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COMPARISON OF HIGH GROUP VELOCITY ACCELERATING STRUCTURES*

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1. Abstract

It is well known that waveguides with no perturbations have phase velocities greater than the velocity of light c. If the waveguide dimensions are chosen so that the phase velocity is only moderately greater than c, only small perturbations are required to reduce the phase velocity to be synchronous with a high energy particle bunch. Such a lightly loaded accelerator structure will have smaller longitudinal and transverse wake potentials and hence will lead to lower emittance growth in an accelerated beam. Since these structures are lightly loaded, their group velocities are only slightly less than c and not in the order of $0.01c$, as is the case for the standard disk-loaded structures. To ascertain that the peak and average power requirements for these structures are not prohibitive, we examine the elastance and the Q for several traveling wave structures: phase slip structures, bellows-like structures, and lightly loaded disk-loaded structures.

2. Introduction

This paper will discuss the variation of several structure figures of merit as a function of group velocity, with emphasis on higher group velocities than are usually considered. The advantages of high group velocity are:

a) Low b/a ratio, where b is the maximum outer structure radius and a is the beam aperture radius. Such structures should be easier to fabricate, especially structures with smoothly changing longitudinal profiles.

b) Large aperture relative to wavelength, hence low wakefields.

c) Lower attenuation and power dissipation.

The chief disadvantage of high group velocity is a higher peak power requirement. However, this can be ameliorated by using pulse compression.

3. Structure Parameters

The properties of traveling wave structures can be obtained from computer codes, for example TWAP.[1] Important output parameters for a given structure geometry are group velocity, synchoronous frequency, and the elastance and internal time constant defined by

$$s = \frac{E^2}{w}; \qquad T_o = \frac{2w}{p_d} \ . \tag{01}$$

Here E is the local accelerating gradient, w is the energy stored per unit length, and p_d is the power dissipation per unit length. At a given group velocity we can define the following normalized structure parameters:

$$a_\lambda \equiv a/\lambda \ , \quad s_a \equiv sa^2 \ , \quad T_g \equiv T_o f^{3/2} \ . \tag{02}$$

where f is the frequency, λ the wavelength and v_g the group velocity.

The section need not by completely filled when the bunch is injected, as long as the bunch is under the rf umbrella provided by a source pulse of width $T_k = l/v_g - l/c$. Therefore, for the acceleration of a single bunch, the average power is reduced by the factor $1 - v_g/c$. This reduction becomes significant as the group velocity approaches the speed of light. Thus as far as average power is concerned, the effective normalized elastance is

$$s_{at} \equiv \frac{sa^2}{1 - v_g/c} \ . \tag{03}$$

Plots of s_{at} and a_λ as a function of group velocity are shown in Fig. 1 for a $2\pi/3$ disk loaded structure with iris thickness of 0.584 cm and a gap of 2.92 cm at 2856 GHz.

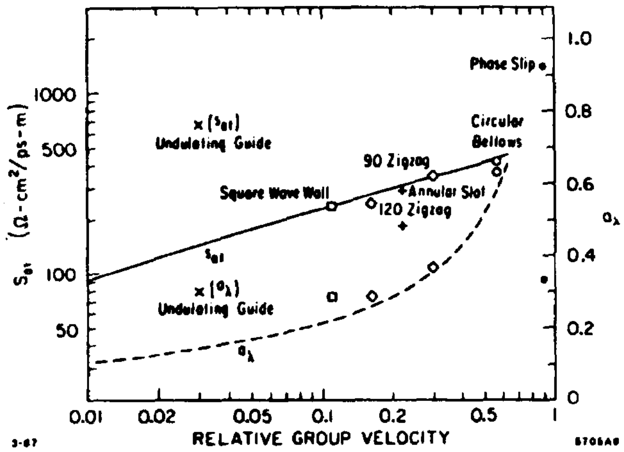

Fig. 1. Aperture invariant structure constants s_{at} and a_λ *versus* relative group velocity.

4. Peak and Average Power

The peak power P_p and average power P_a into an accelerator section of length L required to produce an average gradient E_a in the section are:[3]

$$P_p = \frac{E_a^2 L}{\eta_s s T_f M} = \frac{E_a^2 v_g}{\eta_s s M} = \frac{E_a^2 L}{\eta_s \tau s T_o M};$$
$$P_a = f_r P_p T_f = f_r \frac{E_a^2 L}{\eta_s s \eta_{pc}} \ . \tag{04}$$

The structure efficiency η_s is a function of $\tau = T_f/T_o$. For a constant impedance structure

$$\eta_s = \frac{(1 - e^{-\tau})^2}{\tau^2} \ . \tag{05}$$

The product sT_o equals the twice the shunt resistance. Note that the peak power is proportional to, and the average power is independent of the average group velocity $v_g = L/T_f$. For a high group velocity structure, s is in general lower and T_o is higher at a given frequency. Thus the total average power is increased, but the peak power per unit length will not increase as much. The elastance and the internal TW time constant are shown in Fig. 2 for a $2\pi/3$ disk loaded structure with iris thickness of 0.584 cm and a gap of 2.92 cm at 2856 GHz. The power multiplication factor, M, is the square of the section voltage with pulse compression divided by the square of the section voltage with no pulse compression. The pulse compression efficiency is given by $\eta_{pc} = M/C_f$, where the compression factor C_f is the source pulse length divided by the compressed pulse length.

