SLAC – PUB – 4069
August 1986
T

CP VIOLATION AT THE UNIFICATION SCALE*

GREGORY G. ATHANASIU,**
SAVAS DIMOPOULOS,†‡ AND FREDERICK J. GILMAN**

** *Stanford Linear Accelerator Center*
Stanford University, Stanford, California 94305

and

† *Physics Department, Stanford University, Stanford, California 94305*

ABSTRACT

The three generation phase invariant measure of CP violation is shown to satisfy a simple and solvable renormalization group equation. Its value falls by four to eight orders of magnitude between the weak and grand unification scales in the standard model, as well as in its two Higgs and supersymmetric extensions. Such a small value of CP violation at the grand unification scale can pose a problem for baryogenesis; this is avoided if there are heavy quarks with masses close to their fixed points.

Submitted to *Physical Review Letters*

* Work supported by the Department of Energy, contract DE-AC03-76SF00515 and by the National Science Foundation, contract NSF-PHY-83-10654.

‡ A. P. Sloan Foundation Fellow

Introduction

It has recently been realized that a necessary and sufficient criterion for CP violation in the standard model with three generations can be formulated in a parametrization independent manner. This formulation is stated entirely in terms of the determinant of the commutator of the mass matrices for the charge $2e/3$ and charge $-e/3$ quarks,[1] a quantity invariant under any allowed redefinition of phases of the quark fields. It is not necessary to refer to the Kobayashi-Maskawa matrix,[2] or any parametrization thereof. Instead one can work entirely with mass matrices which involve the fundamental Yukawa couplings of the Higgs boson to the quarks in the basis of weak eigenstates.

With three generations of quarks and leptons CP is violated if and only if the single quantity,[3]

$$\det K = \det[U^\dagger U, D^\dagger D], \tag{1}$$

is non-vanishing. Here U and D are the three by three Yukawa coupling matrices for the charge $2e/3$ and $-e/3$ quarks, respectively.

In the Kobayashi-Maskawa[2] parametrization

$$\det K \propto (m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_u^2 - m_t^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_d^2 - m_b^2) \times \sin^2\theta_1 \sin\theta_2 \sin\theta_3 \cos\theta_1 \cos\theta_2 \cos\theta_3 \sin\delta \tag{2}$$

This vanishes if any quarks with the same charge have the same mass, or any of the angles θ_i assume the values 0 or $\pi/2$, or the phase δ is 0 or π.

In this paper we study the scale dependence of $\det K$. We find that it satisfies a simple renormalization group equation where the change in $\det K$ is proportional to itself. This allows a straightforward computation of its value at the grand unification scale in terms of its value at the weak scale, given an initial set of Yukawa and gauge couplings.

In the standard model with one Higgs doublet and three generations of quarks and leptons det K falls by roughly six orders of magnitude in going from the weak to grand unification scales if the t quark mass is small. When the t quark mass approaches its fixed point value[4,5] of $\sim$ 220 GeV, det K falls by about four orders of magnitude. Similar results hold in extensions of the standard model involving two Higgs doublets or supersymmetry. The decrease in det K is due primarily to the decrease in quark masses that are not near their fixed points and has important consequences for baryogenesis.

Renormalization Group Equation in the Standard Model

The renormalization group equation for det K follows from those for the Yukawa coupling matrices U and D:[6]

$$U^{-1}\frac{dU}{d\tau} = -G_U + 3\ T + \frac{3}{2}(U^\dagger U - D^\dagger D), \tag{3a}$$

and

$$D^{-1}\frac{dD}{d\tau} = -G_D + 3\ T + \frac{3}{2}(D^\dagger D - U^\dagger U). \tag{3b}$$

