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KAON DECAY AMPLITUDES USING STAGGERED FERMIONS*

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ABSTRACT

A status report is given of an attempt, using staggered fermions, to calculate the real and imaginary parts of the amplitudes for $K \to \pi\pi$. Semi-quantitative results are found for the imaginary parts, and these suggest that ϵ' might be smaller than previously expected in the standard model.

INTRODUCTION

This talk describes a calculation of weak interaction matrix elements done in collaboration with Rajan Gupta, Gerry Guralnik, Greg Kilcup and Apoorva Patel (the Los Alamos Advanced Computing Group). Theoretical details can be found in reference [1]; detailed numerical results will appear elsewhere [2].

Present lattice measurements incorporate physics from the range of scales $\pi/L \approx .5\text{GeV} \leq \mu \leq 1/a \approx 2\text{GeV}$. Here a is the lattice spacing, and $L = N_s a$ is the physical size of the spatial box. At the ultraviolet end of this range we hope to match onto perturbative calculations: for weak interaction calculations we use the Renormalization Group machinery to scale down from M_W to $1/a$. This is reliable for small enough a, roughly $1/a > 2\text{GeV}$, corresponding to $g < 1$ on the Wilson axis.

The lower limit to μ is the infrared cut-off provided by the physical size of the lattice. Clearly we cannot simulate processes involving real pions until the smallest non-zero momentum is less than m_π. Further, as stressed here by Ken Wilson, we cannot look in detail at the wavefunctions of hadrons until the smallest momentum is less than the typical transverse momentum of quarks in these particles, i.e. $\simeq 200\text{MeV}$. However, we can overcome the first of these problems using the chiral Lagrangian to extrapolate from the lattice world with $m_\pi \simeq \pi/L$ to the real world with light pions. To do this we have to match our lattice results onto the forms expected for small m_π.

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Combining these two matchings, we are almost in a position to evaluate those matrix elements of the weak interaction Hamiltonian which are relevant to Kaon decays, though only in the quenched approximation. This talk will explain what "almost" means for staggered fermions. I will first discuss the state of the theory, then present our results, and close with some conclusions.

THEORY

We transcribe fermions onto the lattice using the staggered formulation rather than that of Wilson. The pros and cons of staggered, relative to Wilson are:

PRO staggered	*CON staggered*
$U(1)_A$ symmetry for $m \to 0$	4 staggered species per continuum flavor
$\Rightarrow$ Ward Identities	$\Rightarrow$ Continuum theory has
$\Rightarrow$ Restricted Operator mixing	$U(4N_f)_V \times SU(4N_f)_A$ symmetry
	$\Rightarrow$ Extra factors of $N_f = 4$
	Operators with up to 4 links
	$\Rightarrow$ Noisier results
	$\Rightarrow$ Possibly large $O(g^2)$ corrections (?)

One other possible CON — the inability to project onto states of definite parity — should not be a problem for lattice pions (though see below). We have chosen to live with the CONS in order to make use of the PROS; this talk will show how this choice has worked out so far. The only part I will not comment on below is the possible CON of large $O(g^2)$ corrections; these have not been calculated yet for staggered fermions, although simpler calculations give some cause for worry [3]. The absence of these calculations also means that the short distance matching cannot be done in detail, and so only qualitative results can be given.

I will concentrate on the matrix elements (ME) $\langle K|\mathcal{H}_W|\pi\pi\rangle$. Our aim is to calculate their real and imaginary parts, for both charged and neutral kaons. Experimentally, the real part of the K^0 amplitude is 20 times larger than that of the $K^{\pm}$; this is the long-standing puzzle of the $\Delta I = 1/2$ rule. As for the imaginary parts, it is the relative phase between $I = 1/2$ and $I = 3/2$ amplitudes that determines the magnitude of ϵ'. In the standard model both amplitudes get phases from penguin diagrams (strong and electromagnetic) with t and b quarks in the loops.

The direct measurement of $K \to \pi\pi$ amplitudes is beyond present lattice technology. Instead, the standard trick [4] is to use the chiral Lagrangian to relate the amplitudes to those of $\langle K|\mathcal{H}_W{}^{subtracted}|\pi\rangle$. Aside from the fact that this is an approximation, to which I will return later, this trick brings with it a nasty problem – the fact that subtractions have to be done. This is the worst problem to be overcome in order to extract numbers. It might be thought that, given this problem, it is worth putting a lot of effort into a direct calculation of the $K \to \pi\pi$ amplitude, for which no subtractions are needed. This is far from clear. To avoid subtractions, one must calculate on-shell matrix elements. This is hard on the lattice because of the discrete momenta. Furthermore, one has to understand final state interactions; these take a complicated form in Euclidean space since one cannot have a phase.

