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Toponium-Z^0 Mixing*

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ABSTRACT

The subject of Z^0-toponium interference is briefly reviewed. The qualitative features of the Z^0 mixing with one $t\bar{t}$ state are discussed. Effects of mixing with the full $t\bar{t}$ spectrum, of the smearing due to beam spread, and of different potentials, are then shown.

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1. Introduction

In this talk, I would like to discuss a particular aspect of heavy quark phenomenology, of relevance when $2m_t \approx m_{Z^0}$. Why do we expect toponium-Z^0 mixing to be of interest? From the absence of flavor-changing neutral currents in B decay, we are confident that the bottom quark must have an as-yet-unobserved partner. Experimentally, $m_t < 23$ GeV is excluded, while UA1 data suggests a top quark of mass between 30 and 50 GeV. It appears quite possible that $t\bar{t}$ bound states will have masses near that of the Z^0 (93 GeV), and thus vector ($J^{PC} = 1^{--}$) $t\bar{t}$ states (henceforth V) could be nearly degenerate with the Z^0. We expect the effects of $V - Z$ mixing to be seen soon, at both SLC and LEP.

I first present a qualitative way of understanding the nearly complete destructive interference of the Z boson with one V state. I then present the results of the Z mixing with the full spectrum of toponium states (when the Z and V are nearly degenerate); I show the effects of finite beam width on the cross sections and asymmetries. I then display the striking effects that remain if the Z is relatively far away from the V $(10 - 20$ GeV), and conclude by contrasting the effects of the Richardson[4] potential, the Cornell[5] potential, and a non-standard Higgs sector.

This talk is based on work done in collaboration with Fred Gilman and Gregory Athanasiu.[1-3]

2. Mixing of the Z^0 with a Single $t\bar{t}$ State

I shall discuss the simplified case of only two states, the Z and one vector ($J^{PC} = 1^{--}$) toponium resonance, V. I begin with a qualitative argument to show that the interference is indeed destructive. Let us consider the process $e^+e^- \to f\bar{f}$, where f denotes an arbitrary final fermion state. This occurs predominantly as $e^+e^- \to Z_0 \to f\bar{f}$, while another contribution is $e^+e^- \to Z_0 \to V_0 \to Z_0 \to f\bar{f}$ (for now, we neglect the small contributions due to γ

couplings). The first term has an amplitude proportional to the propagator $1/(s-M^2_{Z_0}+i\Gamma_{Z_0}M_{Z_0})$, and therefore to $1/i\Gamma_{Z_0}$ on the peak of the Z_0 resonance. If, for simplicity, we choose the Z_0 and V_0 resonances to be degenerate, the amplitude from the second contribution is similarly proportional to $1/(i\Gamma_{Z_0}i\Gamma_{V_0}i\Gamma_{Z_0})$. Thus we have a relative minus sign between these two amplitudes, i.e., destructive interference.

We can extend this argument by replacing the Z_0 propagator by the iterated series

where the solid line denotes the Z_0 and the double line the V_0. Using a phenomenological $Z_0 - V_0$ coupling a, we get the amplitude to be proportional to

$$\frac{1}{s-M^2_{Z_0}}+\frac{1}{s-M^2_{Z_0}}\cdot\left(a\cdot\frac{1}{s-M^2_{V_0}}\cdot a\cdot\frac{1}{s-M^2_{Z_0}}\right) \tag{2.1}$$

$$+\frac{1}{s-M^2_{Z_0}}\cdot\left(a\cdot\frac{1}{s-M^2_{V_0}}\cdot a\cdot\frac{1}{s-M^2_{Z_0}}\right)^2+\cdots=\frac{s-M^2_{V_0}}{(s-M^2_{Z_0})(s-M^2_{V_0})-a^2}\ .$$

(Here, and often in what follows, we will use $M^2_{Z_0}$ to represent the full expression $M^2_{Z_0}-i\Gamma_{Z_0}M_{Z_0}$.) For energies a few GeV away from a V_0 resonance, $(s-M^2_{Z_0})(s-M^2_{V_0})$ is large compared to a^2; as expected, we recover the Z_0 propagator. On the V_0 resonance we get zero for the amplitude—thus we have complete destructive interference.