The conventions are those of Ref. 5, where $\tau = \frac{1}{16\pi^2}\ln(\mu/M_W)$; $T = T_U + T_D = Tr(U^\dagger U) + Tr(D^\dagger D)$; and G_U and G_D are equal to $8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{20}g_1^2$ and $8g_3^2 + \frac{9}{4}g_2^2 + \frac{1}{4}g_1^2$, respectively. Leptons have been neglected as unimportant. Using Eqs. (3) and the definition of $K = [U^\dagger U, D^\dagger D]$, we find

$$\frac{dK}{d\tau} = \{K, A\}, \tag{4}$$

where

$$A = 6\ T + \frac{3}{2}(U^\dagger U + D^\dagger D) - (G_U + G_D) \tag{5}$$

Noting that

$$\frac{d(\ln\det K)}{d\tau} = \frac{d(Tr\ln K)}{d\tau} = Tr(K^{-1}\frac{dK}{d\tau}) = 2\ TrA, \tag{6}$$

we obtain:

$$(\det K)_\mu = (\det K)_{M_W}\ e^{\int_0^\tau (2\ TrA)d\tau}, \tag{7}$$

where $TrA = \frac{39}{2}T - 3\ (G_U + G_D)$.

The behavior of $\det K$ as a function of the momentum scale μ compared to its value at the weak scale is shown in Figure 1 for various values of m_t. The Yukawa couplings of the other quarks and the values of the gauge couplings are set at M_W to their known values.[7] It is seen that $\det K$ decreases as the scale μ increases. In particular, for t quark masses below about 150 GeV, $(\det K)_\mu/(\det K)_{M_W}$ decreases by over five orders of magnitude when μ is at the grand unification scale (which we take as 10^{15} GeV, corresponding to $\tau = 0.19$). As m_t grows larger, and the corresponding Yukawa coupling approaches its fixed point,[4,5] $(\det K)_{GUT}/(\det K)_W$ approaches $\sim 10^{-4}$.

Figure 2 shows the ratio of $\det K$ at the unification scale to its value at the weak scale as a function of m_t. Here we see more directly that $\det K$ at the unification scale assumes larger values as the t quark Yukawa coupling increases toward its fixed point.[4,5]

This hints that most of the running of $\det K$ is due to the running of the quark masses rather than that of the mixing angles if we decompose $\det K$ into a product of factors, as in Eq. (2). The dashed curve in Figure 2, which shows only the effect of the running of the mass factors in Eq. (2), gives an explicit numerical demonstration that this is the case. An analytic calculation which neglects the contribution of the Yukawa couplings (a good approximation for small quark masses) shows the same result: most of the running is due to the quark masses, which fall by a factor of ~ 3 between the weak and unification scales.

With some hindsight, this is to be expected, as the mixing angles are dimensionless and functions of ratios of quark masses of the same charge. Thus they are insensitive to the running of the gauge couplings, which yield the same factor for all quarks of the same charge; the angles only run if there are large Yukawa couplings.[8]

Extension to Two Higgs Doublets and Supersymmetry

To extend our results to the case with two Higgs doublets, we need only replace Eqs. (3) by[6]

$$U^{-1}\frac{dU}{d\tau} = -G_U + 3\ T_U + \frac{1}{2}(3\ U^\dagger U + D^\dagger D), \tag{8a}$$

and

$$D^{-1}\frac{dD}{d\tau} = -G_D + 3\ T_D + \frac{1}{2}(3\ D^\dagger D + U^\dagger U). \tag{8b}$$

Correspondingly, the matrix A now is replaced by $A_{Two\ Higgs} = 3\ T + \frac{7}{2}(U^\dagger U + D^\dagger D) - (G_U + G_D)$. The form of the remaining equations is the same, as is their solution. The only important difference is that there now exist two vacuum expectation values, v_u and v_d, with the constraint that $v_u^2+v_d^2 = v^2 = (175\ GeV)^2$. Thus in addition to m_t we have an additional variable, which we take to be v_d.

In Figure 3 we plot the value of $\det K$ at the unification scale relative to its value at the weak scale as a function of v_d for various values of m_t. Again, as in Figure 2, this ratio is roughly 10^{-6} when m_t is small and grows to approximately 10^{-4} when the Yukawa coupling of the t quark approaches its fixed point.