So we proceed by calculating $\langle K|\mathcal{H}_W{}^{subtracted}|\pi\rangle$. In the continuum $\mathcal{H}_W$ contains a slew of operators multiplied by coefficients. When transcribed to the lattice, a lot more operators are needed. For staggered fermions this is because the flavor and spin degrees of freedom are spread out over 2^4 points [1]. The operators thus contain varying numbers of gauge links, up to four in each bilinear. We know how this works at $O(g^0)$, but not yet at $O(g^2)$. However, we do know the general features of the operators to all orders. As in the continuum, there is a natural division of the operators into four types: (1) $I = 3/2$, LL operators; (2) $(8_L, 1_R)$, $I = 1/2$, LL operators; (3) $(8_L, 1_R)$, $I = 1/2$ LR operators; (4) $(8_L, 8_R)$ LR operators. As the scale is changed these operators mix; at m_W one has only operators of the first two types, but strong interaction "penguin" diagrams (with t and b loops) produce type (3) operators as the scale is reduced, and electromagnetic penguins produce operators of type (4). At a scale corresponding to about m_c, the real parts of the coefficients are largest for the LL operators, and thus these dominate the K decay rates. Conversely, the imaginary parts are larger for the LR operators, and these probably dominate the contributions to ϵ'.

Before proceeding I want to make a comment about the scale of the lattice calculation. We will be working at a scale $1/a \approx m_c$, for which the charm quark can be ignored, to first approximation. By this I mean that the coefficients are calculated by running down to m_c, but then the charm quark is dropped from the operators. This implies that the usual penguin solution of the $\Delta I = 1/2$ puzzle cannot be tested directly. In this solution, it is the RG scaling below m_c that induces the real part of the coefficient of operators of type (3), and then the enhanced matrix elements of these operators give the $\Delta I = 1/2$ rule. In fact, this idea can never really be tested, because if one runs much below m_c one is outside the range of perturbation theory.

The four types of operator yield four classes of contraction. The first is the eight contraction of the LL operators – known below as LL8. Type (1) operators have only these contractions, and so these are the only contractions contributing to $K^{\pm}$ decays. They also give the main contribution to the imaginary part of $K^0 \leftrightarrow \overline{K^0}$ mixing, i.e. to ϵ. They are straightforward to calculate, partly because they are eights rather than eyes, and partly because they do not require subtractions. However, because of their simplicity, one can make reasonable estimates of them using various continuum approximations, e.g. vacuum saturation. For the lattice to improve upon these estimates, it must give a result accurate to better than a factor of two.

The second class is the eye contractions of the LL operators of type (2), which I call LLI. These are purely $I = 1/2$. At the charm quark scale these are the only possible source of the $\Delta I = 1/2$ rule within the standard model. To the extent that it makes sense to discuss lower scales, these contractions may also be dominant, a view emphasized by Donoghue [5]. These contractions are harder to evaluate for two reasons. First, they require the use of source techniques, which reduces the statistics. Second, they require subtractions. These contractions are also harder to estimate in the continuum: e.g. they vanish in vacuum saturation.

The third contractions are those of the LR octet operators. Unlike the LL operators, these do not retain their spinor structure upon Fierz transformation. For the most important such operator, one has:

$$\bar{s}_a\gamma^\mu(1+\gamma_5)d_b \ \ \bar{q}_b\gamma^\mu(1-\gamma_5)q_a \ = \ -2\ \bar{s}_a(1-\gamma_5)q_a\ \bar{q}_b(1+\gamma_5)d_b$$

Here, a and b are color indices, and q is summed over u, d and s. These operators have both eight and eye contractions, so I refer to them as LR8I. Source methods are again needed, as are subtractions.

The appearance of densities, rather than currents, in the Fierzed form leads one to expect an enhancement over LL operators by $m_K^4/(m_s^2\Lambda^2)$, where $\Lambda \approx 1\text{GeV}$ is the cut-off in the chiral Lagrangian [6]. The factors can be explicitly worked out in the large N_c limit [7][8]. It is this enhancement which has led to all the speculation about the role of penguins in the $\Delta I = 1/2$ rule. However, here I am interested in the penguins as the source of ϵ'.

The final contraction is that of the LR singlet operators. The dominant contribution comes from eight contractions, so I refer to them as LR8. Compared to LL operators, one expects an enhancement of m_K^2/m_s^2 due to the LR structure. They are straightforward to calculate, needing no sources, and no subtractions (at least for the dominant part). They are also easy to estimate in the continuum.

The great advantage of staggered fermions is that these 4 types of contractions separately satisfy exact lattice Ward Identities (WI) [9]. This is true separately for each of the many operators that appear, and is true configuration by configuration. These WI are precise lattice analogues of the continuum WI of PCAC. They constrain the behavior of the ME as one varies m_q. For a general operator $\mathcal{O}$ in $\mathcal{H}_W$:

$$\langle K|\mathcal{O}|\pi\rangle = f^2(\alpha f^2 + \beta m_\pi^2 + \gamma m_K^2 + \delta m_\pi m_K) + O(m_q^2)$$

where f is the value of f_π extrapolated to $m_\pi = 0$, and α, β, γ and δ are dimensionless. The WI imply that for the LL8 $\alpha = \beta = \gamma = 0$, for the LLI and LR8I that $\alpha = \beta = 0$, but no relations for the LR8. For the LR8I (which are made up of contractions like the LR8 together with eyes) it is the addition of eights to eyes that cancels the α and β terms.