The amplitude exactly vanishes only if we make some simplifying assumptions:

(1) I have ignored the virtual photon contribution to the process $e^+e^-\rightarrow\mu^+\mu^-$. This is a good approximation, since the photon, by definition, contributes an R-value of about* one, while the R-value on the Z_0 peak is 200. (Note that

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⋆ The contribution is not exactly one, because R-value is given by the actual photon cross section divided by the QED point cross section with α defined at the electron mass scale.

on the Z_0 peak, the Z amplitude is imaginary while that of the photon is real, so that there is no $\gamma - Z$ interference. However, in general we must compute $Z\gamma V$ mixing. The effect of the photon is small enough to be negligible, except in the determination of the asymmetry parameters.)

(2) I have implicitly assumed that the width of the V_0 is zero. The expression $s - M_{V_0}^2$ really represents $s - M_{V_0}^2 + iM_{V_0}^2\Gamma_{V_0}$ which can only be zero (for a physically allowed value of s) if $\Gamma_{V_0} = 0$. This is also a good approximation, since the expected width of a $t\bar{t}$ $1S$ state (using the Richardson potential) is about 100 keV, compared to Γ_Z=2.7 GeV.

(3) Finally, I have ignored the "direct" couplings of the V_0, that is, the V_0 coupling to fermions through the photon instead of through the Z_0. This approximation is analogous to, and comparable in magnitude with, the second one.

We can also determine the mass and width of the physical eigenstates by diagonalizing the mass matrix (see Ref. 1 for details). The shifts in M_Z and in Γ_V (those in M_V and Γ_Z, respectively, are equal and opposite to the shifts shown) are shown in Fig.1 for the Z mixing with a $1S$ toponium state described by the Richardson potential. While the mass shifts, and shifts in Γ_Z as well, are insignificant, the shift in Γ_V is very impressive. The dashed line indicates the width of the toponium state without mixing.

3. What We Will See: Many States, Smearing, and All That

Considering the Z interfering with the full set of toponium states below threshold we obtain cross sections such as that shown in the first part of Fig. 2. Of course, real machines, such as SLC and LEP, will not resolve these very narrow spikes; we must convolute the curves with a Gaussian (with width related