The situation for the supersymmetric extension of the standard model is similar. Now the Yukawa coupling matrices run according to[6]

$$U^{-1}\frac{dU}{d\tau} = -G_U + 3\ T_U + (3\ U^\dagger U + D^\dagger D), \tag{9a}$$

and

$$D^{-1}\frac{dD}{d\tau} = -G_D + 3\ T_D + (3\ D^\dagger D + U^\dagger U), \tag{9b}$$

and $A_{SUSY} = 3\ T + 7\ (U^\dagger U + D^\dagger D) - (G_U + G_D)$. Because of contributions of super-partners, G_U and G_D are altered to $\frac{16}{3}g_3^2 + 3\ g_2^2 + \frac{13}{9}g_1^2$ and $\frac{16}{3}g_3^2 + 3\ g_2^2 + \frac{7}{9}g_1^2$, respectively. The behavior of $\det K$ is given in Figure 4, and is qualitatively the same as before. However, because the gauge couplings run more slowly, light quark masses decrease by a factor of ~ 5 rather than ~ 3 between the weak scale and the unification scale. Consequently $\det K$ at the unification scale is about three orders of magnitude smaller than for the standard model.

Baryogenesis

CP violation is one of the necessary conditions for baryogenesis.[9] Conversely, baryogenesis is the only probe of the strength of CP violation at the grand unification scale.

A physical quantity involving CP violation does not have to be $\det K$ times pure numbers; it can involve $\det K$ divided by other physical quantities such as quark masses, and therefore have a less dramatic decrease as we move from the weak to the unification scale. For example, analyses of baryon generation in a large class of theories[9–11] lead to a baryon to photon ratio that scales like the product of six Yukawa couplings.

If these Yukawa couplings are unrelated to quark masses, *e.g.*, involve superheavy Higgs bosons that are not in the same $SU(5)$ multiplet as those that give masses to quarks, there is little predictive power. We fix our attention instead on theories where the baryon excess originates in the Yukawa couplings responsible for quark masses, and consequently can be obtained from accessible physics of $\lesssim 10$ TeV.

If the product of the six Yukawa couplings is proportional to light quark masses, one obtains too small a baryon excess.[11] What we need are heavy generations of quarks whose masses are close to their fixed points and thus do not decrease as we go from the weak to the unification scale. Their masses are constrained from above and below: if there exist N heavy generations then no quark can have a mass above the fixed point of $250/\sqrt{N}$ GeV or else perturbative unification is lost.[5] If, on the other hand, they are much lighter than $250/\sqrt{N}$

GeV they are too far from their fixed point; their masses will decrease as we go to the unification scale, and lead to too small a baryon excess.

We conclude that, in this framework, big bang baryogenesis suggests the existence of new heavy quarks close to their fixed points. Such quarks automatically occur[12] in family unified theories.[12,13]

FIGURE CAPTIONS

1. The ratio of $\det K$ at the momentum scale μ to its value at the weak scale plotted versus $\tau = \frac{1}{16\pi^2}\ln(\mu/M_W)$ for various values of m_t.
2. The ratio (solid line) of $\det K$ at the unification scale ($\mu = 10^{15}$ GeV) to its value at the weak scale as a function of m_t. The dotted line shows this ratio due to the effect of the running of the mass factors in Eq. (2) alone.
3. The ratio of $\det K$ at the unification scale to its value at the weak scale in the case of two Higgs doublets as a function of v_d for various values of m_t.
4. The ratio of $\det K$ at the unification scale to its value at the weak scale in the supersymmetric extension of the standard model as a function of v_d for various values of m_t.

REFERENCES

1. C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985); invited talk at the XXIth Rencontre de Moriond, March 9 - 16, 1986 and University of Stockholm preprint, 1986 (unpublished)

2. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

3. This differs from the quantity defined in the first paper of Ref. 1, which is not γ_5 invariant. K contains dimensionless Yukawa couplings rather than masses, and $U^\dagger U$ and $D^\dagger D$, which are hermitian, rather than U and D, which are generally not. The quantity in Ref. 1 and $\det K$ are equivalent as far as indicating CP violation if and only if they are non-vanishing.

4. B. Pendleton and G. Ross, Phys. Lett. 98B, 291 (1981); C. Hill, Phys. Rev. D24, 691 (1981); E. Paschos, Z. Phys. C26, 235 (1984).

5. J. Bagger, S. Dimopoulos, and E. Masso, Nucl. Phys. B253, 397 (1985); Phys. Lett. 156B, 357 (1985); and Phys. Rev. Lett. 55, 1450 (1985).