One can show [4][9] that, to $O(m_q)$, the subtraction needed for the LLI and LR8I will remove the γ term. After subtraction, then, the LLI and LR8I have the same form as the LL8 contractions. One can further show that, to the same order, one can measure γ using

$$\langle K|\mathcal{O}|0\rangle = \gamma\sqrt{2N_f}f^3 m_K^2(m_d - m_s)/(m_d + m_s)$$

where $N_f = 1$ in the continuum, but $N_f = 4$ for staggered fermions. This shows how the subtraction removes the effect of $s \leftrightarrow d$ mixing.

This can all be phrased equivalently in the language of operator mixing [10][9]. All the operators in $\mathcal{H}_W$ mix with other operators of d=6, constrained by the lattice symmetries. In addition, operators of types (2) and (3) mix with the d=4 operator

$$\mathcal{S} = (m_d + m_s)\bar{s}d \ + \ (m_d - m_s)\bar{s}\gamma_5 d$$

which is also a $SU(3)_L$ octet, $I = 1/2$ operator. It is exactly this operator which gives the γ terms in the above equations. This suggests the following method to remove the γ terms. Choose ρ such that

$$\langle K|\mathcal{O}^{subtracted}|0\rangle \equiv \langle K|\mathcal{O} - \rho\mathcal{S}|0\rangle = 0$$

and then

$$\langle K|\mathcal{O}^{subtracted}|\pi\rangle = \delta f^2 m_\pi m_K + O(m_q^2)$$

This method allows a time by time subtraction, and we use it below.

As stressed by the CERN/Rome group [10], the coefficients of the mixing with $\mathcal{S}$ are non-perturbative, i.e. of $O(1/a^2)$. Thus one should use a non-perturbative method of calculating γ, such as that outlined above. But for Wilson fermions this method is not available [10]. One can proceed by performing a perturbative evaluation of γ, as suggested by the UCLA group [11]. However, this is suspect, because non-leading terms will be of $O(g^n/a^2)$, and thus diverge as $a \to 0$. Nevertheless, they claim that for $g \approx 1$ their method might be viable. It seems to me to be important to check first on simpler quantities such as $\langle\overline{\psi}\psi\rangle$.

Whatever method of subtraction one uses, the entire procedure rests upon an expansion in meson masses. This has two consequences. First, as emphasized by Martinelli, the output is only the $O(m^0)$ term in the K decay amplitude; higher order terms cannot be obtained. Second, it is essential that one finds the advertised chiral behavior. Without this, the low energy matching cannot be done. One can also check this in other ways, e.g. by looking at the variation of f_π with m_π.

RESULTS

After a trial run on an $8^3 \times 16$ lattice [1], we have now completed an analysis on a $12^3 \times 30$ lattice [2]. This is long enough to unambiguously expose the lightest states. We use an improved action, that of ref [12] with $K_F = 10.5$. This corresponds to $\beta = 5.96$ on the Wilson axis, and thus is nearly in the scaling region. We have used 25 lattices, and have attempted to address the issue of low energy matching by using small quark masses. To do this, we have calculated with $m_q = .040$ and $.005$. The larger mass corresponds quite closely to the physical strange quark mass, so I refer to it S. The lighter mass is as close as we can get to a realistic u or d quark, and I call it the U quark. This allows us to consider three psuedo-Goldstone bosons: SS ($m \approx$ 700MeV), US ($m \approx$ 500MeV), and UU ($m \approx$ 300MeV). Using these we have measured three ME: $\langle K(SS)|\mathcal{H}_W|\pi(SS)\rangle$, labelled SS; $\langle K(US)|\mathcal{H}_W|\pi(UU)\rangle$, called US; and $\langle K(UU)|\mathcal{H}_W|\pi(UU)\rangle$ named UU. All our propagators have been calculated with antiperiodic boundary conditions (APBC).

First I comment on the chiral behavior of quantities derived from two point correlators. For UU, US and SS respectively, we have $m_\pi = .180, .358, .469$, $f_\pi = .056, .072, .088$, and $Z_\pi = .264, .325, .43$. The Z factor is defined through the two point psuedoscalar correlator C(t)

$$C(t) = e^{-|t|m_\pi} Z_\pi / N_\pi; \quad N_\pi \equiv 2 \sinh m_\pi$$

One expects f_π and Z_π to have the form $a + b\overline{m}$ ($\overline{m}$ is the average quark mass), while $m_\pi^2 = c\overline{m}$. If we are to be in the region where $O(m_q)$ expansions are valid, as we must be to do the low energy matching, the b terms must be small for $\overline{m} = m_s$. We are clearly at the limit of this region with the SS states. Extrapolating our numbers to $m_q = 0$, we find for the physical π and K that $f_K/f_\pi \approx 1.35$, compared to the experimental 1.25. Thus the $O(m)$ terms are in rough agreement with those in the continuum.

