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Upper Bounds on Neutrino Masses*

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ABSTRACT

Using a recent experimental bound on τ-decay into three charged leptons and a weak assumption concerning a general "see-saw" mechanism for neutrino masses, we show that both ν_μ and ν_τ must be lighter than 65 eV. If the "see-saw" is driven by a right-handed W-boson or by a "horizontal" gauge boson, they must be heavier than 50 PeV.

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Left-handed neutrinos are either massless or much lighter than the corresponding charged leptons. If they are *exactly* massless, there must be some fundamental reason (presumably a symmetry principle) which prevents them from acquiring masses when all other quarks and leptons have masses. This can happen in theories in which right-handed neutrinos do not exist and Majorana masses are not allowed (e.g. the simplest version of SU(5) grand unification). If the left-handed neutrinos are *exactly* massless, no cosmological or phenomenological difficulties seem to arise.

If the left-handed ν_e, ν_μ and ν_τ are not massless, they are extremely light. The direct experimental bounds are ~40 eV, 250 keV and 70 MeV respectively[1]. If the light neutrinos are *stable* we have, in addition, a cosmological bound stating that[2,3] $\Sigma m(\nu_i) < 65\ eV$.* A *stable* ν_μ or ν_τ with a mass larger than 65 eV would contribute too much to the matter density of the universe.

The only way of avoiding this limit and having ν_μ and ν_τ masses between 65 eV and their present experimental upper bounds is if ν_μ and/or ν_τ are *unstable*[4]. In such a case, there is another cosmological bound, demanding that the unstable neutrinos decay sufficiently fast and relating the lifetime of a neutrino to its mass. The bound is[4,5]:

$$[m(\nu)]^2 \cdot \tau(\nu) \leq 2 \times 10^{20}\ eV^2 \cdot sec \tag{1}$$

The left-handed ν_μ or ν_τ could be heavier than 65 eV only if they decay with a lifetime which obeys this inequality.

The possible final states for the decay of such an unstable neutrino ν_i are:

$$\nu_j + \gamma\ ;\ \nu_j + \gamma + \gamma\ ;\ \nu_j + scalar\ ;\ \nu_j + \nu_k + \nu_\ell\ ;\ \nu_j + e^+ + e^-. \tag{2}$$

Here ν_j, ν_k, ν_ℓ represent any neutrino or antineutrino lighter than ν_i; the scalar

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$\star$ The actual numerical value of this cosmological bound has been quoted in the literature to be anywhere between 40 eV and 100 eV. The value quoted here is based on the analysis of reference 3, assuming that the age of the universe is at least 10^{10} year.

can be a Goldstone particle, a pseudo-Goldstone or a light Higgs; the $\nu_j e^+ e^-$ decay requires $m(\nu_i) \geq 2m(e)$ and is consequently allowed only for $i = \tau$.

Within the standard model, the above decay modes have been shown to be either forbidden or extremely slow[6,7,8]. They do not obey the cosmological bound on the ν lifetime.** *Outside* the standard model the radiative ν-decays are still hopelessly slow[9] and the scalar decay mode can obey the cosmological bound only when a specific "Majoron" scheme with extremely artificial couplings is introduced[10,11].

The last and only "hope" for a neutrino mass above 65 eV therefore relies on the decay of such a neutrino into three lighter neutrinos, or into $\nu e^+ e^-$, via the exchange of a vector boson or a scalar field which lies *outside the standard model.* The only candidates could be a "horizontal" flavor-changing gauge boson H^0 or an SU(2)-triplet Higgs field[12] Δ_L carrying two units of lepton number and coupling to $\nu_L \nu_L$ rather than to $\bar{\nu}\nu$.

In this paper we consider these possibilities and show that they are either completely excluded or extremely unlikely. In particular, we conclude that ν_μ *must* be[5] lighter than 65 eV, that ν_τ *cannot* have a mass between 65 eV and 900 keV and that it could have a mass between 900 keV and 70 MeV only if an unlikely pattern of Majorana masses exists.

Our overall conclusion is that the three known left-handed neutrinos are almost certainly lighter than 65 eV.

