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O(α^2)W MASS SHIFT FROM A VERY HEAVY TOP QUARK*

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One of the most sensitive tests of the Standard Model[1] (and of electroweak theories in general) to one loop level will be the precision measurement of the W mass to better than 1% accuracy. As is known, the latter is related to the Fermi constant, the Z_0 mass and the electric charge by Sirlin's one-loop formula:[2]

$$M_W^2\left[1-\frac{M_W^2}{M_Z^2}\right] = \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{1-\Delta_r} \quad , \tag{1}$$

where Δ_r is the radiative correction, evaluated to one loop. Δ_r contains the still unknown parameters M_{Higgs} and M_{top}, so that its numerical value can only be given for fixed values of these quantities. Normally, one assumes $M_t \simeq 30$ GeV, $M_H \simeq 100$ GeV and finds[3]

$$\Delta_r\ (M_t = 30\ \text{GeV}\ ,\ M_H = 100\ \text{GeV}\ ) \simeq 0.07 \quad . \tag{2}$$

In practice, this important correction stems mostly from oblique corrections, particularly fermionic vacuum polarization diagrams. More precisely, the value of Eq. (2) is mainly determined by renormalization of the running electric charge where in Euclidean metric with $q^2 = \vec{q}^2 - q_0^2 = -M_Z^2$

$$\alpha_{em}(-M_Z^2) = \frac{\alpha(0)}{1-\Delta_\alpha(-M_Z^2)} \quad , \tag{3}$$

with $\alpha^{-1}(0) \simeq 137.036$ and $\Delta_\alpha(-M_Z^2) \simeq .06$ Actually, one can write

$$\Delta_r = \Delta\alpha(-M_Z^2) - \frac{c_\theta^2}{s_\theta^2}\Delta_\rho(0) + \text{small contributions} \quad , \tag{4}$$

where $c_\theta = M_W/M_Z$, $s_\theta^2 = 1 - c_\theta^2$. The parameter $\Delta_\rho(0)$ gives the correction to the ρ parameter

$$\rho = 1 + \Delta_\rho(0) \quad , \tag{5}$$

and, if the top mass is equal to 30 GeV, $\Delta_\rho(0)$ is sensibly smaller than $\Delta_\alpha(-M_z^2)$.

In fact, $\Delta_\alpha(-M_Z^2)$ gives the leading logarithmic contribution $\sim \ell n(M_Z^2/m_f^2)$ to Δ_r. This is not the case of $\Delta_\rho(0)$, which is quadratic in the fermionic mass and proportional to m_f^2/M_Z^2. Thus for $m_f^2/M_Z^2 \ll 1$ one can discard $\Delta_\rho(0)$ and approximate Δ_r by its leading logarithmic term Δ_α. In this case, renormalization group arguments first introduced by Marciano and Sirlin[4] allow us to compute next order effects in Eq. (1) by simply expanding the Δ_α content of Δ_r through the related geometrical series. Thus, one easily computes the contribution to leading log to Eq. (1) from $\mathcal{O}(\Delta_\alpha^2)$ and finds that it is small; *i.e.*, much smaller than the $\mathcal{O}(\Delta_\alpha)$ term. This is a welcome indication that, as far as Eq. (1) is concerned, assuming $m_t \simeq 30$ GeV, higher order effects can probably be neglected.

The situation might be rather different if the top quark turned out to be substantially heavier; *e.g.*, of the order of $\simeq 200$ GeV. This is still not ruled out by the existing experimental evidence. A straightforward computation shows that in that case the numerical contribution of $\Delta_\rho(0)$ to Eq. (4) becomes almost of the same size (and opposite) to that of $\Delta_\alpha(-M_Z^2)$:

$$-\frac{c_\theta^2}{s_\theta^2}\,\Delta_\rho^{(top)}(0)\,\Bigg|_{m_t=200\ GeV} \simeq -\frac{c_\theta^2}{s_\theta^2}\left[\frac{3\alpha}{16\pi s_\theta^2 c_\theta^2}\,\frac{m_t^2}{M_Z^2}\right] \simeq -0.05 \quad . \tag{6}$$

If this were the case, one would have strong motivation to fear that next order contributions to Δ_r, *e.g.*, of the kind $\sim$ $\Delta_\rho^2(0)$ and $\Delta_\alpha(-M_Z^2)\Delta_\rho(0)$ might be relevant. Since these contributions are *not* of the leading logarithmic kind, their coefficient will differ from that of Δ_α^2. In this case it is *not* correct to expand Eq. (1) including terms $\sim (\Delta_r)^2$ with Δ_r given in Eq. (4). The relevant terms must be evaluated by application of perturbation theory to the proper oblique corrections contributions involving the various vacuum polarizations in a renormalization scheme independent way. We have done this starting from a general approach wihch evaluates higher order corrections which will be illustrated elsewhere.[5] Here we only deal with the specific case of the $O(\alpha^2)$ heavy top corrections to the precise $W^\pm$ mass which will be of special interest for the W mass measurement to be carried through at LEP II.