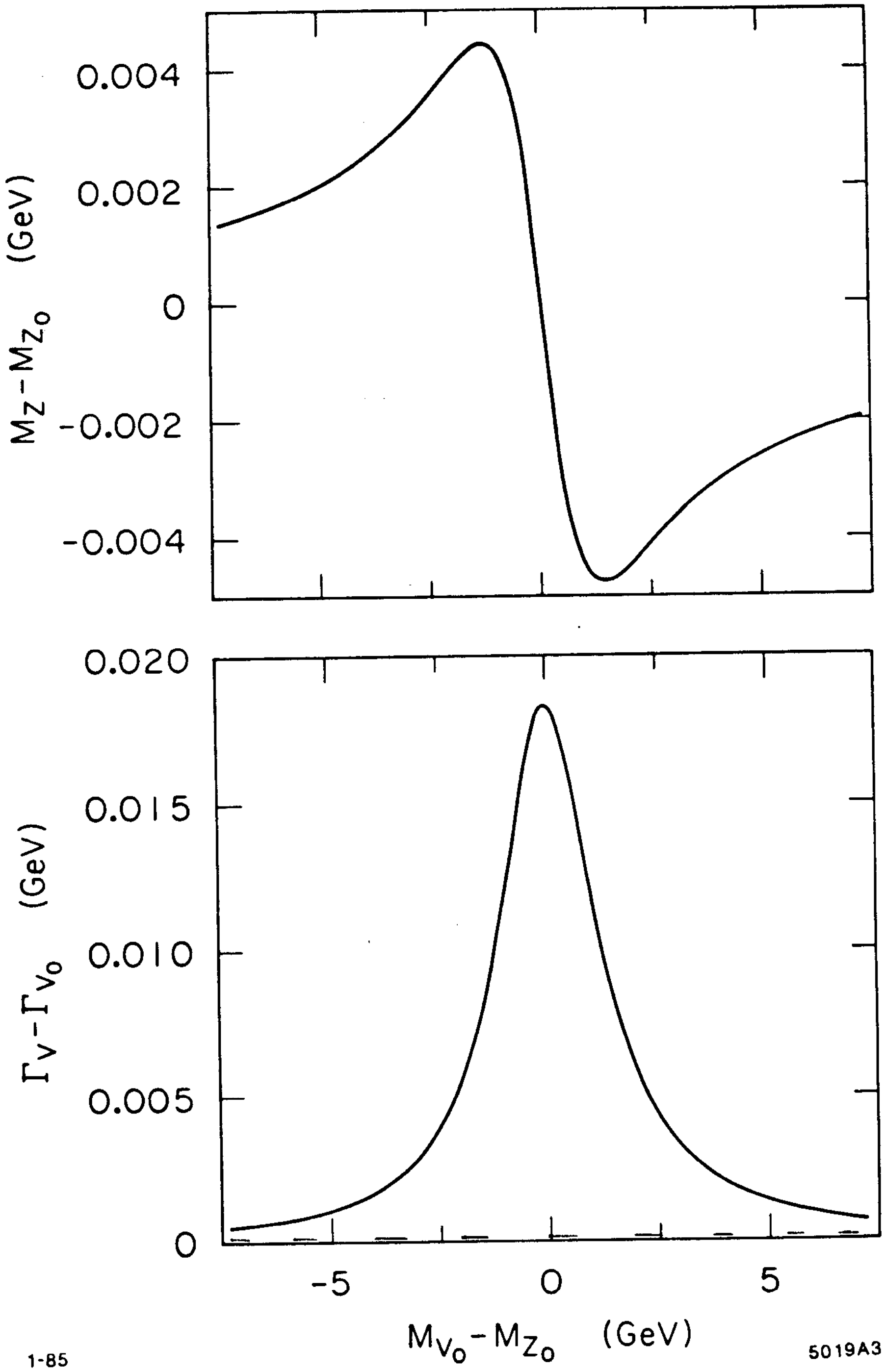

Figure 1. Changes in M_Z and Γ_V due to mixing of the Z state with the ground state of toponium as a function of the mass difference of the bare states (M_{Z_0} is held fixed at 93 GeV, while M_{V_0} is varied; the subscript 0 denotes unmixed states).

to the beam spread) in order to approximate what will be measured. We have

$$\sigma_{SMEARED} = \int_{-\infty}^{\infty} dw' \sigma(w') 1\sqrt{2}\sigma e^{-w^2/2\sigma^2} \tag{3.1}$$

where $\sigma = \sqrt{2}\sigma_{BEAM}$. In the second part of Fig. 2, I show R for the Z alone, and for the Z interfering with toponium states, convoluted with Gaussians appropriate to σ_{beam}= 40 MeV and 100 MeV. LEP is expected to run (without wigglers) at the former beam width; SLC is expected to achieve the latter, and perhaps with special effort, the former.

I next remark that even for a V relatively far away from the Z, the enhancement due to mixing should be quite noticeable (see Fig. 3). The height of the peak does not decrease, though its width does. The smeared height is therefore greatly reduced, but should be compared to the also much reduced background due to the Z. Note that to get comparable statistics to those obtained on the Z, one must run for far longer.

I now present smeared polarization and forward-backward asymmetries for various values of M_{V_0}. These are found by calculating the cross sections (for each individual helicity configuration), smearing them, and then taking the appropriate differences and ratios. Since the asymmetries also crucially depend on the $ZV\gamma$ interference, the results do not seem to have a simple qualitative explanation. In Fig. 4 I show the asymmetries; the effects are in fact more striking for V moderately far away from Z.

All the results I have shown so far used the Richardson potential. I shall briefly show the effects of using the Cornell potential, and the Richardson potential combined with a non-standard Higgs sector. Consider the two-Higgs doublet model of Glashow, Weinberg and Paschos,[6] where one Higgs couples to up-type quarks, and one to down-type. There is a neutral-Higgs (H_0) exchange contribution to the toponium potential, where the H_0 coupling is enhanced by the vacuum-expectation-value ratio ξ/η (ξ being the VEV of the Higgs coupling to