I now turn to the correlators from which we extract the ME. I denote these by $C(t_\pi, t_K)$. Here the operator is at $t = 0$ (or $t = 0,1$ for two timeslice operators), and the π and K are respectively at t_π, t_K. All the figures have $t_\pi = 7$, a distance large enough to remove heavier states, yet small enough to retain a reasonable signal. For eight contractions we have data for all t_π, but for the eyes, what you see is all we've got. The region for $t_K > 15$ has the π and K on opposite sides of $\mathcal{H}_W$, and so corresponds to the ME we want to measure. For $t_K < 15$ we are measuring the off-shell ME $\langle 0|\mathcal{H}_W|K\pi\rangle$, together with final state interaction effects. If one ignores such effects, then the chiral behavior we desire corresponds to the correlator being antisymmetric about the midpoint.

The correlator is related to the ME $\mathcal{M}$, for $t_K > 15$, by

$$C(t_\pi, t_K) = \mathcal{M}\ N_f (e^{-t_\pi m_\pi} Z_\pi / N_\pi)(e^{(t_K - 30)m_K} Z_K / N_K)$$

If the ME has the correct chiral behavior $\mathcal{M} \propto m_\pi m_K$, and we ignore variations in Z_π, etc., then the coefficient of the exponentials should be constant. In the figures this means that the extrapolation of the exponential decay to $t = 30$ (30.5 for two timeslice operators) should not depend on m_π, m_K. But this is not a fair test for our range of m_q. A better approach is to compare the lattice correlators to those calculated in the vacuum insertion approximation (VIA) directly on the lattice. The VIA correlators automatically have the correct chiral behavior, apart from the variation of Z_π etc. with m_q. Comparison of the data with VIA removes this spurious variation. It also makes for simpler comparison between different calculations.

Finally I come to the pictures. In all of them, the y-axis is logarithmic, but I do not show the scale, as it is not relevant here. I should also stress that I am using the $O(g^0)$ transcription of continuum operators onto the lattice. Figure 1 shows the results for the lattice equivalent of the eight contraction in the ME:

$$\langle \pi^+ | O_1 | K^+ \rangle \equiv \langle \pi^+ | \bar{s}_a \gamma_4 \gamma_5 u_a \; \bar{u}_b \gamma_4 \gamma_5 d_b | K^+ \rangle$$

This LL8 contraction is shown for masses UU and SS, along with the VIA results. The bump evident for $t_\pi \approx t_K$ is mainly due to wrap-around effects allowed by the APBC, to which I will return. I note the following: 1. the exponential decays are clear; 2. the correlators are roughly antisymmetric; 3. VIA works well for SS; 4. VIA works poorly for UU. Since the VIA results correspond to the correct chiral behavior, the data appear to be growing too fast as $m_q \to 0$.

In Figure 2 the LL8 results for the operator $O_2 = \bar{s}_a \gamma_4 \gamma_5 u_b \; \bar{u}_b \gamma_4 \gamma_5 d_a$ are compared to those of O_1. In continuum VIA the ME of O_2 are 3 times smaller. For SS this is roughly true, but for UU it is clearly false. The growth of the O_2 ME at small m_q is completely inconsistent with the required chiral behavior. In fact, it is consistent with $\mathcal{M}$ independent of m_π, m_K.

Though I don't have space to show it here, all other LL8 channels show similar violations of VIA and, consequently, the wrong chiral behaviour. A typical example is $\bar{s}_a \gamma_4 u_a \; \bar{u}_b \gamma_4 d_b$. This operator has three links in each bilinear, and is zero in VIA, yet yields a clear signal. This signal is small for SS, but for UU it is $\approx 1/5$ of that for O_1. This violation of chiral behavior

