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COSMOLOGY WITH DECAYING VACUUM ENERGY*

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ABSTRACT

Motivated by recent attempts to solve the cosmological constant problem, we examine the observational consequences of a vacuum energy which decays in time. In both radiation and matter dominated eras, the ratio of the vacuum to the total energy density of the universe must be small. Although the vacuum cannot provide the "missing mass" required to close the universe today, its presence earlier in the history of the universe could have important consequences. Element abundances from primordial nucleosynthesis require the ratio $x = \rho_{vac}/(\rho_{vac} + \rho_{rad}) \leq 0.1$ in the radiation epoch; even a nonzero $x \gtrsim 0.01$ at nucleosynthesis, however, can allow the number of neutrino (or equivalent light) species to exceed $N_\nu > 4$, a case ruled out in the standard cosmological model. If the vacuum decays into low energy photons, the lack of observed spectral distortions in the microwave background gives tighter bounds, $x < 4 \times 10^{-4}$. In the matter-dominated era, the presence of a vacuum term may allow more time for growth of protogalactic perturbations.

I. INTRODUCTION

In the standard cosmological model, the early universe is thought to have passed through a series of symmetry breaking phase transitions at various energy scales M_x. As the temperature drops below M_x, the vacuum energy density associated with the Higgs field changes by $O(M_x^4)$. It is therefore puzzling that the upper bound on the present value of the vacuum energy density, $\rho_v < (0.003 \text{ eV})^4$ [1], is much smaller than any of the energy scales associated with particle physics. Even if such a cancellation can be arranged classically, there is at present no known low energy symmetry which prevents quantum corrections from inducing a large value for ρ_v: the cosmological constant does not appear to be a naturally small parameter.

There is some hope that a fundamental quantum theory of gravity will require $\rho_v = 0$, but such a theory must give rise to a cosmological term which is precisely cancelled by "low energy" contributions (*e.g.,* at the electroweak scale) if our present understanding of the dynamics of phase transitions is correct.

Another class of approaches to the problem relies on dynamical mechanisms to reduce ρ_v to a very small value over a period of time. In this category fall various models which introduce new fields, typically with very small mass scales, into particle physics [2]. This class also includes a set of ideas suggested by studies of the dynamical effects of quantum fields in de Sitter space-time. In the semi-classical approximation to quantum gravity, the backreaction of quantum fields on the metric may render de Sitter space unstable to conformal perturbations [3]. At present, the significance for cosmology of such an instability is unclear, since it is not known how the system would evolve away from the initial de Sitter solution. If either of these dynamical mechanisms proves viable, the standard inflationary scenario is in need of revision.

An intriguing possibility is that the universe eventually evolves to a state in which the effective cosmological term (ρ_v) is small and continues to decrease with time. In this paper, we consider the consequences for observational cosmology of a continuously *decaying* vacuum energy density.

In such a general form, the problem is underdetermined: a fundamental calculation is necessary to specify how rapidly the vacuum energy decays and how it couples separately to non-relativistic matter and to radiation. In this preliminary study, we adopt a phenomenological approach instead and consider a class of cosmological models characterized by a new parameter,

$$x = \rho_v/(\rho_r + \rho_v) \ , \tag{1}$$

the ratio of vacuum to the sum of vacuum and radiation (ρ_r) energy density.

The modified cosmological model is presented in general in Sect. II. Stipulating that the evolution of such a universe not differ too drastically from the standard model, we discuss the radiation epoch in Sect. III. There, we present a detailed discussion of the limits on the vacuum component obtained by (i) consideration of element abundances from Big Bang Nucleosynthesis, (ii) bounds on the spectral distortion of the microwave background, and (iii) constraints on the production of entropy since the epoch of baryogenesis. In Sect. IV we consider the matter-dominated epoch under two different assumptions: (i) the vacuum couples only to radiation, and (ii) the vacuum couples directly to nonrelativistic matter, leading to the spontaneous creation of matter in the universe. For the latter we investigate constraints arising from the growth of density perturbations and the diffuse gamma-ray background. In both cases, the contribution of the vacuum energy at the present epoch must be several orders of magnitude less than the nonrelativistic matter energy density, i.e., at most of order of the radiation content of the present universe. We note that this constraint is much stronger than the corresponding bound for models with *constant* vacuum density[1].

Our major conclusion is that dynamical models of decaying vacuum energy of a rather general variety are consistent with observational cosmology; however, the deviation from the standard model must be small.

II. THE MODEL

Suppose that in addition to ordinary matter and radiation energy density, ρ, there is an energy density associated with the vacuum, ρ_v. This additional component enters the Einstein equation for the Robertson-Walker scale factor $a(t)$,

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \rho_v), \quad \rho = \rho_m + \rho_r \ , \tag{2}$$

where we have assumed flat spatial sections ($k = 0$) and a line element,

$$ds^2 = -dt^2 + a^2(t)\ d\vec{x}^2.$$

If we require the energy momentum tensor of the vacuum component to be Lorentz invariant in the flat space limit, then the homogeneous, isotropic pressure of the vacuum is $p_v = -\rho_v$. The pressure of the non-vacuum component is

$$p = w\rho \ , \tag{3}$$

where w is given by

$$w = \frac{1}{3}\frac{\rho_r}{\rho_m + \rho_r} \ . \tag{4a}$$

Since the variation of $w(t)$ is slow compared with the expansion of the universe, except near the time $t_{\rm eq}$ when matter and radiation energy densities are equal, we will approximate $w(t)$ as a step function:

$$w = \begin{cases} \frac{1}{3}, & t < t_{\rm eq} \quad \text{(radiation dominated)} \\ 0, & t > t_{\rm eq} \quad \text{(matter dominated)} \end{cases} \ . \tag{4b}$$

Note that the terminology "radiation dominated" or "matter dominated" refers to the relative contributions of these two components to the energy density of the universe, independently of the value of ρ_v.

The conservation equation for energy-momentum takes the form

$$\dot{\rho}_v + \dot{\rho} + 3\ (1+w)\ \frac{\dot{a}}{a}\rho = 0\ , \tag{5a}$$

or

$$\dot{\rho}_v + \frac{1}{a^3}\ \frac{d}{dt}\ (\rho_m a^3) + \frac{1}{a^4}\ \frac{d}{dt}\ (\rho_r a^4) = 0\ . \tag{5b}$$

From the latter we notice immediately that if $\dot{\rho}_v \neq 0$, at least one of the ordinary adiabatic relations $\rho_r \sim a^{-4}$, $\rho_m \sim a^{-3}$ ceases to be valid. If $\dot{\rho}_v < 0$ then entropy or matter *must* be generated in the expansion. In the radiation-dominated epoch ($\rho_m < \rho_r$) the second term in Eqn. (5b) is small and can be neglected. In Sect. IV, we show that the creation of nonrelativistic particles by the decay of ρ_v during the matter-dominated epoch is severly constrained by annihilation limits from the gamma ray background. As a result, the contribution of the term $\frac{d}{dt}(\rho_m a^3)$ to Eqn. (5b) must be negligible, and we drop it from now on.