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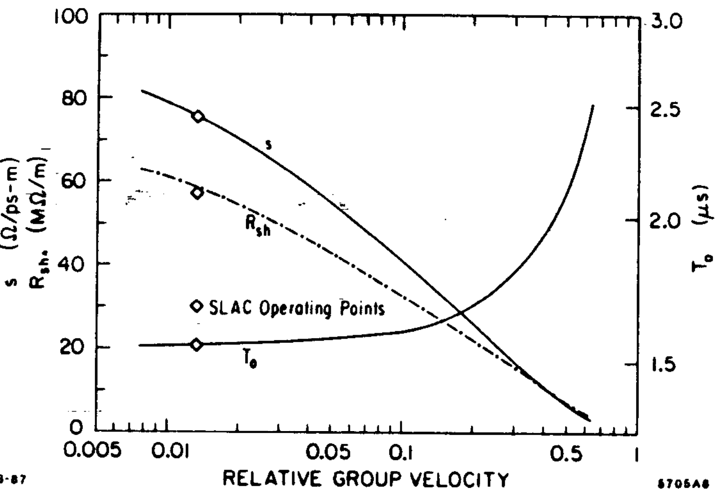

Fig. 2. Elastance, shunt resistance and attenuation time *versus* relative group velocity at 2856 MHz.

Pulse compression reduces the required source peak power but, because it is not 100% efficient, it increases the source average power. However, this increase can be more than offset by the increase in η_s due to higher v_g. The increase in η_s is due to the decrease in T_f and to some extent the increase in T_o. In a traveling wave structure the pulse width of the RF input to the structure has to be much less than the internal time constant of the structure in order to limit the RF lost to the structure walls. This results in high peak power requirement at high gradients. With pulse compression, the power is enhanced outside the accelerating structure in a device whose internal time constant is much larger than that of the accelerating structure.

5. Disk-Loaded Structures

There is a choice of four interdependent parameters: the two structure parameters aperture and frequency, and the two section parameters length and attenation. For a section with fixed L and τ :

$$T_f = \frac{L}{v_g}, \quad T_o = \frac{T_f}{\tau}, \quad f = \left[\frac{T_g}{T_o}\right]^{2/3}, \quad a = a_\lambda \lambda, \quad s = \frac{s_a}{a^2} .$$

A plot of frequency and aperture *versus* relative group velocity of a $2\pi/3$ disk-loaded section for $\tau = 0.61$, $L = 1$ m, is shown in Fig. 3. The average and peak power *versus* group velocity required by that section when operating at a gradient of 185 MV/m is shown in Fig. 4. At $v_g/c = 0.03$ the frequency is 11.4 GHz, (four times the SLAC frequency) the aperture is 3.8 mm, the average power is 7 KW/m and the peak power

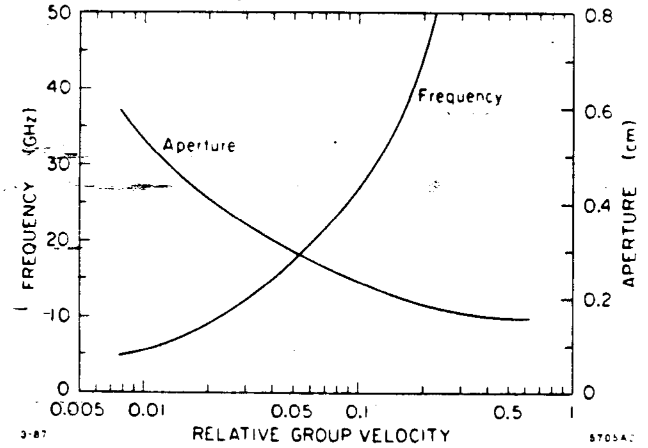

Fig. 3. Frequency and aperture with $\tau = 0.61$, $L = 1$ m *versus* relative group velocity.