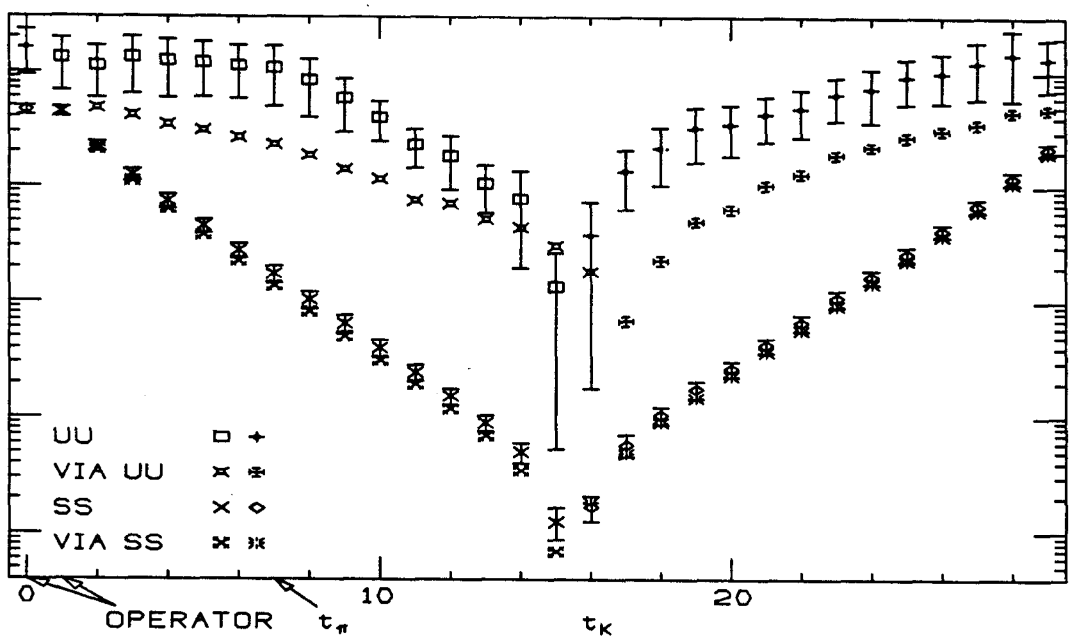

Fig 1. LL8 data for $\langle K | O_1 | \pi \rangle$. The errors in the VIA data have been removed for clarity. They are comparable to those on the data they approximate. Of the two symbols, the first is for positive data, the second for negative.

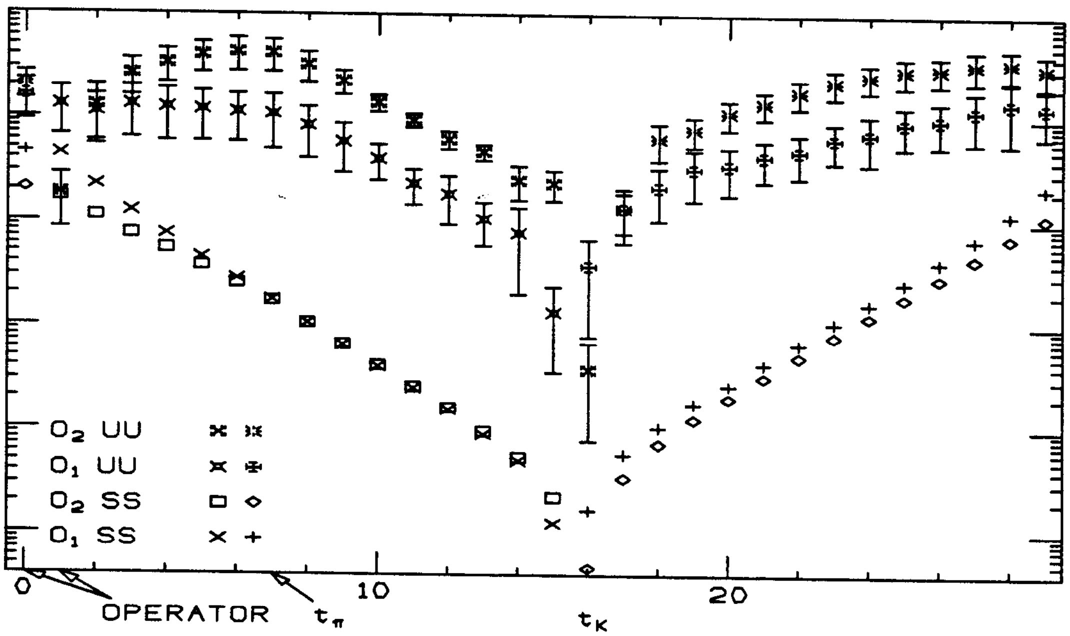

Fig 2. Comparison of $\mathcal{O}_1$ and $\mathcal{O}_2$ LL8 data. Error bars for SS data are about the size of the symbols.

and VIA is both good and bad. It is good because VIA gives a poor description of Kaon decays [7]. It is bad since the wrong chiral behavior can mean only two things: (a) we do not yet have small enough m_q; (b) the APBC effects are dominant. These wrap-around effects do not violate the WI, but do affect the argument leading from the WI to the chiral behavior.