Assuming therefore that the vacuum couples only to radiation, the vacuum component of the energy density can be parameterized by the quantity x, defined in Eqn. (1). Equations (1) and (5) may be used to derive the evolution equation for $x(t)$ [4]:

$$\frac{\dot{x}}{x} = \frac{\dot{\rho}_v}{\rho_v} + 4\frac{\dot{a}}{a}\ (1-x)\ . \tag{6}$$

There are three possibilities for the behavior of $x(t)$ at large t: (i) for $x \to 1$, the vacuum term dominates, and the universe becomes de-Sitter-like as the radiation is redshifted away. This case is ruled out at the level of the bounds of Ref. 1; (ii) the vacuum density falls more rapidly than the radiation density, *i.e.*, $x \to 0$, and we recover the standard cosmological model; (iii) the only genuinely new cosmology is obtained if x approaches a non-zero constant betwen 0 and 1, which corresponds to the vacuum and radiation densities redshifting at the same rate.

For this case, which we consider in the remainder of this paper, we find from Eqn. (6)

$$\rho_v(t) = \left(\frac{x}{1-x}\right)\rho_r \sim a^{-4(1-x)} \ . \tag{7}$$

Note that x may assume two *different* constant values, one in the radiation dominated and one in the matter dominated epoch, if the expansion rate $\dot{a}/a$ changes at $t_{\rm eq}$. Again we assume a step function behavior for x at time $t_{\rm eq}$.

From Eqn. (7), we see that the radiation density drops more slowly as a function of the scale factor than in the standard cosmology, whereas the matter density approximately redshifts in the usual way, $\rho_m \sim a^{-3}$ [see discussion following Eqn. (5)]. With this scaling of the two components, the constraint that an early radiation epoch be followed by a matter dominated era requires that $x < \frac{1}{4}$ in both the matter and radiation epochs.

In the radiation era ($\rho_r \gg \rho_m$), Eqns. (2) and (7) are easily solved to yield

$$\left.\begin{aligned} a &\sim t^{\{1/[2(1-x)]\}} \\ \rho_v &= \frac{3x}{32\pi G(1-x)^2 t^2} \end{aligned}\right\} \mathrm{RD}\ , \tag{8}$$

while in the matter epoch ($\rho_m \gg \rho_r$),

$$\left.\begin{aligned} a &\sim t^{2/3} \\ \rho_v &\sim t^{[-(8/3)(1-x)]} \end{aligned}\right\} \mathrm{MD}\ . \tag{9}$$

Increasing x towards unity speeds up the expansion rate of the universe in the radiation era.

In the dynamical decay process of Ref. 3, a form like (8) might be expected, at least in the radiation dominated epoch. Since the time scale for the instability of de Sitter space is the Hubble time $(G\rho_v)^{-1/2}$, $\dot{\rho}_v$ might be expected to be proportional to $(G\rho_v)^{1/2}$, multiplied by ρ_v for spontaneous vacuum decay, or by ρ_r for induced decay in the presence of massless radiation. A decay law of the form (8) has also been suggested by various authors in Ref. 2. The value of x is presumably determined by the particular physical model of vacuum decay.

We have not yet specified the forms of radiation into which the vacuum decays. As long as the created radiation reaches thermal equlibrium, it can be characterized by its temperature, with

$$\rho_r = \frac{\pi^2}{30}\, g_{\rm eff}\, T^4 \; ; \tag{10}$$

here $g_{\rm eff}$ is the number of relativistic degrees of freedom. In this case, from Eqns. (7,8), we find

$$T(t) = \left[\frac{16\pi^3 G\; g_{\rm eff}(1-x)}{45}\right]^{-1/4} t^{-1/2} \; , \tag{11}$$

for the radiation temperature as a function of time in the radiation-dominated epoch. Although the issue is model-dependent, for the rest of this paper we assume that essentially all of the radiation emitted by the vacuum is in the form of photons and neutrinos. As we show in Sect. III (ii), the electromagnetic radiation created by vacuum decay thermalizes completely up to at least a time $t_T \approx 10^5$ sec (and, in particular, throughout primordial nucleosynthesis); thus it makes sense to describe the radiation by a Planck spectrum with a temperature given by (11).

The assumption of thermal equilibrium determines how the radiation number density and energy per particle change with time. From Eqns. (7) and (10), as long as the photons remain in thermal equilibrium, we have $T \sim a^{x-1}$. To maintain a Planck distribution, the energy per particle must redshift like the temperature, so $E_\gamma \sim a^{x-1}$ as well. From Eqn. (7), this implies the photon number density scales as $n_\gamma \sim a^{-3(1-x)}$.

The assumption that photons are created in vacuum decay also implies that the baryon to photon ratio, n_B/n_γ, decreases as the universe expands.

Since baryons are not created, $n_B \sim a^{-3}$, and the baryon to photon ratio thus scales as

$$\eta = \frac{n_B}{n_\gamma} \sim a^{-3x} \sim T^{[3x/(1-x)]} , \tag{12}$$

at least up to t_T. This fact will be used repeatedly in the discussions of observational constraints which follow in Sect. III. In this paper, we have not considered the case where the vacuum decays, in whole or in part, into noninteracting, nonthermal particles such as gravitons, shadow photons, etc; in that case, Eqns. (11) and (12) are clearly inapplicable.

III. THE RADIATION ERA

i) Nucleosynthesis

Since a non-zero vacuum component changes both the expansion rate through Eqn. (8) and the temperature-time relation, Eqn. (11), it can alter the delicate balance with nuclear reaction rates at the time of helium and deuterium synthesis that holds in the standard cosmology. First we brieflly review the heuristic arguments for element abundances obtained at the epoch of Big Bang Nucleosynthesis and illustrate qualitatively how these will change in the presence of a vacuum component. A correct treatment of this highly nonlinear problem with many coupled nuclear reactions, however, requires a numerical analysis. For comparison to observation we therefore rely on the calculation of primordial element abundances in Wagoner's code[5], modified to include a vacuum component. (For reviews, see Ref.6.)