6. T. Cheng, E. Eichten, and L. Li, Phys. Rev. D9, 2259 (1974); M. Machacek and M. Vaughn, Nucl. Phys. B236, 221 (1984).

7. The Yukawa coupling matrices at the weak scale are computed from the quark masses and the Kobayashi-Maskawa matrix with $\sin\theta_1 = 0.22$, $\sin\theta_2 = 0.03$, $\sin\theta_3 = 0.02$ and $\delta = \pi/4$, which are values consistent with the measurements of the magnitudes of the Kobayashi-Maskawa matrix elements and with the magnitude of ϵ in the neutral K system. For the gauge couplings we use $g_3^2 = 1.70$, $g_2^2 = 0.45$, and $g_1^2 = 0.21$ and take $\Lambda_{\overline{MS}} = 250$ MeV. We have checked that our results for the running of $\det K$ are insensitive to the choice of the Kobayashi-Maskawa angles and to the values of the light quark masses.

8. In $\det K$ there are twelve powers of Yukawa couplings. In the analogous quantity defined in the first paper of Ref. 1 there are six powers. Consequently, if we had chosen to consider the value of the latter quantity at the

grand unification scale compared to its value at the weak scale, we would have found a decrease by about three orders of magnitude. Which quantity is relevant (if either is) depends on the physics of the mechanism of CP violation in the particular problem at hand.

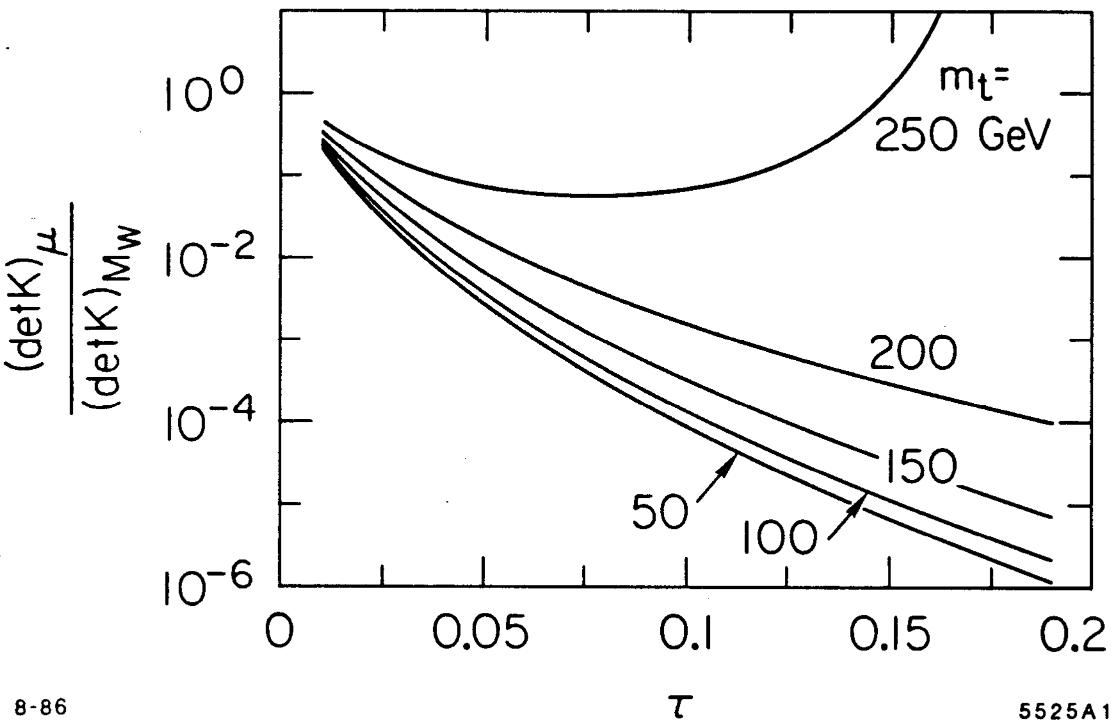
9. A. D. Sakharov, Zh. Eksp. Teor. Fiz. Pisma 5, 32 (1967) [JETP Lett. 5, 24 (1967)]; M. Yoshimura, Phys. Rev. Lett. 41, 281 (1978); S. Dimopoulos and L. Susskind, Phys. Rev. D18, 4500 (1978); D. Toussaint, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D19, 1036 (1979).