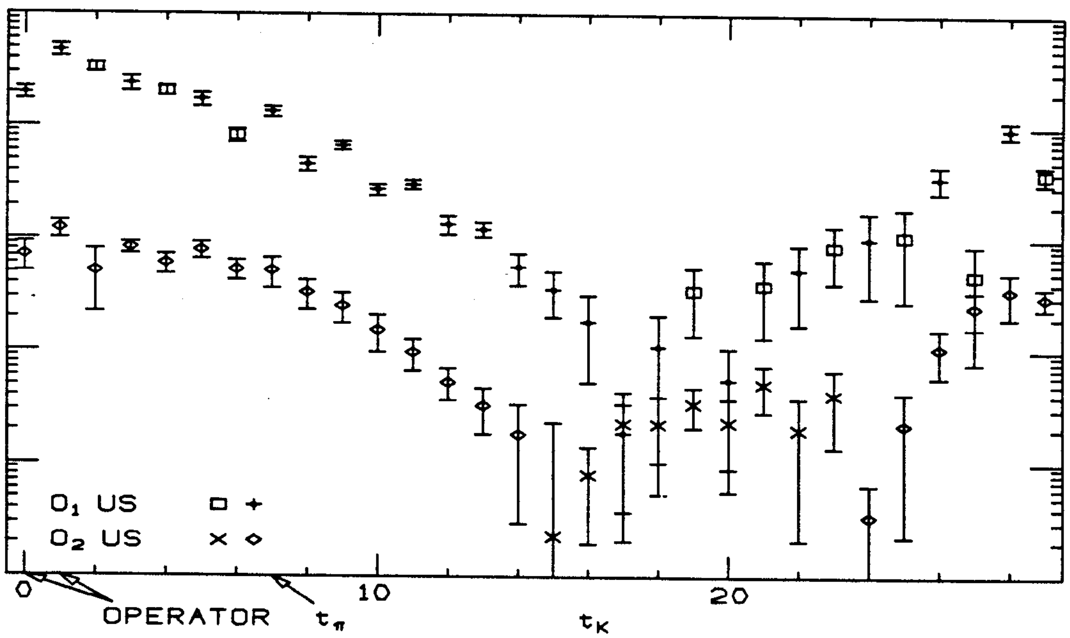

Fig 3. LLI data for $\mathcal{O}_1$ and $\mathcal{O}_2$ after subtraction.

Next I turn to the LLI contractions. Here we must make the subtraction, and this can only be done for the masses US. Figure 3 show the results for the eye contractions of operators $\mathcal{O}_1$ and $\mathcal{O}_2$, after subtraction. We want the correlator to be antisymmetric, and there are signs of this. However, the $\mathcal{O}_2$ data is too poor to extract a number, and the $\mathcal{O}_1$ data shows a dominant oscillatory behavior. This, we think, is due to wrap-around effects. So one of the staggered fermion CONs has really come home to roost. For what it is worth, the typical magnitude of the $\mathcal{O}_2$ ME *is* large enough to yield the $\Delta I = 1/2$ rule.

So much for the bad news. For the LR8I contractions we *can* extract some useful conclusions. Here we can do the subtraction for all m_q. We show results for the dominant part of the operators:

$$\mathcal{O}_5 = \bar{s}_a(1+\gamma_5)d_a \ \ \bar{q}_b(1-\gamma_5)q_b \ ; \ \mathcal{O}_6 = \bar{s}_a(1+\gamma_5)d_b \ \ \bar{q}_b(1-\gamma_5)q_a$$

Figure 4 shows the SS results for $\langle K|\mathcal{O}|0\rangle$ with operators $\mathcal{O}_5$, $\mathcal{O}_6$, S, and the VIA to $\mathcal{O}_6$. It is apparent that (a) VIA does very well; (b) the continuum VIA expectation that $\mathcal{O}_6 = 3\mathcal{O}_5$ works extremely well; and (c) the determination of the subtraction coefficient ρ can be done easily.

Figure 5 shows the SS results for $\langle K|\mathcal{O}_6|\pi\rangle$, $\langle K|\mathcal{O}_6^{sub}|\pi\rangle$ and their VIA values. This shows how the subtraction removes a large symmetric part to expose the antisymmetric residue. The data, however, agrees extremely well with VIA, and, though not shown, it remains true that the $\mathcal{O}_6 = 3\mathcal{O}_5$.

For the UU LL8I results VIA does less well. It falls above the data for $\langle K|\mathcal{O}_6|0\rangle$ by $10-20\%$. The $\langle K|\mathcal{O}_6|\pi\rangle$ data are shown in Figure 6. Here the VIA result is much cleaner than the actual data, the latter showing signs

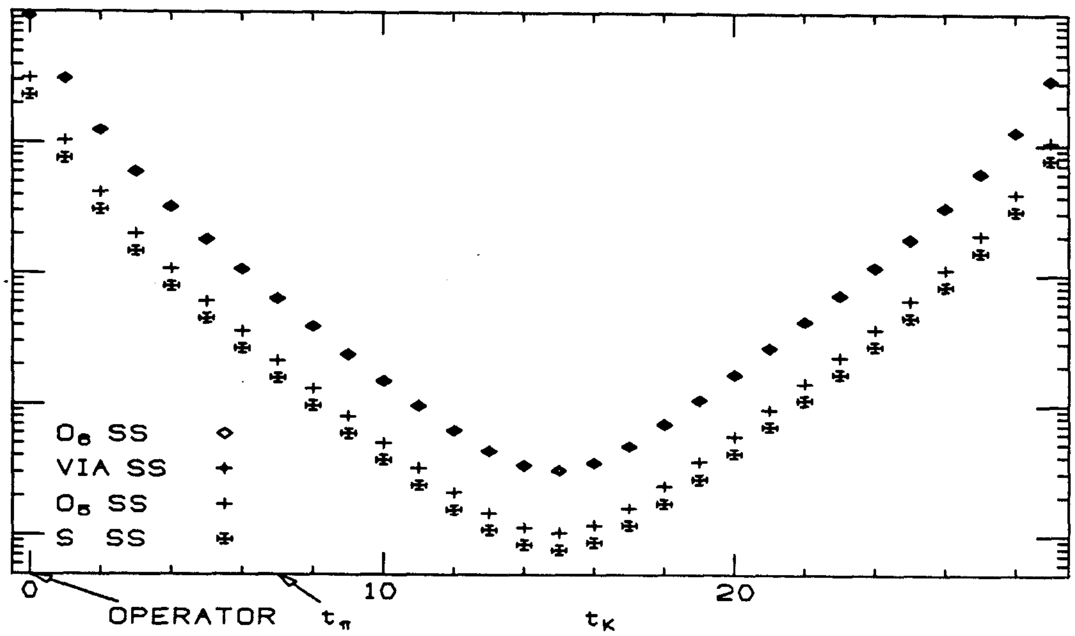

Fig 4. $\langle K|\mathcal{O}|0\rangle$ data for LR8I contractions.