At the high temperatures in the early universe, the ratio of neutrons to protons is determined by its thermal equilibrium value,

$$n/p = e^{-Q/kT} , \quad T \geq T_F \tag{13}$$

where the neutron-proton mass difference $Q = 1.293$ MeV and k is Boltzmann's constant. The neutrons drop out of equilibrium below a freeze-out temperature

T_F, where the weak interaction rates can no longer keep up with the expansion of the universe. Below T_F the n/p ratio continues to fall due to β-decay on the time scale of the neutron half-life τ_n. In the standard model, nucleosynthesis, (subscript D), takes place at a temperature approximately given by

$$T_D = \frac{2.2 \text{ MeV}}{-\ln \eta} \,. \tag{14}$$

Once T_D is reached, deuterium becomes stable against photodissociation and nucleosynthesis takes place very rapidly, efficiently converting essentially all of the available neutrons into ^{4}He. In this approximation, the primordial helium abundance Y_p is given by

$$Y_p = \left(\frac{2n}{n+p}\right)_D = \left(\frac{2n}{n+p}\right)_F \exp\left[-\Gamma(t_D - t_F)\right] \approx \frac{2e^{-\Gamma t_D}}{1+\exp\left[Q/kT_{\text{Freese}}\right]} \,, \tag{15}$$

where the final approximation is valid since $\Gamma^{-1} = \tau_n/\ln\, 2 \gg t_F \sim 1$ sec.

In the presence of a small vacuum component $x \ll 1$ (we will see from the numerical results that x must be less than 0.1), we can illustrate heuristically the deviation from the standard model. The x dependence of T_F may be obtained by recalling how T_F is determined. Freeze-out occurs when a typical $n \Leftrightarrow p$ weak interaction rate $G_w^2 T_F^5$ is equal to the expansion rate $[G(\rho_r + \rho_v)]^{\frac{1}{2}}$. Since $\rho_v + \rho_r = \frac{1}{1-x}\rho_r$, the effective speed up of the expansion rate due to the vacuum component is

$$\xi_{\text{eff}} = (1-x)^{-1/2} \geq 1 \,. \tag{16}$$

Then, since $T_F^5 \sim T_F^2\, \xi_{\text{eff}}$ we obtain

$$T_F = \bar{T}_F (1-x)^{-1/6} \,, \tag{17}$$

where an overbar indicates the standard model value ($x = 0$). By itself this would tend to increase Y_p by increasing the n/p ratio at freeze out. However, Eqns. (11) and (14) indicate that

$$t_D = \bar{t}_D(1-x)^{-\frac{1}{2}} , \tag{18}$$

which increases the available time for neutrons to β-decay. This turns out to be the larger effect in the domain of interest, and we see that Y_p is a decreasing function of x. The numerical analysis does indeed follow this general trend, although the quantitative results are somewhat different.

In our modification of the nucleosynthesis code, we have directly included a nonzero vacuum term in Einstein's Eqns. (2) and (5). As a consequence the temperature-time relation (11) and therefore all timescales (*e.g.,* T_F and t_D) are automatically modified. One of the most dramatic changes from the standard model (not included in the previous heuristic discussion) is the behavior of entropy, which in the standard model is constant throughout and after nucleosynthesis (except for the infusion of e^+e^- pairs). In the decaying vacuum model the entropy per baryon can change drastically through nucleosynthesis and continues to change afterward according to Eqn. (12). As a check on our use of the code we verified that the temperature-time relation and the temperature dependence of the entropy are indeed given by Eqns. (11) and (12).

The code we used assumes that neutrinos maintain a thermal distribution which parallels the photon distribution [Eqn. (11)], modulo the effects of e^+e^- annihilation. This will certainly be true at early times $t \lesssim t_F$ before neutrinos decouple. If neutrinos are created in vacuum decay with a thermal spectrum, they will be described by Eqn. (11) for later times, and throughout nucleosynthesis, as well. If, however, neutrinos are created with a non-thermal spectrum, then for $t \gtrsim t_F$ they do not have a simple Planck distribution (10), but the neutrino density continues to redshift according to Eqn. (7). Since neutrinos affect nucleosynthesis through the their contribution to the expansion rate [Eqn. (2)], our numerical results should apply with reasonable accuracy to this case as well.

We present our results graphically for ^{4}He and D abundances in Figs. 1-3 for $N_\nu = 3, 4$, and 5 neutrino species. For comparison with observation we have chosen a temperature $T_N = 10^8$ K to signal the end of nucleosynthesis; after this time, element abundances from the code no longer change significantly even for nonzero x, although the entropy continues to drop according to Eqn. (12). Requiring that $0.22 \leq Y_p \leq 0.26$ and $10^{-5} \leq D/H \leq 10^{-4}$[7], we find the (η, x) plane at T_N is restricted as shown, and conclude that even for $N_\nu = 5$ the vacuum component must satisfy $x < 0.1$:

N_ν	x_{max}
3	0.08
4	0.09
5	0.10

Although at most four neutrino (or equivalent numbers of light) species can be accommodated in the standard model, for $x \geq 0.01$ five neutrinos (or perhaps even more) are consistent with the observed element abundances (see Fig. 3). To turn this result around, if the number of neutrino species is found to exceed four, decaying vacuum energy offers a scenario with agreement between calculated element abundances and observations.

We have also confirmed consistency with observation of the ^{7}Li abundance obtained from the code for this range of parameters. The abundances of ^{4}He, D, and ^{7}Li are all lower than in the standard model, whereas the H abundance is slightly higher. This result is consistent with Eqn. (18) which indicates that there is more time for free neutrons to β-decay into protons. The constraints on $\eta(T_N)$ become more restrictive in the presence of a nonzero x, but remain within the same range as in the standard model $(10^{-10} \leq \eta(T_N) \leq 10^{-9})$. Thus we reach these important conclusions in this section:

1) Primordial nucleosynthesis in the presence of a vacuum component with $x \leq 0.1$ is consistent with observations of abundances of ^{4}He, D and the other light nuclei.

2) If $x > 0$, the preferred values of η at nucleosynthesis lie within the same range as in the standard model.

3) If $x > 0$, Y_p decreases. Although more restrictive observational upper bounds on the ^{4}He abundance (or $N_\nu > 4$) would lead to inconsistencies in the standard model, these could be resolved by the presence of a vacuum component at nucleosynthesis.

ii) Microwave Background Distortions

As Eqns. (5b) and (7) show, an interesting feature of models with $x =$ const. and $\rho_v \sim 1/t^2$ is that some fraction of the microwave background photons in the present universe was created by the decay of the vacuum. The spectrum of radiation emitted by the decaying ρ_v is model-dependent: in general it may be quite different from the Planck distribution appropriate for fully equilibrated radiation. If this is the case, and if the processes involved in the relaxation of the injected photon spectrum toward equilibrium are not 100% efficient, then distortions of the Planck spectrum may arise[8].