For the sake of definiteness, we first consider the three-neutrino decay within the framework of the minimal version[13] of the Left-Right Symmetric (LRS) extension of the standard model. We will then show that our results are much more general. In the minimal LRS model we have Higgs fields ϕ, Δ_L and Δ_R transforming under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ like the the $(\frac{1}{2}, \frac{1}{2})_0, (1,0)_2$ and

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** In the case of the $e^+ e^- \nu$ final state, the cosmological bound could actually be obeyed within the standard model, but the required decay rate would then lead to other unacceptable cosmological consequences (references 7, 8).

$(0,1)_2$ representations, respectively. The ϕ field is essentially the standard-model Higgs. Its vacuum expectation values are k, k' and they are responsible for the masses of W_L, quarks and leptons. The neutral components of Δ_L and Δ_R obtain v.e.v's v_L and v_R. It is always assumed that $|v_R| \gg |v_L|$ and:

$$|v_R|^2 > |k|^2 + |k'|^2 \quad ; \quad |v_L|^2 < |k|^2 + |k'|^2 \tag{3}$$

Consequently, $M(W_R) > M(W_L)$ and the Weinberg mass relation is obeyed to a good approximation.

The v.e.v. of Δ_R contributes not only to the W_R-mass but also to a Majorana mass for the right-handed neutrino. It produces the well-known "see-saw" matrix[14] for neutrino masses:

$$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \tag{4}$$

where m_D is a neutrino Dirac-mass due to $< \phi >$, M_R is a Majorana mass due to $< \Delta_R >$ and the zero represents the negligible contribution of $< \Delta_L >$. Assuming $m_D \sim m(\ell), M_R \sim M(W_R)$ we get:

$$m(\nu_{\ell L}) \sim \frac{m_D^2}{M_R} \sim \frac{m^2(\ell)}{M(W_R)} \; ; \; m(\nu_{\ell R}) \sim M_R \sim M(W_R) \tag{5}$$

where (ν_ℓ, ℓ) is an $SU(2)_L$-doublet of leptons.

We have a definite lower bound[15] for the W_R mass: $M(W_R) > 1.7$ TeV. However, there are good reasons to believe that $M(W_R)$ is actually *at least* in the $5 - 10$ TeV range, possibly much higher[16].

A Higgs field Δ_L^0 coupling to $\nu_L \nu_L$ must carry $I_{3L} = -1$ and belong to an $SU(2)_L$-triplet together with a Δ_L^+ and a Δ_L^{++}. The masses of the three Higgs particles of the Δ_L-triplet must be almost degenerate[17]. The Higgs potential[13] contains terms of the form $\Delta_L^2 \Delta_R^2$, leading to Δ_L masses of order v_R. Any mass-splitting *within* the Δ_L-triplet must be due to $< \phi >$. For a general Higgs potential we expect:

$$\left| \frac{m(\Delta_L^0) - m(\Delta_L^+)}{m(\Delta_L)} \right| \sim O \left[\frac{< \phi >}{< \Delta_R >} \right]^2 \sim O \left[\frac{M(W_L)}{M(W_R)} \right]^2 < 2.5 \times 10^{-3} \tag{6}$$

and similarly for the $\Delta_L^{++} - \Delta_L^0$ splitting. In some special cases, the ratio may be larger (*e.g.* $O\left[\frac{M(W_L)}{M(W_R)}\right]$), but still much smaller than one. It is therefore natural† to assume that, within a few percents, $\Delta_L^{++}, \Delta_L^+$ and Δ_L^0 are degenerate.