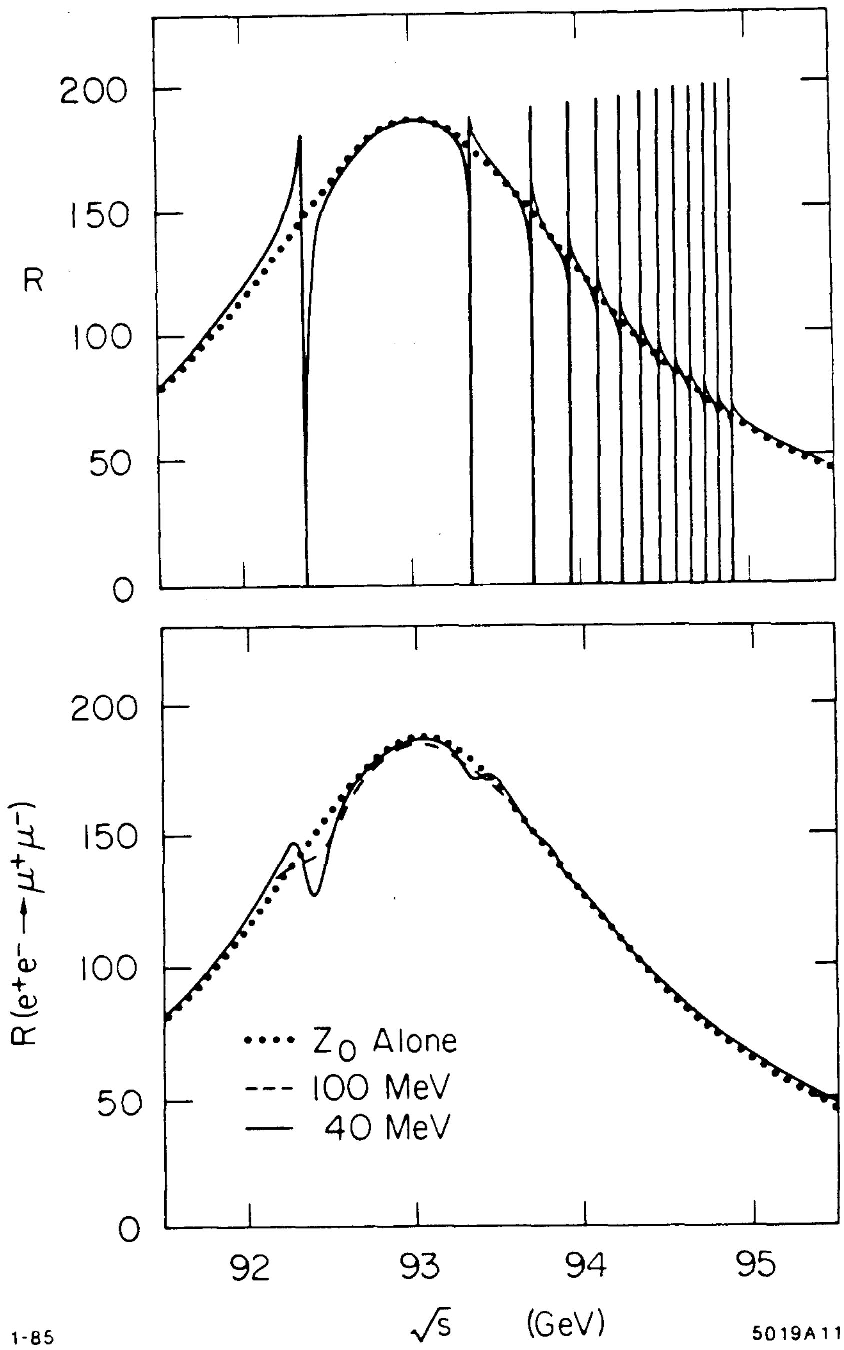

Figure 2. (top) $R(e^+e^- \to \mu^+\mu^-)$ for several toponium states mixing with the Z (Richardson potential, $m_t = 47$ GeV). The dotted line is the Z_0 alone.

(bottom) $R(e^+e^- \to \mu^+\mu^-)$, smeared, for various expected beam widths.