On the other hand, if the energy *is* injected with a thermal spectrum, then there will be no distortions in the present relic radiation spectrum and the bounds we discuss below do not apply. Such a situation might arise, for example, if the vacuum produces photons through an induced decay mechanism. Then the thermal photons already present might induce the vacuum to produce photons with a thermal spectrum. Equations (11) and (12) of Sect. II would then continue to apply down to the present epoch.

In this subsection we explicitly assume that the vacuum does not decay into photons fully equilibrated to a Planck spectrum. The most likely possibility seems to be that the emission is peaked at long wavelengths ($E_\gamma < kT$).* In that case

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* In the dynamical decay scenarios of Refs. 2 and 3, the only lengthscale in the problem appears to be the Hubble radius or possibly the Compton wavelength associated with a very small mass.

the photons would be efficiently absorbed by the free electron plasma via inverse bremsstrahlung, since the cross section for this process rises like $1/\omega^3$. At frequencies lower than the plasma frequency

$$\omega_p \cong 3 \times 10^6 \, (\mathrm{T/eV})^{3/[2(1-x)]} \, \eta_{-10}^{1/2} \, 10^{6x/(x-1)} \, \mathrm{sec}^{-1} \quad ,$$

any electromagnetic radiation produced by the decaying vacuum is rapidly damped and its energy transferred to the plasma by ohmic heating. The result in either case would be to increase the electron energy density relative to the radiation density. However, at very early times, *i.e.*, for redshifts greater than $z_{\mathrm{T}} = 6.3 \times 10^4 (\Omega_B h^2)^{-6/5}$, the injected energy is completely thermalized by double Compton and bremsstrahlung process: no distortions survive. [In this and all the following we take $H_0 = 100h$ km/sec Mpc^{-1} for the present value of the Hubble parameter, $T_0 = 2.7\,^{\circ}\mathrm{K}$ for the present radiation temperature and we assume three massless neutrino species;Ω_B is the density parameter for the ionized gas. We also neglect the small x-dependent factors in all redshifts defined in this section.]

When $z < z_T$, the injected radiation energy heats the electron plasma, and photon production by the electron gas continues in the far Rayleigh-Jeans region ($\hbar\omega \ll kT$). At higher energies, however, photon production by the hot electrons becomes inefficient, and Compton scattering cannot redistribute the excess low energy radiation toward the peak. Thus, except at the very low end of the spectrum, we have $T_e > T_r$, and multiple scattering off the electrons shifts the background radiation to higher frequencies without changing the total number of photons. The spectrum then takes on a Bose-Einstein form, with a nonzero chemical potential μ. If $(T_e - T_r)/T_r \ll 1$, the resulting value of μ (also small compared to kT_r) depends only on the total amount of energy injected into photons, $\Delta\rho_r$, and is *independent* of their initial frequency distribution:

$$\mu = 1.4 \; kT_r \, \frac{\Delta\rho_r}{\rho_r} \; . \qquad (19)$$

For a continuous injection of photons by vacuum decay, $(\Delta\rho_r)/\rho_r$ is the time integral of $-(\dot{\rho}_v)/\rho_r$ (the contribution of neutrinos to ρ_r cancels from the integral),

$$\frac{\Delta\rho_r}{\rho_r} = -\int_{z_i}^{z_f} \frac{\dot{\rho}_v}{\rho_r}\frac{dt}{dz}dz = \frac{x}{1-x}\ln\left(\frac{\rho_r(z_i)}{\rho_r(z_f)}\right)$$
$$= 4x\ln\left(\frac{z_i}{z_f}\right) , \tag{20}$$

by Eqn. (7). Thermalization to a Bose-Einstein spectrum requires multiple photon scattering, but the average number of scatterings per photon decreases as the universe expands. For times later than $t_1 \cong 10^{11}$ sec (or redshifts less than $z_1 = 8.5\times 10^3(\Omega_B h^2)^{-1/2}$), multiple scattering becomes inefficient, and the background subsequently evolves too slowly to relax to a Bose-Einstein spectrum. So we take $z_i = z_T$ and $z_f = z_1$ for these μ distortions. Since observations of the microwave background spectrum require $\mu < 0.01\,kT_r$ [9] we obtain the bound on x:

$$x < 4\times 10^{-4} , \tag{21}$$

where we have taken $\Omega_B h^2 = 2.5\times 10^{-2}$ here and below.

At later times $t > t_1$, energy injected and efficiently absorbed by the electron plasma produces a different distortion of the microwave background spectrum. Compton scattering shifts the photons to higher energies, creating an excess in the Wien region and a shortage in the Rayleigh-Jeans part of the spectrum. The resulting spectrum is parametrized by a new variable y, which can again be related to the total energy injected:

$$y = \frac{1}{4}\frac{\Delta\rho_r}{\rho_r} = x\ \ln\left(\frac{z_i}{z_f}\right) , \tag{22}$$

by Eqn. (20).

If we assume that the injected photons are too low in energy to reionize the gas after it has recombined, then z_f for this distortion should be taken to be of the order of the redshift of recombination, $z_2 \cong 10^3$. For $t > t_2$, energy injection will continue to raise the temperature of the residual ionized gas and heat the intergalactic medium, without distorting the microwave spectrum.

Taking $z_i = z_1$ and $z_f = z_2$ in Eqn. (22) and using the observational bound $y < 0.02$ [9] yields the following bound on x:

$$x < 5 \times 10^{-3} \, , \tag{23}$$

so that the μ bound is the most stringent constraint on x we have obtained. These constraints are so severe because the background radiation is being subjected to the injection of energy over many expansion times, when the processes responsible for restoring equilibrium are inefficient. We reiterate that the key assumptions used in deriving these bounds are (i) that the vacuum produces photons which are out of equilibrium with the pre-existing radiation and (ii) that essentially all of the energy injected by the vacuum decay goes into heating the electron gas to $T_e > T_r$. If the vacuum decays into some non-interacting form of dark matter instead, then (ii) need not be true and we would again lose the very stringent bounds of Eqns. (21,23).

iii) Entropy Generation

According to models of grand unification(GUTs), an excess of baryons over antibaryons is produced at a very early epoch[10], typically at a temperature corresponding to GUT symmetry breaking, of order $T_{\rm GUT} \simeq 10^{15}$ GeV. After baryon-antibaryon annihilation, this baryon excess corresponds to a primordial baryon-to-photon ratio, $\eta(T_{\rm GUT})$. Since entropy is produced by the decay of the vacuum, η subsequently decreases with temperature according to Eqn. (12). The vacuum energy density can be constrained by ensuring that the value of