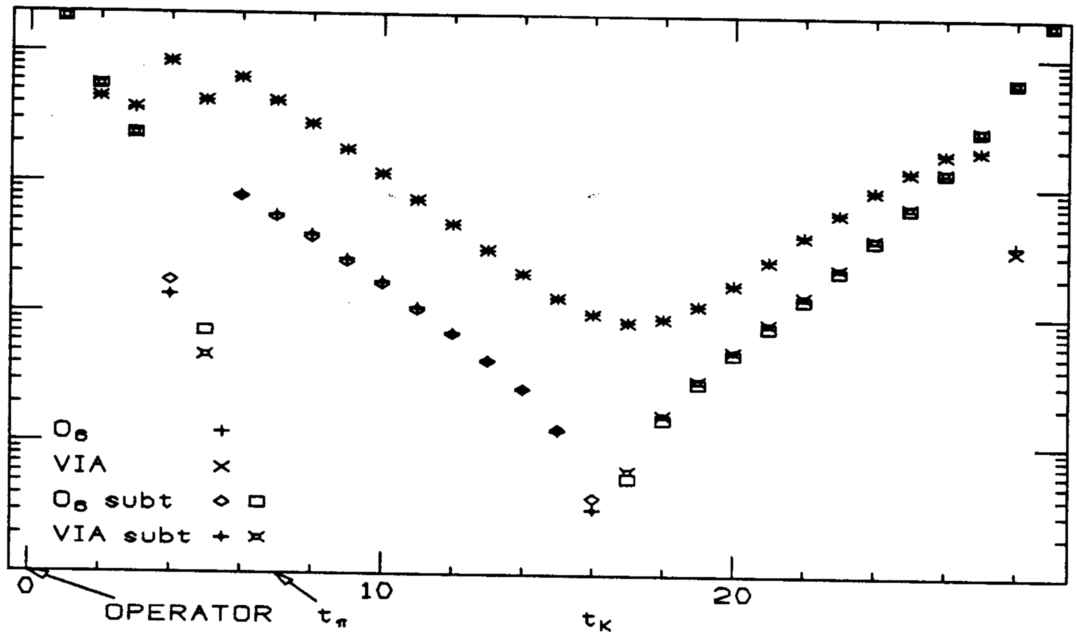

Fig 5. $\langle K|\mathcal{O}|\pi\rangle$ data for LR8I, with and without subtractions.

of oscillations again. Nevertheless, the data are much better than the LLI, with the antisymmetry being clear. Because of the oscillations, it is hard to extract quantitative conclusions, but it is clear that the data gives a ME substantially smaller than that in VIA. This is in striking contrast to the LL8 (and the LR8) data. Furthermore, the chiral behavior is fine, if anything a little to soft.

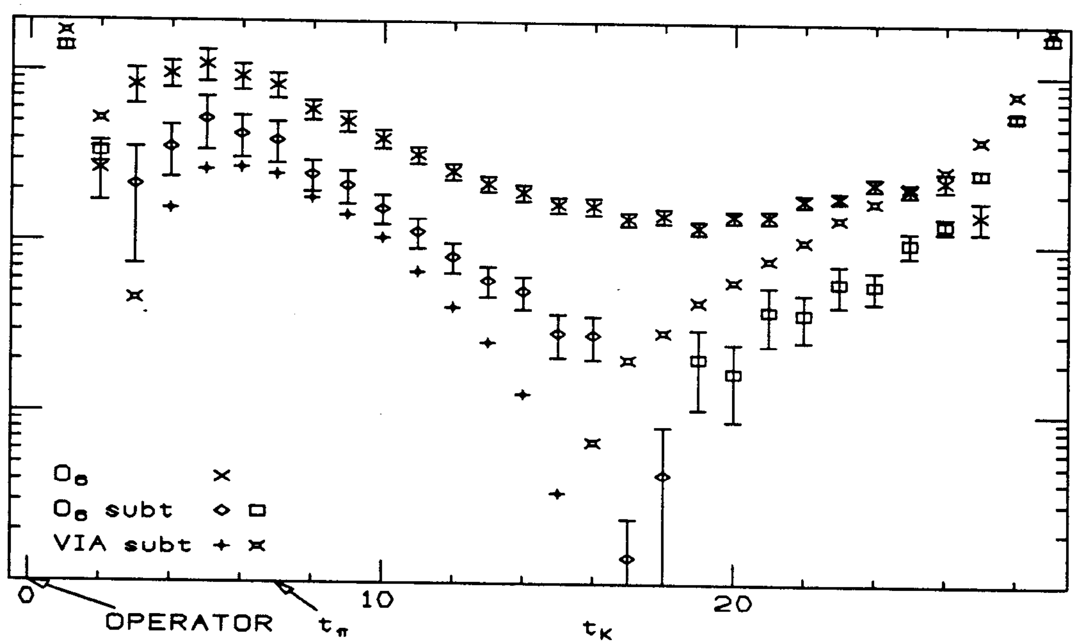

Fig 6. Same as Fig 5, but for UU.