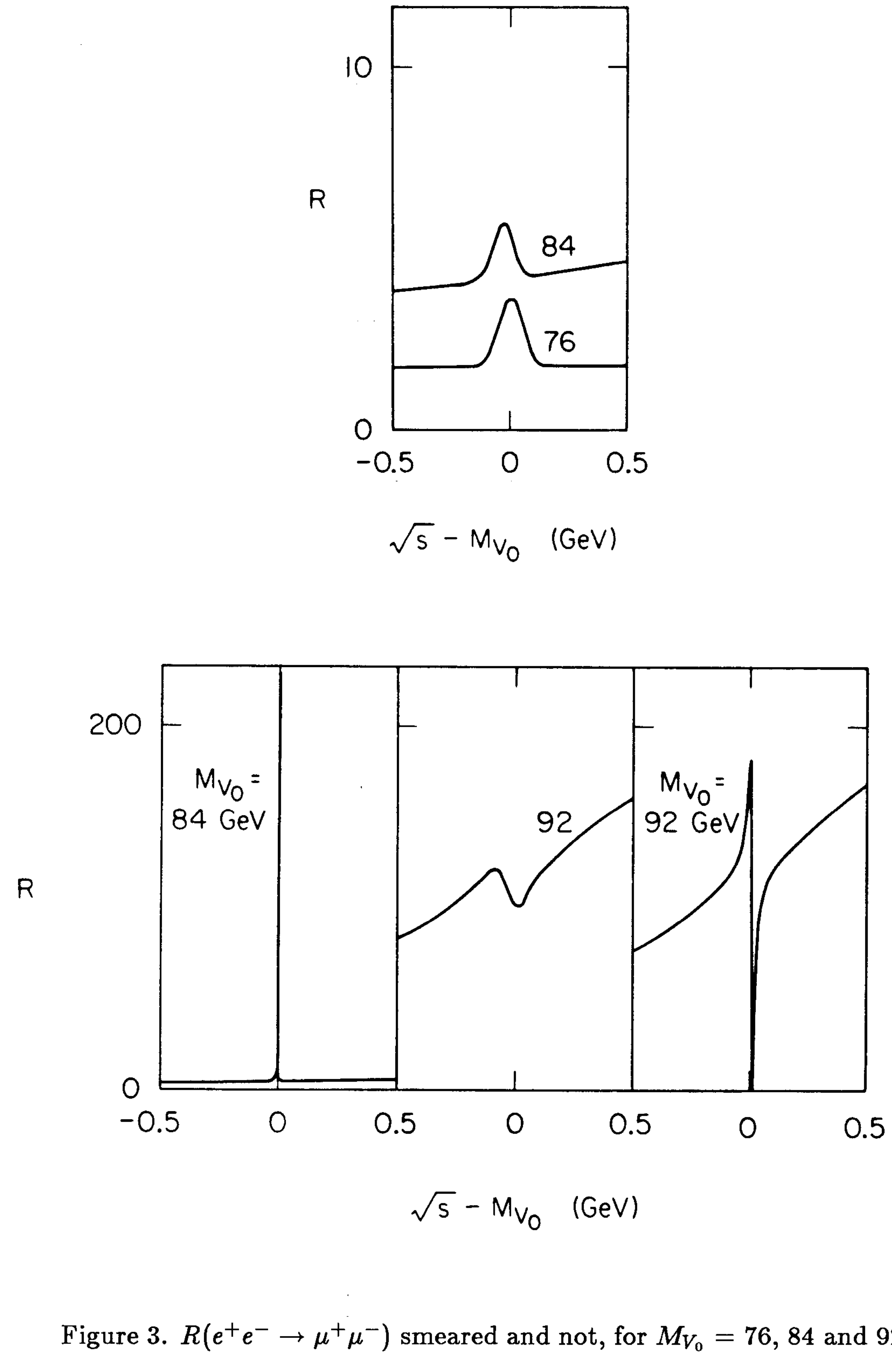

Figure 3. $R(e^+e^- \rightarrow \mu^+\mu^-)$ smeared and not, for $M_{V_0} = 76$, 84 and 92 GeV.