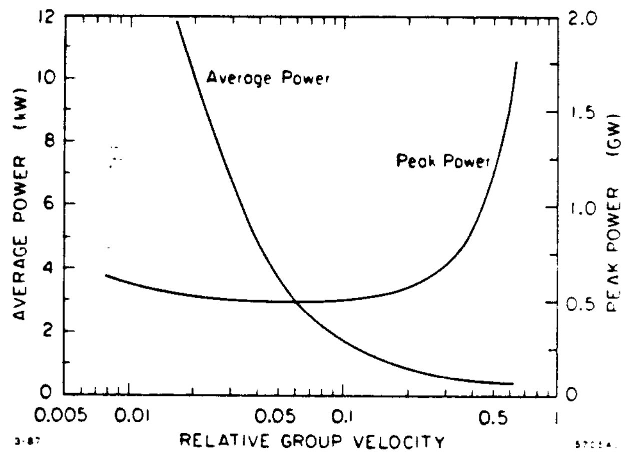

Fig. 4. Peak and average powers $\tau = 0.61$, $L = 1$ m *versus* relative group velocity.

is 500 MW/m. If the relative group velocity is increased to 0.2, the average power is reduced 7 fold and the peak power is increased slightly. The frequency is 45 GHz and the aperture is reduced to 2 mm.

At a fixed aperture of 0.382 cm there is a reasonable trade-off between peak and average powers at a frequency of 11.4 GHz. This is shown in Fig. 5, in which the peak and average power is plotted as a function of frequency for $L = 1$ m and $E_a = 185$ MV/m. This frequency is near the upper limit of the X-band wavelength range, and of the microwave range wavelength of 1-30 cm. The microwave wavelength range from 1-30 cm should really be called the practical wavelength range. Within this range, components are easy to handle and are readily available. At longer wavelengths components are bulky. At shorter wavelengths we enter the jewelers' domain.

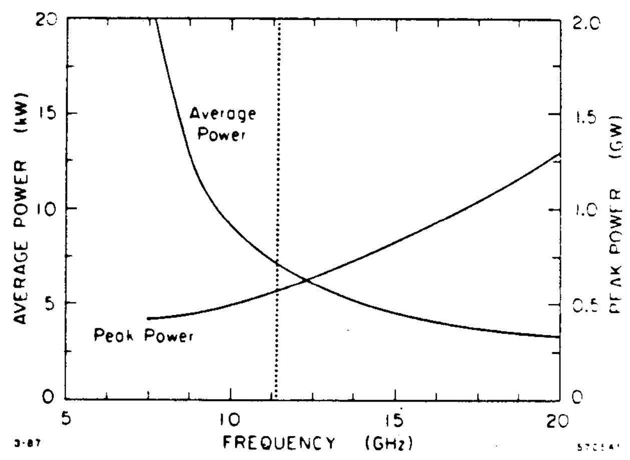

Fig. 5. Average and peak powers for $a = 0.382$ cm *versus* frequency.

6. Other Structures

So far only disk-loaded structures have been considered. However, other structures may have advantages, such as ease of fabrication or lower wake fields. A generic structure is the phase slip structure shown in Fig. 6. In each section of TM_{01} guide the bunch slips in phase by $\Delta\phi$ with respect to the RF wave. The phase is then reset at the RF bypasses. This structure is therefore a smooth TM_{01} guide with added discrete RF path lengths to reduce the effective RF phase velocity to the speed of light. The normalized elastance and aperture are readily calculated from the well known properties of modes in a smooth guide.

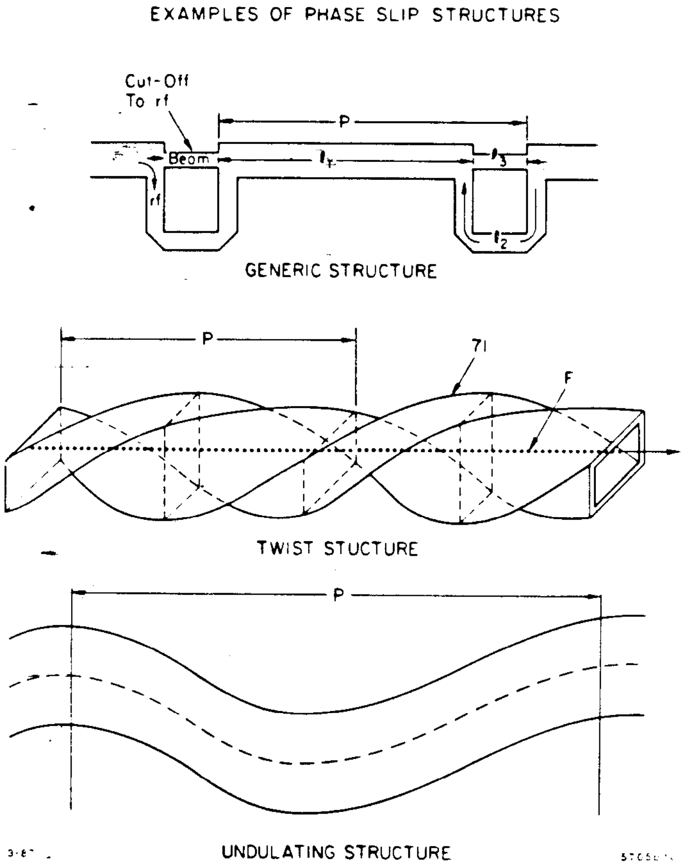

Fig. 6. Examples of phase slip structures.