Here we work in the renormalization scheme which uses $\alpha(0)$, the muon lifetime coefficient, $G_\mu(0)$ and the physical Z° mass M_Z as physical input parameters and start from the coupled Dyson's equations for the various gauge bosons propagators:

$$\begin{aligned} G_{WW} &= \frac{1}{M_W^2 + q^2 - \widetilde{\pi}_{WW}(q^2)} \quad , \\ G_{ZZ} &= \frac{1}{M_Z^2 + q^2 - \widetilde{\pi}_{ZZ}(q^2) - \dfrac{\widetilde{\pi}_{ZA}^2(q^2)}{q^2 - \widetilde{\pi}_{AA}(q^2)}} \quad , \\ G_{AA} &= \frac{1}{q^2 - \widetilde{\pi}_{AA}(q^2) - \dfrac{\widetilde{\pi}_{ZA}^2(q^2)}{M_Z^2 + q^2 - \widetilde{\pi}_{ZZ}(q^2)}} \quad , \\ G_{ZA} &= \frac{\widetilde{\pi}_{ZA}(q^2)}{[q^2 - \widetilde{\pi}_{AA}(q^2)]\,[M_Z^2 + q^2 - \widetilde{\pi}_{ZZ}(q^2)] - \widetilde{\pi}_{ZA}^2(q^2)} \quad , \end{aligned} \tag{7}$$

where the $\widetilde{\pi}_{ij}$'s are the $1PI$ vacuum polarizations for vector bosons $i_j j = W^\pm$, Z, A (photon) which we write as

$$\widetilde{\pi}_{ij} \equiv \pi_{ij} + \text{counterterms} \ , \tag{8}$$

with π_{ij} calculated with the *bare* coupling constants. The specific choices of physical parameters are then used to fix the numerical value of diferent quantities which enter the oblique radiative corrections. In particular, we find:

$$\begin{aligned} \mathrm{Re}\ \frac{\widetilde{\pi}_{ZA}(-M_Z^2)}{M_Z^2} &= \frac{\Delta_p(-M_Z^2)}{s_\theta\, c_\theta} \left[1 + \frac{\Delta_p(-M_Z^2)}{1-2s_\theta^2} + \mathcal{O}(\alpha^2)\right] ; \\ \Delta_p(-M_Z^2) &\simeq \frac{s_\theta^2\, c_\theta^2}{1-2s_\theta^2}\ \left[\Delta_\alpha(-M_Z^2) - \Delta_\rho(0)\right] \quad . \end{aligned} \tag{9}$$

Defining the W mass as the pole of the W propagator and using consistently Eq. (7) leads us then to the following result:

$$\begin{aligned} M_W^2 \simeq \tilde{c}_\theta^2 M_Z^2 \Bigg\{ 1 &- \frac{\tilde{s}_\theta^2}{1-2\tilde{s}_\theta^2} \left[\Delta_\alpha(-M_Z^2) - \frac{\tilde{c}_\theta^2}{\tilde{s}_\theta^2}\,\Delta_\rho(0)\right] \\ &- \frac{\tilde{s}_\theta^2(1-3\tilde{s}_\theta^2+3\tilde{s}_\theta^4)}{(1-2\tilde{s}_\theta^2)^3}\ \Delta_\alpha^2(-M_Z^2) + \frac{\tilde{c}_\theta^4(1-3\tilde{s}_\theta^2)}{(1-2\tilde{s}_\theta^2)^3}\ \Delta_\rho^2(0) \\ &+ \frac{2\tilde{c}_\theta^2\,\tilde{s}_\theta^4\,\Delta_\alpha(-M_Z^2)\,\Delta_\rho(0)}{(1-2\tilde{s}_\theta^2)^3} \Bigg\} ; \end{aligned} \tag{10}$$

$$\tilde{c}_\theta^2 = 1 - \tilde{s}_\theta^2 = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4\alpha\pi}{\sqrt{2}\, G_\mu M_z^2}}\right) . \tag{11}$$

which allows us to compute those effects coming from a heavy top quark to $\mathcal{O}(\alpha^2)$ (one loop $\times$ one loop) terms.[♯1]

Note that the coefficient of Δ_α^2 in Eq. (10) is, as we expected from Marciano and Sirlin's arguments, that which corresponds to the geometrical series

♯1 One particle irreducible two-loop effects within $\Delta_\rho(0)$ have been computed[6] and found to be negligibly small.

expansion of the Δ_α content of $1/(1-\Delta_r)$. But the coefficients of Δ_ρ^2 and of $\Delta_\alpha \Delta_\rho$ are, as one might expect, quite different. For a top quark mass of 200 GeV, we find from Eq. (11)

$$\Delta M_W^{[top;\mathcal{O}(\alpha^2)]} \simeq +18 \ \text{MeV} \ , \tag{12}$$

and of these $\sim$ 18 MeV, $\sim$ 10 come from the interference $\sim \Delta_\alpha \Delta_\rho$, while $\sim$ 8 come from Δ_ρ^2. This $\mathcal{O}(\alpha^2)$ contribution should be compared to that coming, for the same value of $m_t = 200$ GeV, from the $\mathcal{O}(\alpha)$ term, which is of approximately +1 GeV.[2,3,7] Thus we conclude that such $\mathcal{O}(\alpha^2)$ effect is completely negligible even at the required level of accuracy, which we assume to be of the order of $\sim$ 50 MeV. This result is rather important since, *a priori*, a larger effect might have been found[♯2] and thus it may be feared that a large uncertainty in the Standard Model prediction for the $W^\pm$ mass could come from higher order effects.

♯2 An *incorrect* calculation done by expanding Eq. (1) including terms $\sim (\Delta_r)^2$ with Δ_r given in Eq. (4) would have yielded the *incorrect* result $\Delta M_W^{[top;\mathcal{O}(\alpha^2)]} \simeq -40$ MeV.

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