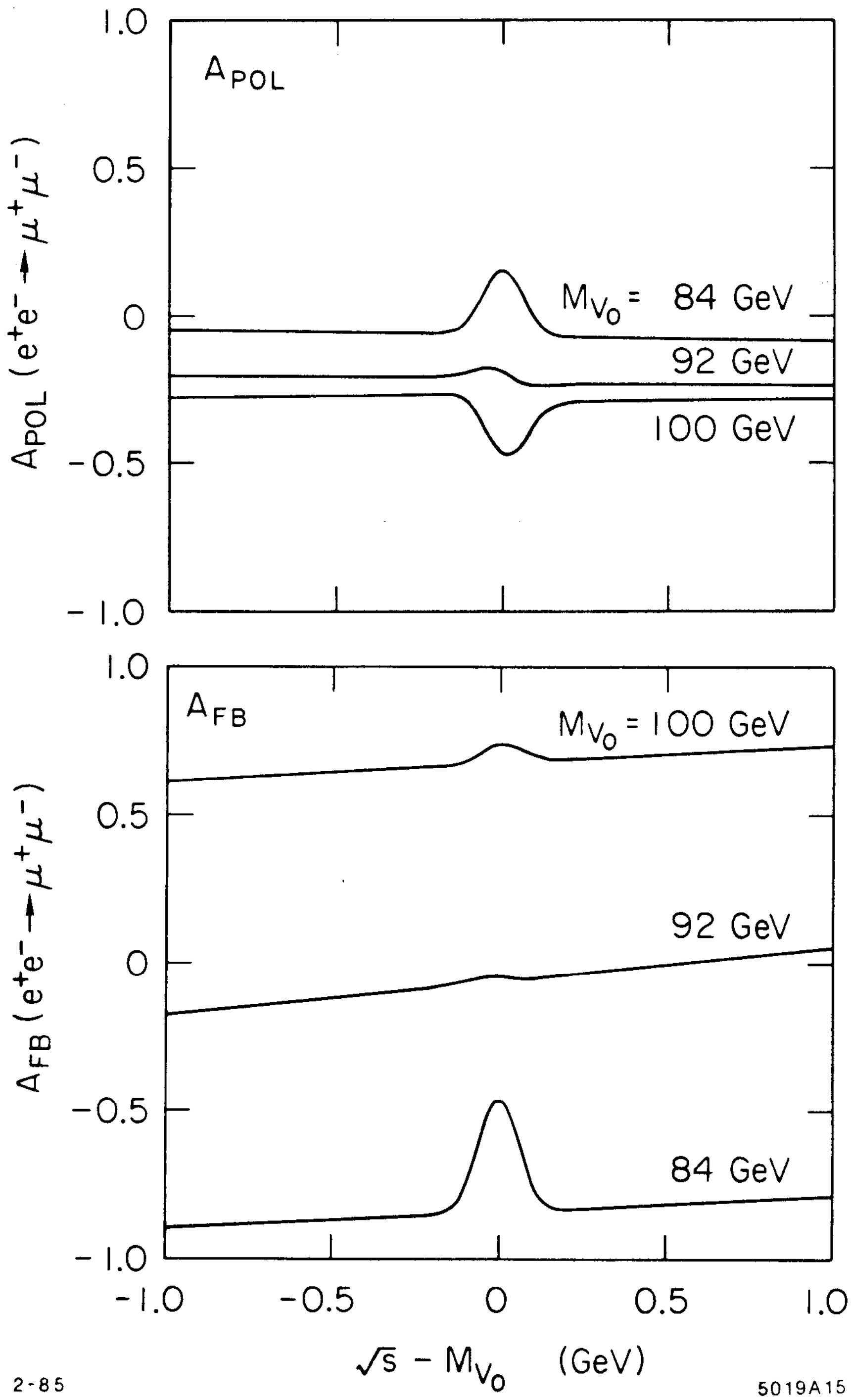

Figure 4. A_{pol} and A_{fb} for three different values of M_{V_0}.

down type quarks and η to up-type). The extra contribution is an attractive Yukawa, in momentum space

$$-\left(\frac{\xi}{\eta}\frac{gm_t}{2M_W}\right)^2 \frac{1}{m_H^2+q^2} \quad \text{or} \quad -\left(\frac{\xi}{\eta}\frac{gm_t}{2M_W}\right)^2 \frac{e^{-rm_H}}{4\pi r} \tag{3.2}$$

in coordinate space. This addition has the effects of increasing the wavefunctions at the origin, since it pulls in the wavefunctions, and of lowering states (increasing binding energies); it changes the level spacings, since it affects the lowest lying states the most. Finally, if the Higgs term is strong enough⋆ it has a very curious effect—it causes the 2S state to lie below the 1P. This effect does not happen for any standard quarkonium potential, and is related[7] to the fact that $\Delta V(r) < 0$ for the Higgs potential and not so for any standard quarkonium potential. In Fig. 5, I show $R(e^+e^- \to \mu^+\mu^-)$, smeared ($\sigma_{\text{beam}} = 40$ MeV), for Richardson alone, Cornell alone, and Richardson with Higgs† . Note the qualitative similarity between the second and third figure.