The Δ_L-exchange contribution to the amplitude for $\nu_i \to \bar{\nu}_j \nu_k \nu_\ell$ is proportional to

$$\frac{h_{\Delta ij} h_{\Delta k\ell}}{\left[m(\Delta_L^0)\right]^2} \tag{7}$$

where $h_{\Delta ij}$ is the unknown coupling constant in the Yukawa term $h_{\Delta ij} \Delta_L \nu_i \nu_j$. In the case of ν_μ, there is only one decay channel: $\nu_\mu \to \bar{\nu}_e \nu_e \nu_e$. The Δ_L^0 contribution to this decay mode is similar[5] to the Δ_L^{++} contribution to the decay $\mu \to e^+ e^- e^-$. For ν_μ and/or ν_τ masses above 65 eV we may safely neglect the generalized leptonic Cabibbo angles. This follows from neutrino oscillation experiments which lead to strong upper bounds[19] on these angles, for sufficiently large masses. Neglecting the generation mixing, we must have equal $\Delta_L^0 \nu_i \nu_j$ and $\Delta_L^{++} \ell_i \ell_j$ couplings, where ν_i, ℓ_i are corresponding leptons in the same generation. Since the Δ_L^0 and Δ_L^{++} masses are approximately equal, one concludes that[5], within a few percents:

$$\frac{\Gamma(\nu_\mu \to \bar{\nu}_e \nu_e \nu_e)}{\Gamma(\mu^- \to e^+ e^- e^-)} = \left[\frac{m(\nu_\mu)}{m(\mu)} \right]^5 \tag{8}$$

Assuming no other contributions to ν_μ-decay and using the present experimental

† See reference 18 for a discussion of the unnatural possibility of a large Δ_L mass difference.

bound on $\Gamma(\mu \rightarrow 3e)$ one gets:

$$[m(\nu_\mu)]^5 \cdot \tau(\nu_\mu) \geq 1.2 \times 10^{46}\ eV^5 \cdot sec \tag{9}$$

Combining this with the cosmological bound on the lifetime of ν_μ we obtain:

$$m(\nu_\mu) \geq 400\ MeV \tag{10}$$

in clear conflict with the experimental bound $m(\nu_\mu) \leq 250\ keV$.

It is therefore clear that the decay $\nu_\mu \rightarrow \bar{\nu}_e \nu_e \nu_e$ cannot proceed via Δ_L^0 exchange and $m(\nu_\mu)$ cannot exceed the 65 eV limit[5].

Our "last chance" for a neutrino heavier than 65 eV within the LRS model is the possibility that $m(\nu_\tau)$ is anywhere between 65 eV and 70 MeV and ν_τ decays to three lighter neutrinos via Δ_L^0 exchange, obeying the cosmological upper bound on the lifetime.

In order to repeat a similar analysis to the one used above for ν_μ, we note that ν_τ could have six different decay modes to three neutrinos, via Δ_L^0 exchange. The allowed final states are:

$$\nu_\tau \rightarrow \bar{\nu}_\mu \nu_\mu \nu_\mu,\ \bar{\nu}_\mu \nu_\mu \nu_e,\ \bar{\nu}_\mu \nu_e \nu_e,\ \bar{\nu}_e \nu_\mu \nu_\mu,\ \bar{\nu}_e \nu_\mu \nu_e,\ \bar{\nu}_e \nu_e \nu_e. \tag{11}$$

Their relative strengths depend on the unknown Yukawa couplings of Δ_L. However, there are also six possible decay modes of τ into three charged leptons:

$$\tau \rightarrow \mu^+\mu^-\mu^-,\ \mu^+\mu^-e^-,\ \mu^+e^-e^-,\ e^+\mu^-\mu^-,\ e^+\mu^-e^-,\ e^+e^-e^-. \tag{12}$$

The same argument as in the case of ν_μ decay now yields:

$$\frac{\Gamma(\nu_\tau \rightarrow \bar{\nu}_\ell \nu_\ell \nu_\ell)}{\Gamma(\tau^- \rightarrow \ell^+\ell^-\ell^-)} = \left[\frac{m(\nu_\tau)}{m(\tau)}\right]^5 \tag{13}$$

where $\ell = e$ or μ and each of the two partial widths is summed over the corresponding six channels.

The ARGUS collaboration has recently reported[20] a new experimental upper bound for all channels of $\tau \to 3\ell$. They obtain: $BR(\tau \to \ell^+\ell^-\ell^-) \leq 3.8 \times 10^{-5}$. Using this bound together with the known mass and lifetime of τ we obtain:

$$[m(\nu_\tau)]^5 \cdot \tau(\nu_\tau) \geq 1.5 \times 10^{38} \; eV^5 \cdot sec \tag{14}$$

Combining this result with the cosmological bound on the lifetime, we obtain:

$$m(\nu_\tau) \geq 900 \; keV \tag{15}$$

At this stage we therefore conclude that, in order to obey the cosmological bounds, $m(\nu_\tau)$ must be either below 65 eV or between 0.9 and 70 MeV.

