposed in Ref. 17 could account for all the observed binding energy differences, it is certainly important to investigate this point thoroughly.

Finally, with our present knowledge of the proton from elastic and inelastic processes at low and high momentum transfer, it should be possible to reduce the theoretical uncertainty in the Coulomb correction of the p-p scattering length. A theoretical effort in this direction is worth attempting to have a completely unambiguous comparison with a_{nn}.

Note Added:

After finishing this work, we learned of a proposal by a Los Alamos—Oak Ridge collaboration for a direct n-n collision experiment from two simultaneous fusion-fision sources [D.W. Glasgow *et al.*, Proc. Int. Conf. on Nucl. Data for Basic and Applied Science, Santa Fe (1985); D.W. Glasgow, private communication]. The neutron beams transported through evacuated lines-of-sight collide at 3.8°, and the pulse of scattered events is confined within a kinematic forward cone. Center-of-mass (CM) energies of 2–38 keV are obtained for a 1–14 MeV fusion-fision spectrum. This has the advantage of having very slow (S–wave) colliding neutrons in the CM, but involving the detection of scattered neutrons at high laboratory energies, which outrace the background neutrons. The first stage of this experiment, necessarily carried out under extreme conditions, has been performed. It is expected that the experiment will be repeated and data obtained in the near future.

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APPENDIX

We give in this appendix various useful formulas related to the trace calculations of the transition probabilities used in the evaluation of the energy spectrum. The transition probability is proportional to

$$\overline{\sum} \mid \Im \mid^2 N \tag{A.1}$$

where N is the appropriate phase space factor and $\overline{\sum}$ indicates the average of polarizations for the initial state and the sum of spin in the final state. The photon polarization vector $\vec{\epsilon}^{\,(\lambda)}$ satisfies the following conditions

$$\vec{\epsilon}^{\,(\lambda)} \cdot \vec{k} = 0 \quad , \tag{A.2}$$

$$\vec{\epsilon}^{\,*(\lambda)} \cdot \vec{\epsilon}^{\,(\lambda')} = \delta_{\lambda\lambda'} \quad , \tag{A.3}$$

$$\sum_{\lambda=1}^{2} \epsilon_i^{*(\lambda)} \epsilon_j^{(\lambda)} = \delta_{ij} - k_i k_j / \mid \vec{k} \mid^2 \quad . \tag{A.4}$$

The analogous condition for the deuteron polarization vector $\vec{\eta}^{\,(\mu)}$ are:

$$\vec{\eta}^{\,*(\mu)} \cdot \vec{\eta}^{\,(\mu')} = \delta_{\mu\mu'} \quad , \tag{A.5}$$

$$\sum_{\mu=1}^{3} \eta_i^{*(\mu)} n_j^{(\mu)} = \delta_{ij} \quad , \tag{A.6}$$

since the pionic capture in (1.1) occurs from rest in the laboratory frame. Using the above equations and the explicit form of $\Im$ given in Table 1 of I (Ref. 35) we obtain the following result

$$\overline{\sum} \mid \Im \mid^2 = \frac{1}{3} \sum_{\lambda=1}^{2} \sum_{\mu=1}^{3} T_R \left[\Im^{+}_{\lambda,\mu} \Im_{\lambda,\mu} \right]$$
$$= a + b \cos\theta + c \sin^2\theta + d \cos\theta \sin^2\theta \quad , \tag{A.7}$$

where θ is the angle between the relative momentum of the two outgoing neutrons and the photon direction and

$$a = \frac{4}{3} \; [\mid f_1 \mid^2 + \mid f_2 \mid^2 + \mid f_4 \mid^2 + 2 \mid f_7 \mid^2 + \mid f_{11} \mid^2 + \mid f_{12} \mid^2$$
$$- 2\mathrm{Re}\,(f_7 f_{11}^* + f_7 f_{12}^* - f_{11} f_{12}^*)] \quad ,$$

$$b = \frac{8}{3} \; \mathrm{Re}\,(f_2 f_7^* - f_4 f_7^* + f_4 f_{11}^* + f_4 f_{12}^*) \quad ,$$