η produced at baryogenesis does not fall too low subsequently. Requiring that η lies within the bounds $10^{-10} \leq \eta(T_N) \leq 10^{-9}$ at the time of nucleosynthesis (see Sect. III i) and using $\eta(T_N) = \eta(T_{\rm GUT})\,(T_N/T_{\rm GUT})^{3x/(1-x)}$, we find the constraint

$$x \lesssim 0.02\left(1 + \log\ \eta_{-9}^{(G)}\right) , \tag{24}$$

where $\eta(T_{\rm GUT}) = \eta_{-9}^{(G)} 10^{-9}$ and we have assumed $T_{\rm GUT} \simeq 10^{15}$ GeV. For a large primordial baryon asymmetry, $\eta_{\rm GUT} \lesssim 10^{-4}$, this bound is comparable to that of Sect. III (i), $x \lesssim 0.1$.

Similarly, x is constrained by the evolution of η after nucleosynthesis. The most stringent bound is obtained if the vacuum decays to a thermal spectrum of radiation for all time. In this case, Eqn. (12) applies through the present epoch,

$$\eta(2.7\ {\rm K}) = \eta(T_N)\left(\frac{2.7\ {\rm K}}{T_N}\right)^{3x/(1-x)} . \tag{25}$$

Taking $\eta(2.7\ {\rm K}) \gtrsim 2\times 10^{-11}$ [7] for a conservative lower bound today and $\eta(T_N) \leq 10^{-9}$ at nucleosynthesis, we find

$$x \leq 0.07 . \tag{26}$$

On the other hand, if, as considered in III (iii), the radiation produced by vacuum decay only thermalizes up to a time t_T, then η remains constant for $T < T_T \simeq \left(3.3\times 10^5 (\Omega h^2)^{-6/5}\right)^{1-x}$ 2.7 K. In this case, we obtain the less stringent bound

$$x < 0.15 . \tag{27}$$

IV. THE MATTER DOMINATED ERA

When $\rho_m > \rho_r$, *i.e.*, for $t > t_{\rm eq}$, the vacuum can decay either to matter (the dominant component) or to radiation. Thus there are two possibilities to consider:

i) $\frac{d}{dt}\,(a^3\,\rho_m) = 0$

ii) $\frac{d}{dt}\,(a^3\,\rho_m) \neq 0.$

The first case expresses exact conservation of particle number; the second would result if massive particles are created by the decay of ρ_v. Vacuum coupling primarily to matter (case ii) would imply the continuous creation of baryon-antibaryon pairs. Naively, one would expect this effect to be greatly suppressed relative to radiation production. More to the point, observations so restrict the size of such a matter creation term that case (ii) becomes indistinguishable from case (i), as we now demonstrate.

Denote by Δn_B the net baryon (or antibaryon) number density created by the vacuum during the matter dominated epoch. Since the vacuum decay presumably does not violate baryon number, equal numbers of baryons (B) and antibaryons ($\bar{B}$) would be produced. By Eqn. (5b),

$$\Delta n_B = \Delta n_{\bar{B}} = -\frac{1}{m_p a^3(t)} \int\limits_{t_{eq}}^{t} a^3(t)\ \dot{\rho}_v\ dt\ , \tag{28}$$

where m_p is the proton mass. For this argument, we replace x by $\tilde{x}$ where

$$\tilde{x} = \frac{\rho_v}{\rho_m + \rho_v}\ , \tag{29}$$

and consider the $\tilde{x} \to$ constant case. We can then solve Einstein's equations (2) and (5) to obtain

$$a(t) \sim t^{2/[3(1-\tilde{x})]} \tag{30a}$$

$$\rho_v = \frac{1}{6\pi G}\frac{\tilde{x}}{(1-\tilde{x})^2}\frac{1}{t^2} , \qquad (30b)$$

and the integral in Eqn. (28) can be evaluated to find

$$\Delta n_B(t) = \frac{1}{6\pi G m_p t^2 (1-\tilde{x})}\left[1 - \left(\frac{t_{\rm eq}}{t}\right)^{2\tilde{x}/(1-\tilde{x})}\right] . \qquad (31)$$

The baryon and antibaryon pairs annihilate with one another and produce an observable gamma ray flux. Any such pair annihilation in the Galaxy is extremely constrained by observations so the most likely place to hide antimatter would be in the intergalactic medium. There, observations of the isotropic gamma ray flux lead to the constraint [11]

$$(\Delta n_B)(\Delta n_{\bar{B}}) \lesssim 10^{-18}\ {\rm cm}^{-6} . \qquad (32)$$

Substituting $\Delta n_B(t_0)$ from Eqn. (31) into Eqn. (32) and taking the present age of the universe (10^{10} yr $< t_0 < 2 \times 10^{10}$ yr), we find the constraint

$$1200\left[1 - \left(\frac{t_{\rm eq}}{t_0}\right)^{2\tilde{x}/(1-\tilde{x})}\right] \lesssim 1 . \qquad (33)$$

The formation of galaxies by the present epoch requires that density perturbations must have had time to grow. Since they can only start to grow at the earliest when $t = t_{\rm eq}$, we will obtain a maximum value for $t_{\rm eq}/t_0$ and hence an upper limit to $\tilde{x}$.

The density contrast $\delta \equiv (\delta\rho/\rho)_m$ in the linearized approximation satisfies the equation [12]:

$$\ddot{\delta} + 2\,\frac{\dot{a}}{a}\,\dot{\delta} = 4\pi\, G\, \rho_m\, \delta . \qquad (34)$$

From (2) and (29) the power law solutions to this equation are of the form

$\delta \sim t^n$ where

$$n = \left(\frac{1}{2} - \beta\right) \pm \left(\beta^2 + \frac{1}{4}\right)^{1/2} , \tag{35}$$

where $\beta = 2/[3(1-\tilde{x})]$ is the power law index of the scale factor. The range of n is $n \in (1/2, 2/3)$ as $\tilde{x}$ varies between 1 and 0. Requiring $\delta(t_{\rm eq}) = 10^{-q}$ and $\delta(t_0) = 1$, *i.e.*, that the perturbations were able to grow by a factor of 10^q since the beginning of matter domination, we find $(t_{\rm eq}/t_0)^n = 10^{-q}$ which leads to the constraint $\tilde{x} \lesssim \frac{1}{q}(1.2 \times 10^{-4})$ from (33). Since agreement with the isotropy of the microwave background requires $q \gtrsim 4$[13], we find $\tilde{x} \lesssim 3 \times 10^{-5}$.