Until now, we have not invoked the "see-saw" mechanism. We now assume such a mechanism and use it in order to study the ratio $\frac{m(\nu_\tau)}{m(\nu_\mu)}$. In the simplest three-generation "see-saw" matrix, we would expect $M_{Re} \sim M_{R\mu} \sim M_{R\tau}$, predicting:

$$\frac{m(\nu_\tau)}{m(\nu_\mu)} \sim \left[\frac{m(\tau)}{m(\mu)}\right]^2 \tag{16}$$

An alternative reasonable guess for the Majorana masses of different generations would be:

$$\frac{M_{R\tau}}{M_{R\mu}} \sim \frac{m(\tau)}{m(\mu)} \tag{17}$$

leading to:

$$\frac{m(\nu_\tau)}{m(\nu_\mu)} \sim \frac{m(\tau)}{m(\mu)} \tag{18}$$

Most other "reasonable" models for the three-generation "see-saw" predict[21] a neutrino mass-ratio of the order of $\left[\frac{m(\tau)}{m(\mu)}\right]^n$ with $1 \leq n \leq 2$.

Assuming $n \leq 2$ we then conclude:

$$m(\nu_\tau) \leq m(\nu_\mu) \cdot \left[\frac{m(\tau)}{m(\mu)}\right]^2 \tag{19}$$

Using $m(\nu_\mu) \leq 65\ eV$, the above inequality based on a "reasonable see-saw" yields:

$$m(\nu_\tau) \leq 20\ keV \tag{20}$$

in gross disagreement with our "definite" bound:

$$m(\nu_\tau) \geq 900\ keV \tag{21}$$

We therefore conclude that, within an LRS model with a "reasonable see-saw", the decay $\nu_i \to \nu_j \nu_k \nu_\ell$ will not obey the cosmological bound on the lifetime. Consequently, in such a model, both ν_μ and ν_τ must be lighter than 65 eV. Models containing the LRS group as a subgroup will lead to similar conclusions.

The case of a "horizontal" gauge symmetry is similar and somewhat simpler. In such theories one assumes the existence of a new "horizontal" gauge boson H^0 with flavor-changing couplings. The "horizontal" group commutes with the gauge group of the standard model. Consequently:

$$g_{H^0 \ell_i \ell_j} = g_{H^0 \nu_i \nu_j} \tag{22}$$

where (ν_i, ℓ_i) is an $SU(2)_L$ doublet. The decay $\nu_i \to \nu_j \nu_k \nu_\ell$ can proceed by H^0 exchange with unknown gauge couplings and an unknown H^0 mass. However, the same relations as before exist between $\Gamma(\nu_i \to \nu_j \nu_k \nu_\ell)$ and $\Gamma(\ell_i \to \ell_j \ell_k \ell_\ell)$. The entire analysis proceeds along the same lines, reaching the same conclusions.

Both in the case of Δ_L-exchange and in the case of H^0-exchange, it is easy to show that the decay width $\Gamma(\nu_\tau \to e^+ e^- \nu_{e,\mu})$ is smaller than the corresponding partial widths for $\nu_\tau \to \bar{\nu}_e \nu_e \nu_{e,\mu}$. In the first case the $e^+ e^- \nu$ decay proceeds by

Δ_L^+-exchange while in the second case it proceeds by the exchange of the same H^0 as the 3ν final state. In both cases the coupling constants of the two processes are simply related. At the lower end of the (0.9, ... , 70) MeV range for $m(\nu_\tau)$, the $e^+e^-\nu$ decay is further suppressed by phase space considerations. We therefore conclude that this decay will not change any of our conclusions.[††]