$$c = \frac{2}{3} \; [3 \mid f_3 \mid^2 + \mid f_5 \mid^2 + \mid f_6 \mid^2 + 2 \mid f_8 \mid^2 + \mid f_9 \mid^2 + 2 \mid f_{10} \mid^2 - \mid f_{11} \mid^2$$
$$+ 2\mathrm{Re}\,(f_1 f_5^* + f_2 f_3^* + f_2 f_4^* + f_2 f_6^* + f_3 f_4^* + f_3 f_6^*$$
$$+ f_3 f_8^* + f_4 f_6^* - f_4 f_8^* - 2 f_7 f_{10}^* + f_7 f_{11}^* + f_{10} f_{12}^* - 2 f_{11} f_{12}^*)]$$

$$d = \frac{4}{3} \; \mathrm{Re}\,(f_5 f_9^* + f_6 f_{12}^* + f_8 f_{10}^* - f_8 f_{11}^*) \quad . \tag{A.8}$$

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Table 1. Dipole, quadrupole and octopole transition amplitudes to a ${}^{2S+1}L_J\, n-n$ final state from an S state of the pionic deuterium atom.

Dipole	Quadrupole	Octopole
$E1({}^1S_0)$	$E2({}^3P_1)$	$M3({}^3P_2)$
$M1({}^3P_0)$	$E2({}^3P_2)$	$E3({}^1D_2)$
$M1({}^3P_1)$	$M2({}^1D_2)$	$M3({}^3F_2)$
$M1({}^3P_2)$	$E2({}^3F_2)$	$M3({}^3F_3)$
$E1({}^1D_2)$	$E2({}^3F_3)$	$M3({}^3F_4)$
$M1({}^3F_2)$		$E3({}^1G_4)$
		$M3({}^3H_4)$

Table 2. Tensor decomposition of total angular momentum eigenstates of the n–n system.

Final n–n State	J	2 × 2 Matrix
$^1S_0$	0	I
$^3P_0$	0	$\vec{\sigma} \cdot \hat{p}$
$^3P_1$	1	$\vec{\sigma} \times \hat{p}$
$^3P_2$	2	$\frac{3}{2}\left(\sigma_i \hat{p}_j + \sigma_j \hat{p}_i\right) - (\vec{\sigma} \cdot \hat{p})\, \delta_{ij} = U_{ij}$
$^1D_2$	2	$\frac{1}{2}\left(3\hat{p}_i \hat{p}_j - \delta_{ij}\right) = T_{ij}$
$^3F_2$	2	$\frac{1}{2}\left(5\hat{p}_i \hat{p}_j\, (\vec{\sigma} \cdot \hat{p}) - \sigma_i \hat{p}_j - \sigma_j \hat{p}_i - (\vec{\sigma} \cdot \hat{p})\, \delta_{ij}\right) = V_{ij}$
$^3F_3$	3	0
$^3F_4$	4	$\frac{1}{8}\left(35 p_i p_j p_k \sigma_\ell \ldots\ldots\ldots\right)$

FIGURE CAPTIONS

Fig. 1. Radial n-n difference function $\Delta w_S(p,r)$ for $p = 10$ MeV/c. The dotted line corresponds to a Yamaguchi rank-one potential, the dot-dot-dashed line to a Bargmann rank-two potential, and the solid line to a Reid Soft Core potential. The n-n scattering parameters are fixed in this example to $a_{nn} = -17.1$ fm and $r_{nn} = 2.8$ fm.

Fig. 2. Effect of final-state interactions on the shape of the spectrum. The solid and dotted lines correspond to the models described in Fig. 1. The dash-dot line is the zero range approximation, and the dashed line represents the dispersion relation solution.