Thus, any vacuum component must be at most at the level of the radiation energy density in the present epoch, and from (30a) the expansion rate of the universe is indistinguishable from the usual ($\tilde{x} = 0$) matter-dominated case. Before we drop this possibility entirely, it is worthwhile to point out the loopholes in the above agrument. If the creation of matter and antimatter could somehow be separated at large scales or if it takes place only in dense sources such as black holes (from which gamma rays cannot escape without at least being degraded in energy), case (ii) might still be possible (though it would remain implausible theoretically).

If the vacuum component decays only to radiation [case (i)], then there are again two distinct possibilities [see Sect. III (ii)]: (a) the vacuum produces photons with a thermal spectrum or (b) the vacuum produces photons in a non-equilibrium distribution. In the matter dominated era, when the radiation energy density is small, we recover most of the features of the standard model with $x = 0$. In particular, $a \sim t^{\frac{2}{3}}$ for either of the cases (a) or (b) above. There is an important difference between the two cases, however.

We first consider case (a). When the vacuum photons are thermal, the radiation energy density falls like $a^{-4(1-x)}$ [*cf.*, Eqns. (7) and (11)] throughout the matter dominated era. Since the matter density falls like a^{-3}, the redshift when matter and radiation were equal is given by

$$z_{\rm eq} = \left(\frac{\rho_{m,0}}{\rho_{r,0}}\right)^{1/(1-4x)} > \left(\frac{\rho_{m,0}}{\rho_{r,0}}\right) = (z_{\rm eq})_{std} = 2.5 \times 10^4\,\Omega h^2\ . \tag{36}$$

Thus the time $t_{\rm eq}$ occurs ***earlier*** than in the standard cosmological model. Moreover, since the stringent bounds on x from microwave distortions do not apply in this case, $z_{\rm eq}$ could be significantly earlier. For example, if x is as large as 0.07, as allowed by nucleosynthesis and entropy generation, the redshift of equality becomes $(z_{std})^{1/(1-4x)} = 1.9 \times 10^5$ (for $\Omega h^2 = 0.25$).

This reduction in t_{eq} could have important consequences for models of galaxy formation based on adiabatic density perturbations with cold dark matter[14]. For example, for a scale-invariant (Zeldovich-Harrison) spectrum, the scale $\lambda_{\rm eq} = ct_{\rm eq}(1+z_{\rm eq})$ below which the spectrum flattens is decreased,*

$$\lambda_{\rm eq} = \frac{13Mpc}{\Omega h^2}\,(2.5 \times 10^4 \Omega h^2)^{-2x/(1-4x)}$$

Since the spectrum is usually normalized at a scale $\lambda_{\rm n} \simeq 8h^{-1}Mpc$, less than $\lambda_{\rm eq}(x = 0)$, the large-scale perturbation amplitude, and thus the large-angle

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* The power on scales less than $\lambda_{\rm eq}$ will be slightly modified from the standard ($x = 0$) model, because the growth of perturbations which enter the horizon during the radiation-dominated era is altered. Eqn. (34) may be written as

$$\frac{d^2\delta}{dy^2} + \frac{3y + (2+4x)y^{4x}}{2y(y+y^{4x})}\frac{d\delta}{dy} - \frac{3\delta}{2y(y+y^{4x})} = 0$$

where $y = a(t)/a(t_{eq})$. At $y << 1$, i.e., well before the matter era, the approximate solutions are

$$\delta_1 \simeq 1 + \frac{3y^{1-4x}}{2(1-4x)(1-2x)}$$

and

$$\delta_2 \simeq y^{-2x} + \frac{3y^{1-6x}}{2(1-6x)(1-4x)}$$

Note these solutions are degenerate at $x = 0$. The corresponding solutions for $x = 0$ were found in Refs.15. At small y, the growing mode solution δ_1 is an increasing function of x, but its coefficient must be determined by including both solutions in the match at horizon crossing[16].

anisotropy in the cosmic microwave background, will be reduced. (This is just a reflection of the fact that pushing t_{eq} back allows more time for non-baryonic perturbation growth, so the initial perturbation amplitude and hence the corresponding microwave anisotropy can be smaller than in the standard model.) This reduction in power on large scales may strain the ability of such models to produce the structure recently observed or inferred on large scales [17]. Further, for $x = 0.07$ with $\Omega = 1$ and $h = 0.5$, $\lambda_{\rm eq} < \lambda_{\rm n}$ so that the perturbation amplitude on small scales ($< \lambda_{\rm n}$) is increased. In this case, the small angle ($4.5'$) microwave anisotropy, which is sensitive to perturbations on scales $\sim \lambda_{\rm n}$, may be slightly enhanced.

For a universe dominated by hot dark matter which becomes nonrelativistic at late times, *e.g.,* massive neutrinos, the effects are quite different. (We here assume the neutrinos are also produced with a thermal spectrum.) From Eqn. (11), the time $t_{nr} \sim T_{nr}^{-2} \sim m_{\nu}^{-2}$ at which neutrinos become nonrelativistic is essentially unchanged from the standard model (it is increased by the negligible factor $(1-x)^{-1/2}$). Since $t_{nr} \simeq t_{eq}$ for massive neutrinos, in this case t_{eq} is also unchanged. Further, the redshift z_{nr} is increased, so the free streaming scale $\lambda_{fs} \simeq ct_{nr}(1+z_{nr})$, below which neutrino perturbations are damped out, is slightly increased. (For discussion of neutrino models, see [14].)

On the other hand, if the vacuum decays to nonthermal radiation [case (b)], the limits on x from microwave constraints apply (*cf.,* Sect. III), and the value of x must be very small. As a result, $z_{\rm eq}$ will not differ appreciably from its value in the standard ($x = 0$) model.

IV. CONCLUSION

We have investigated the cosmological constraints on and consequences of a vacuum energy density which dynamically decays in time. We conclude that such a scenario can be consistent, but the universe cannot be vacuum-dominated for times later than about $t \sim 1 sec$. In fact, from the concordance of big bang nucleosynthesis with observations of element abundances and constraints on baryon-antibaryon annihilation from the gamma ray background, we find that the vacuum density must remain below the radiation density in both the radiation and matter eras. For vacuum decay to a non-thermal radiation distribution, the microwave background spectrum provides the strongest constraint, $x < 4 \times 10^{-4}$. On the other hand, if the radiation produced by the vacuum retains a Planck spectrum for all time, the requirement that the baryon-to-photon ratio not drop too low after nucleosynthesis gives the strongest bound, $x < 0.07$. In the latter case, saturation of the bound may have important (and possibly adverse) consequences for theories of galaxy formation.