Can we have other mechanisms for ν_τ decay? In order to evade the consequences of our analysis one would need a new flavor-changing vector or scalar particle x which couples to charged leptons much more weakly than to the corresponding neutrinos. We cannot exclude with complete generality the artificial introduction of such a particle for the sole purpose of inducing a $\nu \rightarrow 3\nu$ decay, but there is no natural place for it in any of the known theories which go beyond the standard model. In order to allow *e.g.* $m(\nu_\tau) \sim 20\ keV$ (so that we may have a chance of obeying $m(\nu_\tau) \leq m(\nu_\mu)\cdot\left[\frac{m(\tau)}{m(\mu)}\right]^2$), we would need $\tau(\nu_\tau) \leq 5\times 10^{11}\ sec.$ The x-exchange mechanism would give a lifetime:

$$\frac{1}{\tau(\nu_\tau)} \propto [m(\nu_\tau)]^5 \cdot \frac{g_{xij}^2 g_{xk\ell}^2}{[M(x)]^4} \tag{23}$$

where g_{xij} is the $x\nu_i\nu_j$ coupling constant. For $g_{xij} \sim g_{xk\ell} \sim g_{weak}$ we obtain *e.g.*:

$$\frac{\tau(\mu)}{\tau(\nu_\tau)} = C\left[\frac{m(\nu_\tau)}{m(\mu)}\right]^5 \left[\frac{M(W_L)}{M(x)}\right]^4 . \tag{24}$$

Here C is a constant of order one, depending on whether x is vector, scalar, axial vector, etc. Using the above bound on the lifetime we obtain $M(x) \leq 40\ GeV$. For weaker couplings g_{xij}, the upper bound on $M(x)$ decreases accordingly.

We therefore conclude that, barring unlikely and exotic possibilities, all three left-handed neutrinos are lighter than 65 eV (and may be stable). In order to

†† The decay $\nu_\tau \rightarrow e^+e^-\nu_{e,\mu}$ for $m(\nu_\tau)$ above a few MeV has important implications for the deuterium-hydrogen ratio in the universe (see reference 8). For a given neutrino mass in that range one can get bounds on the lifetime which are stronger than the usual cosmological bound discussed above. However, in models which go beyond the standard model, we cannot use the deuterium in order to derive a stronger bound on $m(\nu_\tau)$.

avoid this we must have either a Majoron scheme with arbitrarily concocted couplings[11] or an unnatural set of Δ_L masses requiring substantial fine tuning[18] or a new flavor-changing neutral boson below 40 GeV with normal weak couplings to neutrinos but much smaller couplings to charged leptons or a "see-saw" matrix with an extremely peculiar set of Majorana masses for right-handed neutrinos of different generations.

All of these possibilities are extremely awkward, they do not seem to solve any other problem and they do not arise naturally in any known model.

Assuming that all three known left-handed neutrinos are indeed lighter than 65 eV and that their masses are due to some kind of a "see-saw" mechanism, we can now derive a *lower limit* on the Majorana masses of the corresponding right-handed neutrinos. We obtain ($1\ PeV = 10^3\ TeV$):

$$M_R \sim \frac{[m(\tau)]^2}{m(\nu_\tau)} \geq 50\ PeV \tag{25}$$

For a "see-saw" driven by the GUT scale, this bound is useless. However, for LRS theories it implies (assuming $h_{\Delta\nu\nu} \leq g_{weak}$):

$$M(W_R) \geq\ 50\ PeV \tag{26}$$

and for a "see-saw" driven by a horizontal symmetry we obtain:

$$M(H^0) \geq\ 50\ PeV \tag{27}$$

Both of these limits are very significant. In the case of LRS theories they imply that no right-handed W or Z will be produced in experiments within the next several decades and that most effects (including CP-violation) which are due to right-handed currents are negligible. Previous bounds on the scale of right-handed currents[15,16] were in the range of a few TeV's, well below our new bound. In the case of horizontal symmetry, the new bound is stronger than previous bounds[22] obtained from rare processes such as $\mu N \to eN$, $\mu \to 3e$, $\mu \to e\gamma$, $K^0 \to e\mu$, $K^+ \to \pi^+\mu e$ and $\Delta M(K_S^0 - K_L^0)$.

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