The final result I want to discuss is for the LR8 contractions, though I have no space for pictures. These are dominated by a symmetric part corresponding to $\mathcal{M}$ being independent of the meson masses. We find that the relation $3\mathcal{O}_5 = \mathcal{O}_6$ works well for all masses. VIA works very well for SS, but lies significantly below the UU data.

CONCLUSIONS

The two major problems with our study, in purely numerical terms, are poor statistics and wrap-around contributions. The latter occur for all types of lattice fermion, but are exacerbated by the use of staggered fermions. These two problems conspire to make an extraction of even a qualitative result on the $\Delta I = 1/2$ rule impossible. Together with the lack of a perturbative operator mixing calculation, they also do not allow even a semi-quantitative result for the B parameter of $K^0 - \overline{K^0}$ mixing. This last point is true for both LL and LR operators.

We can, however, make some general comments. For both LL8 and LR8, the data agree with VIA at large m_q, but exceed VIA for small m_q. Clearly, fluctuations are damped at large m_q (recall that VIA is exact on a single configuration). An optimistic interpretation is that there is region for small m_q where VIA is violated, but in such a way that the data has the correct chiral behavior. We need more low mass data to check this. A more pessimistic possibility is that the bad chiral behavior is intrinsically related to our use of APBC. The wrap-around contributions cannot invalidate the WI, but can remove the connection between the WI and the chiral behaviour of the ME. Even assuming the optimistic scenario, one should not forget the caveat raised here by Mütter. Some of the fluctuations at small m_q are artifacts of the quenched approximation. They will be damped out by the fermion determinant in the full theory.

I have placed much stress on the utility of a comparison with VIA. Of course, VIA cannot work for all values of the lattice spacing, because the anomalous dimensions of the true operators and their approximants differ. Nevertheless, at the present stage, when the calculations are not quantitative, this is a small effect. The usefulness of VIA is most clear for the LR8 contractions. One expects these to be enhanced by factors $\propto 1/m_s^2$ relative to LL8. But what is m_s? In this calculation, and in other calculations in the quenched approximation, one finds $m_s \approx 50\text{MeV}$. This is at least a factor of 3 smaller than the continuum $m_s(\mu = 1\text{GeV})$. This discrepancy *cannot* be explained by saying that the appropriate scale is not $\mu = 1/a$ but $\mu = (\Lambda_{mom}/\Lambda_{lat})(1/a)$ [13]. The logarithmic scaling of masses is too slow. The only reasonable explanation is that the small m_s is due to the quenched approximation. In essence, one fixes the combination $m_s\langle\overline{\psi}\psi\rangle$ to be correct. Since $\langle\overline{\psi}\psi\rangle$ is too large in the quenched approximation, m_s must be too small. Returning to the LR8I, this means that naively extracted ME are ≈ 10 times too large. The correct approach is to compare the calculation to the VIA, which can then be evaluated in the continuum with the correct m_s. The same comments apply to the LR8.

The continuum VIA to the LR8I has been worked out in references [7] and [8]. Using $m_s(\mu \approx m_c) = 125\text{MeV}$, one finds roughly that $\epsilon'/\epsilon = .002\ (\tilde{c}_6/.1)$. Here, $\tilde{c}_6$ is the imaginary part of the Wilson coefficient evaluated at $\mu \approx m_c$. Its value is controversial, but is not likely to be much larger than .1, though it could be smaller. The electromagnetic penguins also contribute to ϵ', and in VIA reduce it by about 20%. There are also isospin violating effects which combine to reduce ϵ' by another $30-40\%$ [14]. These numbers should be compared to a present experimental limit of $\approx .005$, and a future sensitivity of .001. Our results suggest that a bad situation may be worse still. If the penguin contribution is smaller than VIA, yet the electromagnetic penguin is larger than VIA, ϵ' will be further reduced. A more detailed discussion will be given in [2].

In summary, progress has been made towards the calculation of ϵ'. For this the use of staggered fermions is essential. To improve this calculation, and to extract values for the decay rates of Kaons, ϵ and the B parameter, we need to do the following. (1) Replace APBC with fixed BC; (2) Increase the number of small quark masses used; (3) Increase the statistical sample; (4) Check asymptotic scaling; (5) Increase the size of the infrared cut-off; (6) Include dynamical fermions; and, last but not least, (7) Do the perturbative operator mixing calculation.

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