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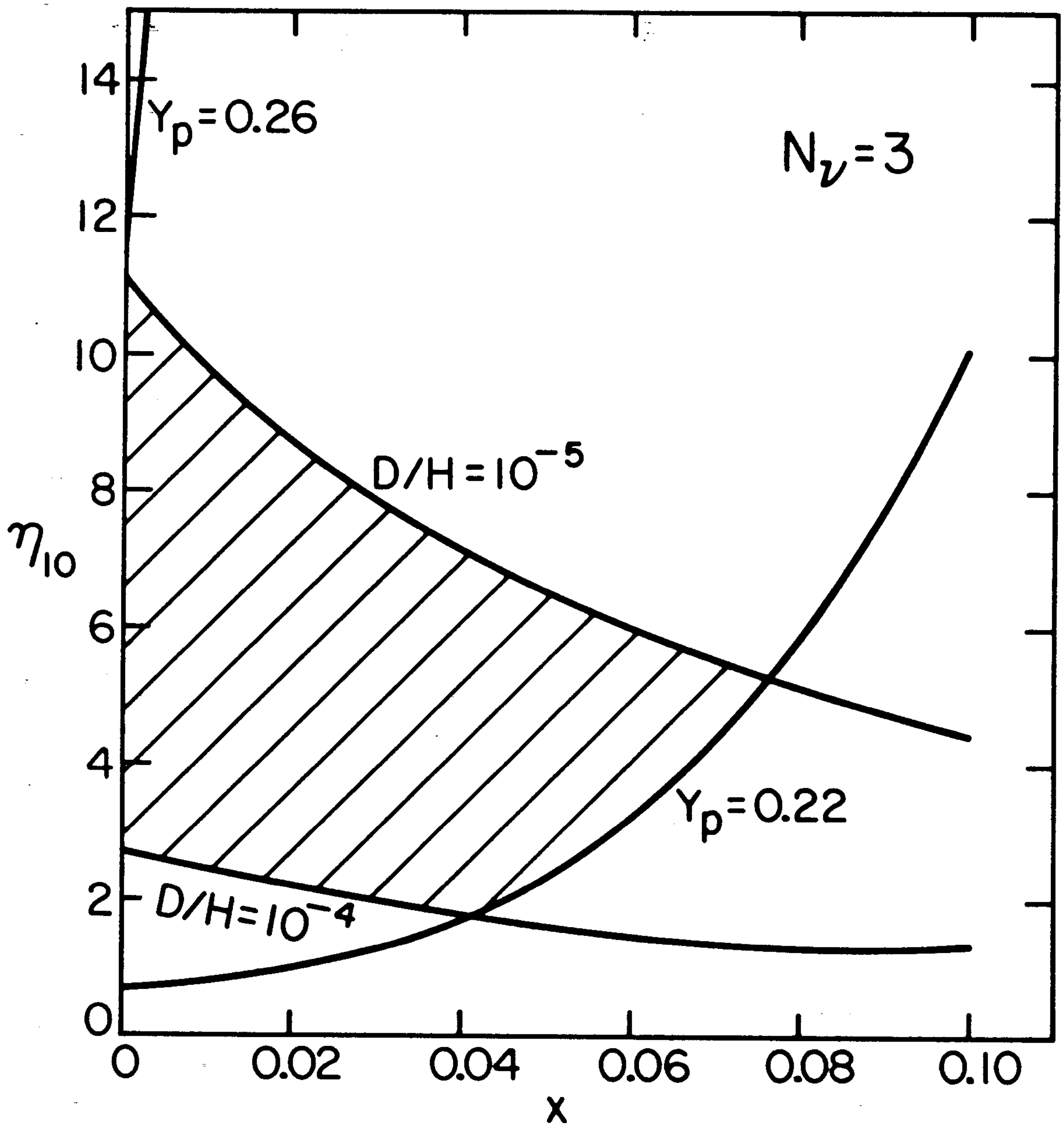
FIGURE CAPTIONS

1. Element abundances as a function of the vacuum energy density parameter x and the baryon-to-photon ratio at T_N, $\eta = \eta_{-10}10^{-10}$ for $N_\nu = 3$ neutrino species. The primordial ^{4}He abundance satisfies $0.22 < Y_p < 0.26$ and the ratio $10^{-5} < D/H < 10^{-4}$. Cross-hatching indicates the allowed region.