The minimum aperture can be taken as the radius of round pipe just below cut off for the TM_{01} mode: $a \approx \lambda/3$. Aperture and elastance for a typical example are plotted in Fig. 1.

Another example of a phase slip structure is the undulating structure shown in Fig. 6. Take any smooth waveguide operating in a TM mode and cause the longitudinal profile to undulate sinusoidally with amplitude δ and period p. Acceleration can be produced if the phase velocity v_p is adjusted such that the wave slips ahead of an electron bunch traveling at velocity v_e by one guide wavelength during the time it takes for the bunch to travel distance p. If $v_e \approx c$, this synchronism condition is

$$p/\lambda = [1 - (\lambda/\lambda_g)]^{-1} = [1 - (v_g/c)]^{-1} = [1 - (1 - \alpha^2)^{1/2}]^{-1},$$

where α is the ratio of the free space wavelength to the cutoff wavelength for the guide. As a simple case that can be readily calculated, consider a rectangular waveguide operating in the TM_{21}, mode in which the centerline of the guide undulates in the x-plane by amplitude $\pm\delta$ with respect to the electron beam. The elastance is

$$s = \frac{8F^2(\delta/a)\alpha^2}{ab\epsilon_0} = \frac{8F^2(\delta/a)\eta c\alpha^2}{ab},$$

where a and b are the x and y dimensions of the guide and F is the ratio of the average gradient to the peak longitudinal field. The maximum value of F is ≈ 0.55 at $\delta/a \approx 1/4$. The optimum ratio for b/a can be shown to be $1/2$. Thus

$$s_{max} \approx \frac{2.4\alpha^4}{\epsilon_0\lambda^2} \approx \frac{2.4\eta c\alpha^4}{\lambda^2}.$$

For v_g/c =0.03, $\alpha \approx 1$, p = 1.03λ and the normalized free aperture in both planes $a_\lambda \approx b/2\lambda \approx a/4\lambda \approx 1/2\sqrt{2}$, $s_{at} = 0.0336\ \Omega - m/ps$ and $s_{max} = 25\ \Omega/ps - m$ at 2856 MHz. These results can be compared to the case of a disk loaded structure at the same v_g/c where $a_\lambda = 0.14$, $s_{at} = 0.0138\ \Omega - m/ps$ and $s = 64\ \Omega/ps - m$ at 2856 MHz.

The use of a smooth waveguide with a periodic longitudinal perturbation as an accelerating structure has a long history. In 1946 E. J. Gorn[3] filed a patent for a structure constructed from a rectangular TM_{21} waveguide with periodic helical twist. More recently, J. Nation[4] has proposed an undulating circular waveguide, operating in the TM_{02} mode. He has described this type of structure as a longitudinal inverse free electron laser because of its similarity in operating principle to an FEL. Results for a_λ and s_{at} for the undulating structure discussed above, and for some additional structures listed below, are shown in Fig. 1. Unless stated otherwise the mode is $2\pi/3$.

a) Square wave wall, gap = disk = 1.75 cm at 2856 MHz

b) ZIGZAG Structure

c) ZIGZAG Structure, $\pi/2$ mode

d) Annular slot, gap = 0.584 cm, disk = 2.92 cm.

e) Circular Bellows

The zigzag structure has faces making a 70° angle with the axis. The annular slot structure is the inverse of the disk loaded structure, with radial outward slots instead of irises; *i.e.*, gaps and disks are interchanged. The circular bellows consists of connected semi circles. Note that a_λ and s_{at} for most of these structures are not much different than for the disk-loaded structure. Because they have smoother walls, however, they may have the advantages of lower wake fields and ease of fabrication. In the case of the annular slot structure, the small diameter is the maximum diameter over a large fraction of the cell length and therefore small aperture focusing magnets can be installed.

7. Conclusion

High group velocity structures represent a transition between the usual structures in use at present and structures for laser driven accelerators. The shorter RF pulse length associated with high group velocities results in lower average power, lower power dissipation and higher breakdown fields. The low dissipation and large bandwidths associated with high group velocities are especially important for long accelerators, leading to less restrictive tolerances and lower fabrication costs. Lower wakefields are also of great importance for the next generation of linear colliders.

Acknowledgment

We wish to thank John Lawson for calling to our attention Ref. 3 and to other accelerating structures of historical interest based on the phase slip principle.

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