In summary, we have seen that toponium and the Z_0 almost completely destructively interfere. Toponium states pick up a large width from mixing—the 1S state, with a bare width of 100 keV, can acquire a width of as much as 20 MeV (using the Richardson potential). While the beam widths of machines such as SLC and LEP will greatly blur the sharp spikes that we find, effects will be visible as wiggles in cross sections and asymmetry parameters. The exact potential for toponium (and thus exactly what we will see) is not very well known. The Higgs (in a two-Higgs model) can have noticeable effects, but it may be hard to distinguish these effects from those of different potentials; the 2S-1P level inversion is a possible qualitative difference, if the Higgs couplings are rather large.

⋆ that is, ξ/η equals about 5, if we are using the Cornell potential, or 10, for Richardson.

† the parameters have been chosen to be dramatic; they are all but excluded by $B\bar{B}$ mixing[3]

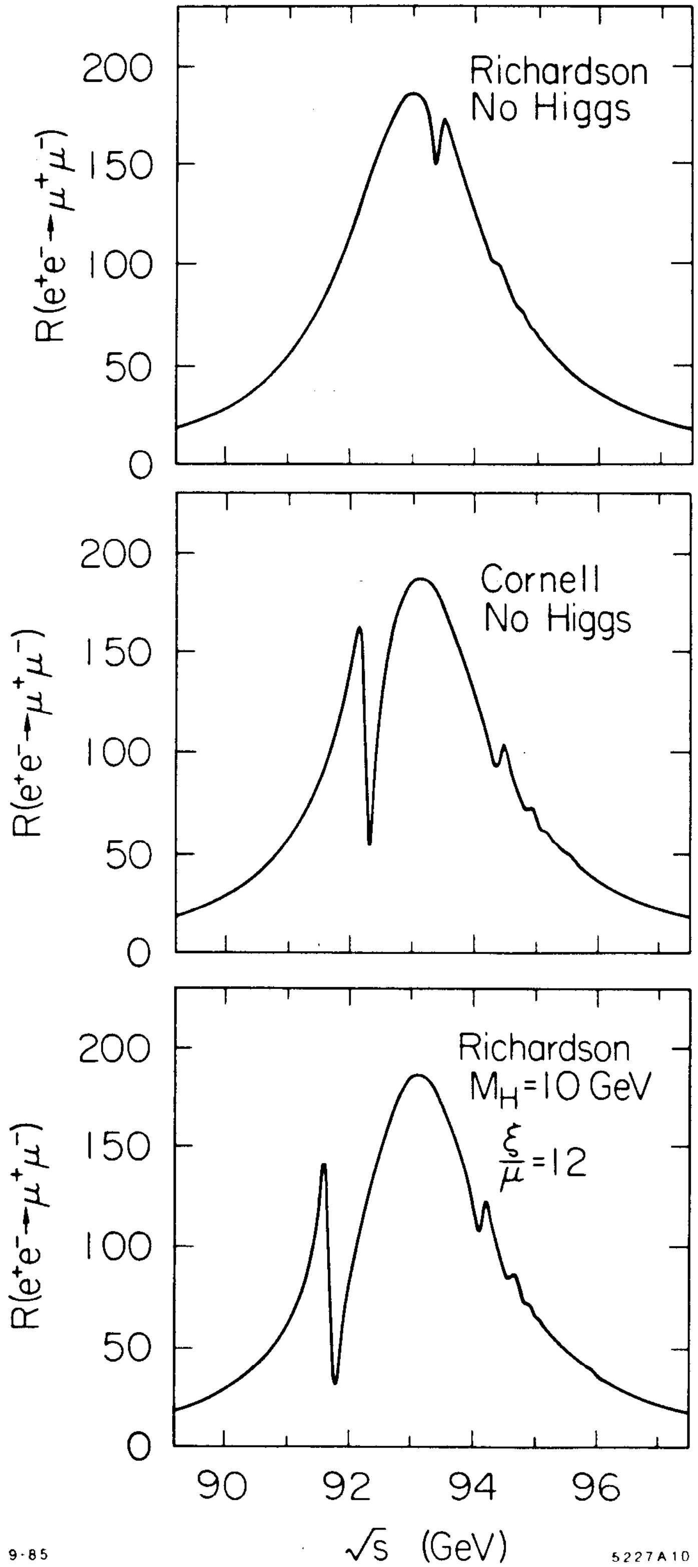

Figure 5. Effects of varying quarkonium potential.

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