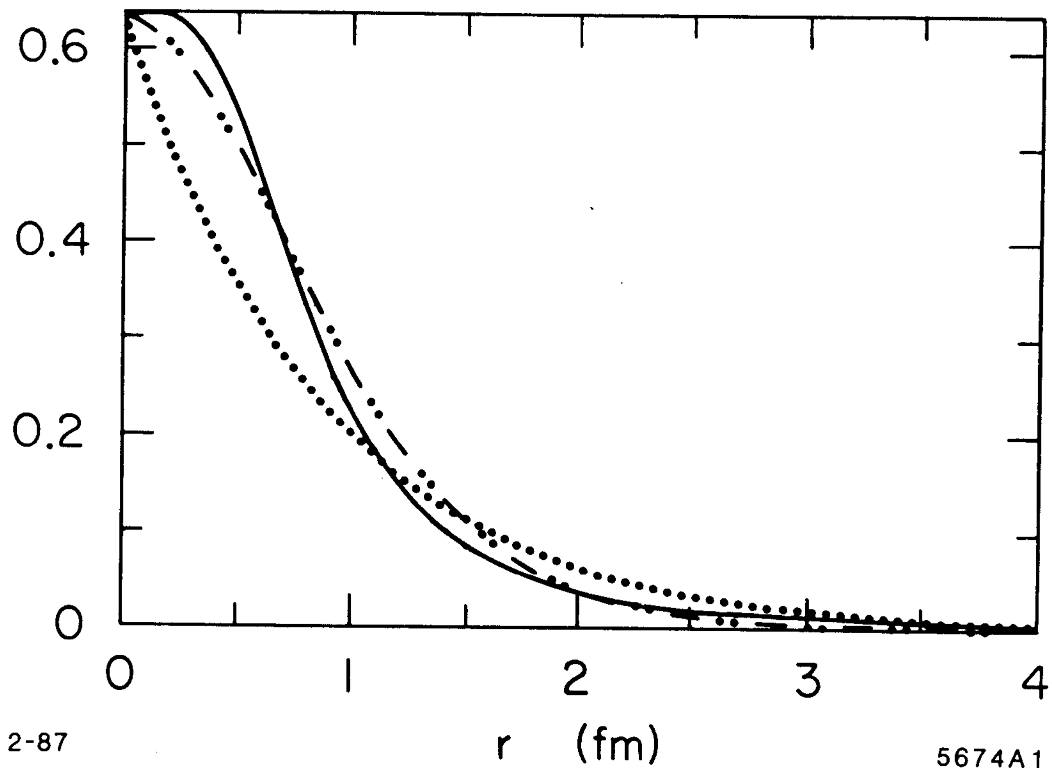

Fig. 1

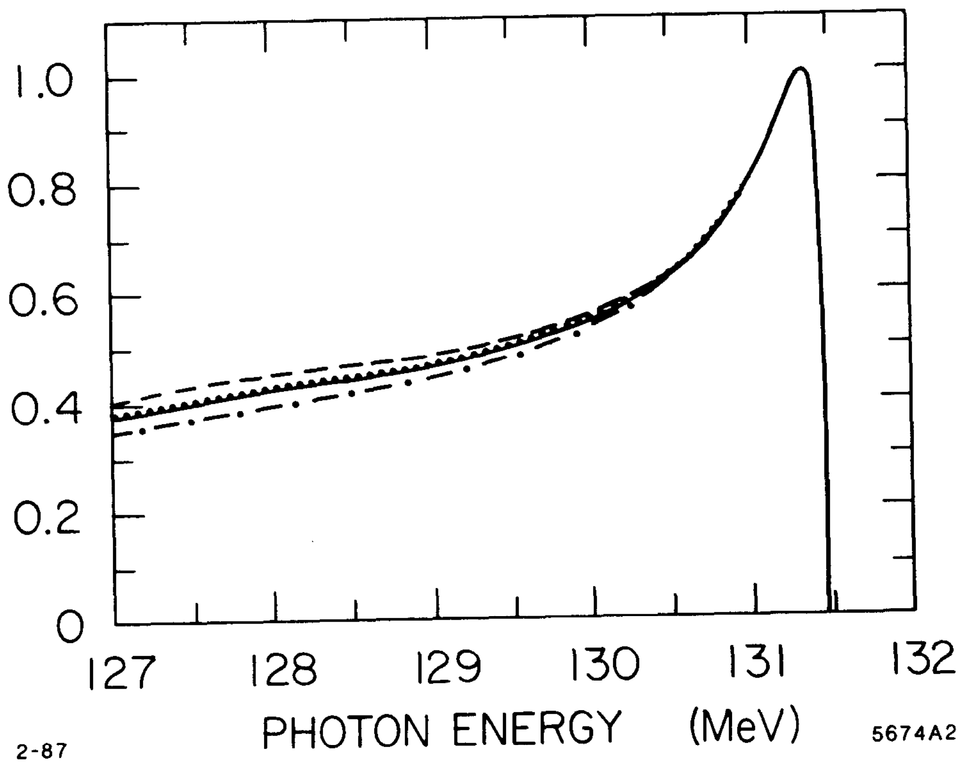
1.0
0.8
0.6
0.4
0.2
0
127
128
129
130
131
132
PHOTON ENERGY (MeV)
2-87
5674A2

Fig. 2