2. Same as Fig. 1 for $N_\nu = 4$ neutrino species.

3. Same as Fig. 1 for $N_\nu = 5$ neutrino species. Note that models with $x > 0$ can accomodate $N_\nu > 4$.



14
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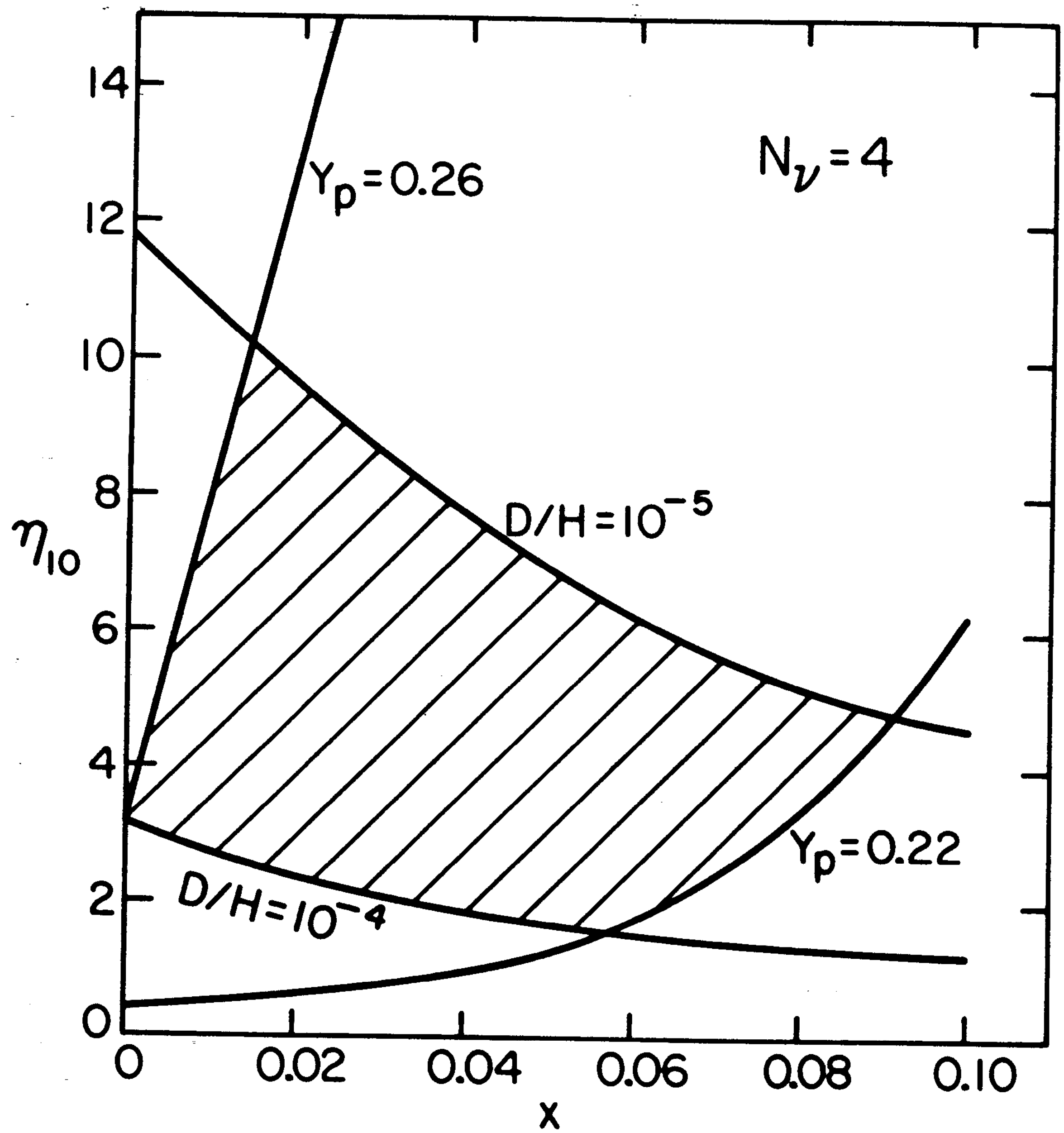
10
8
6
4
2
0
η_{10}
$Y_p = 0.26$
$N_\nu = 3$
$D/H = 10^{-5}$
$Y_p = 0.22$
$D/H = 10^{-4}$
0
0.02
0.04
0.06
0.08
0.10
X

Fig. 1



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$Y_p = 0.26$
$N_\nu = 4$
$D/H = 10^{-5}$
$Y_p = 0.22$
$D/H = 10^{-4}$
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0.04
0.06
0.08
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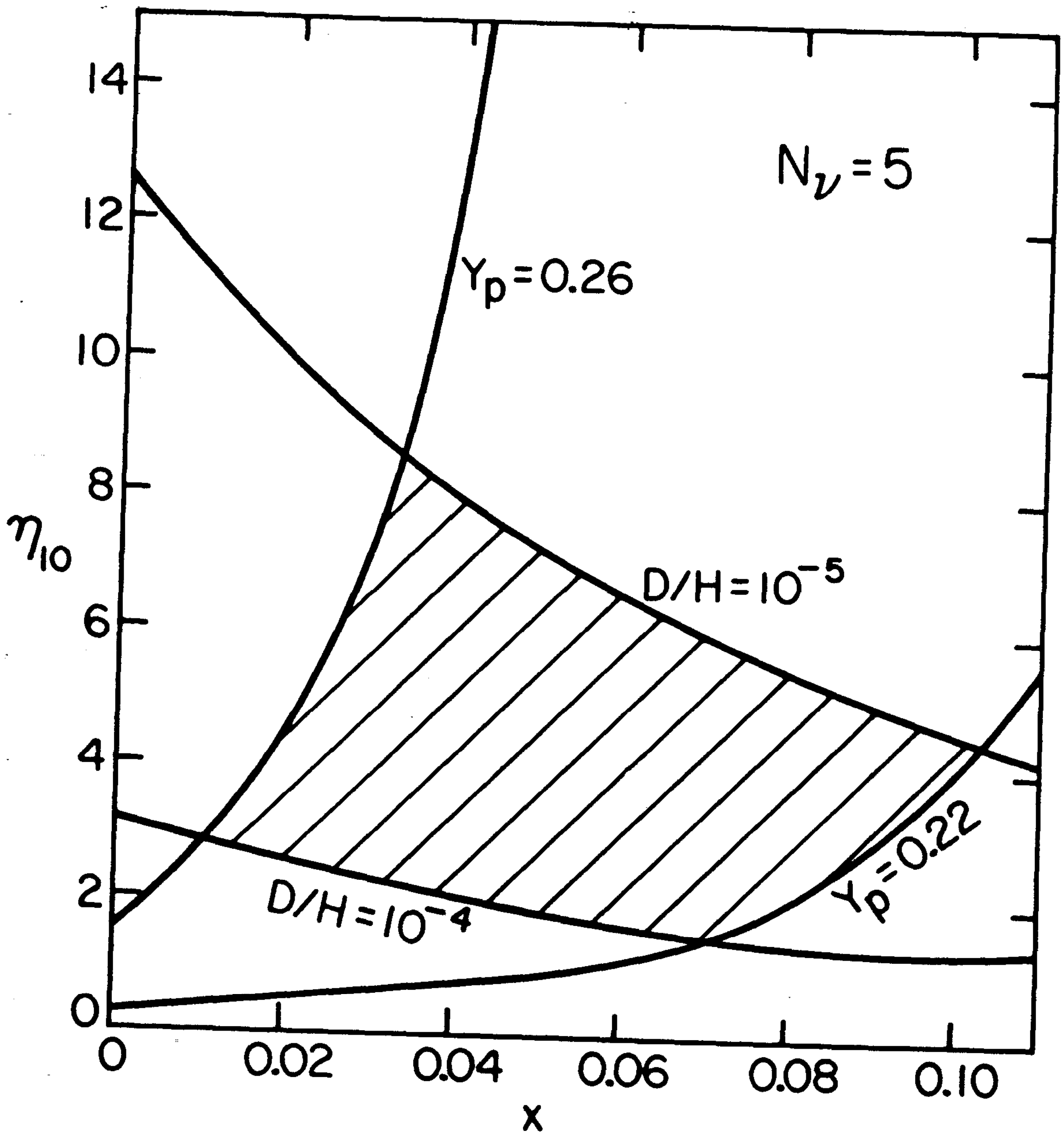
X

Fig. 2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$N_\nu = 5$
$Y_p = 0.26$
$D/H = 10^{-5}$
$D/H = 10^{-4}$
$Y_p = 0.22$
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0.02
0.04
0.06
0.08
0.10
x

Fig. 3