10. See, for example, the review by E. W. Kolb and M. S. Turner, Ann. Rev. Nucl. Sci. 33, 645 (1983) and references therein. We thank them for discussions on the history of these calculations.

11. J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Phys. Lett. 80B, 360 (1979) and 82B, 464 (1979) (*Erratum*); S. Weinberg, Phys. Rev. Lett. 42, 850 (1979); D. V. Nanopoulos and S. Weinberg, Phys. Rev. D20, 2484 (1979); S. Barr, G. Segre, and H. A. Weldon, Phys. Rev. D20, 2494 (1979); G. Segre and M. S. Turner, Phys. Lett. 99B, 399 (1981).

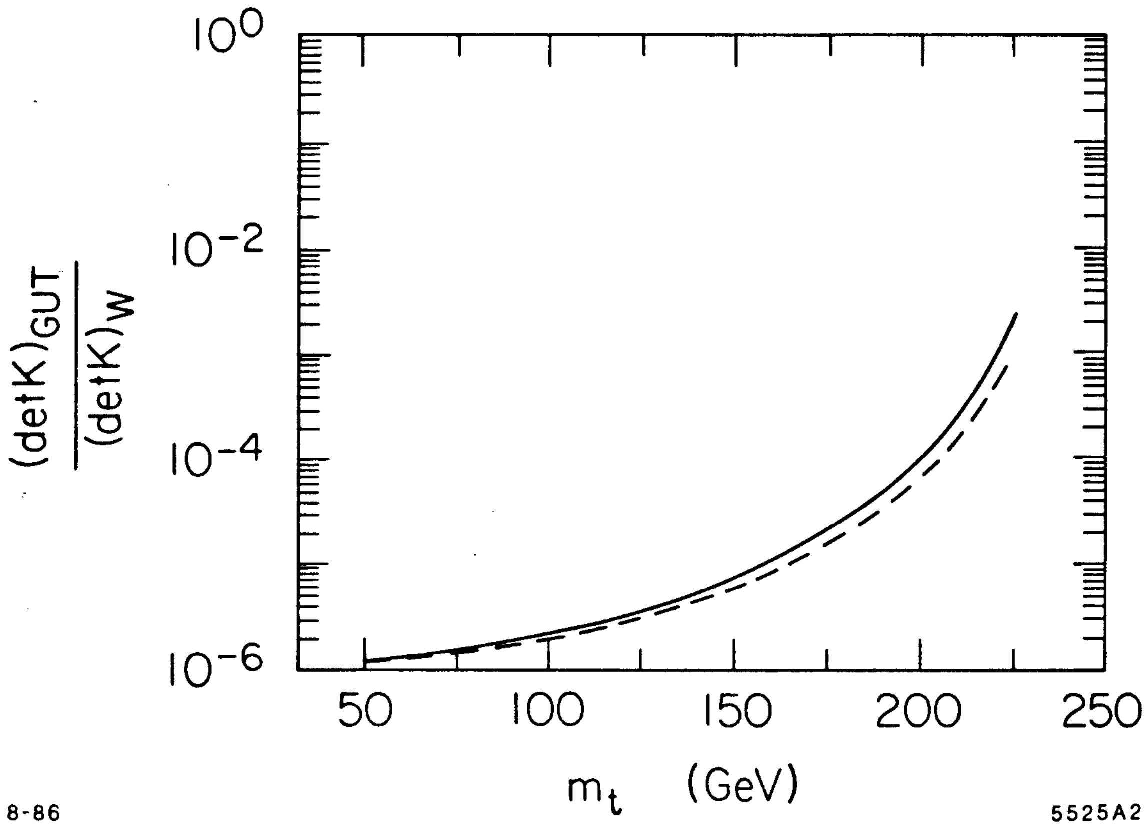
12. J. Bagger and S. Dimopoulos, Nucl. Phys. B244, 247 (1984); J. Bagger, S. Dimopoulos, E. Masso, and M. H. Reno, Phys. Rev. Lett. 54, 2199 (1985); Nucl. Phys. B258, 565 (1985).

13. M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam, 1979), p. 317; F. Wilczek and A. Zee, Phys. Rev. D25, 553 (1982).



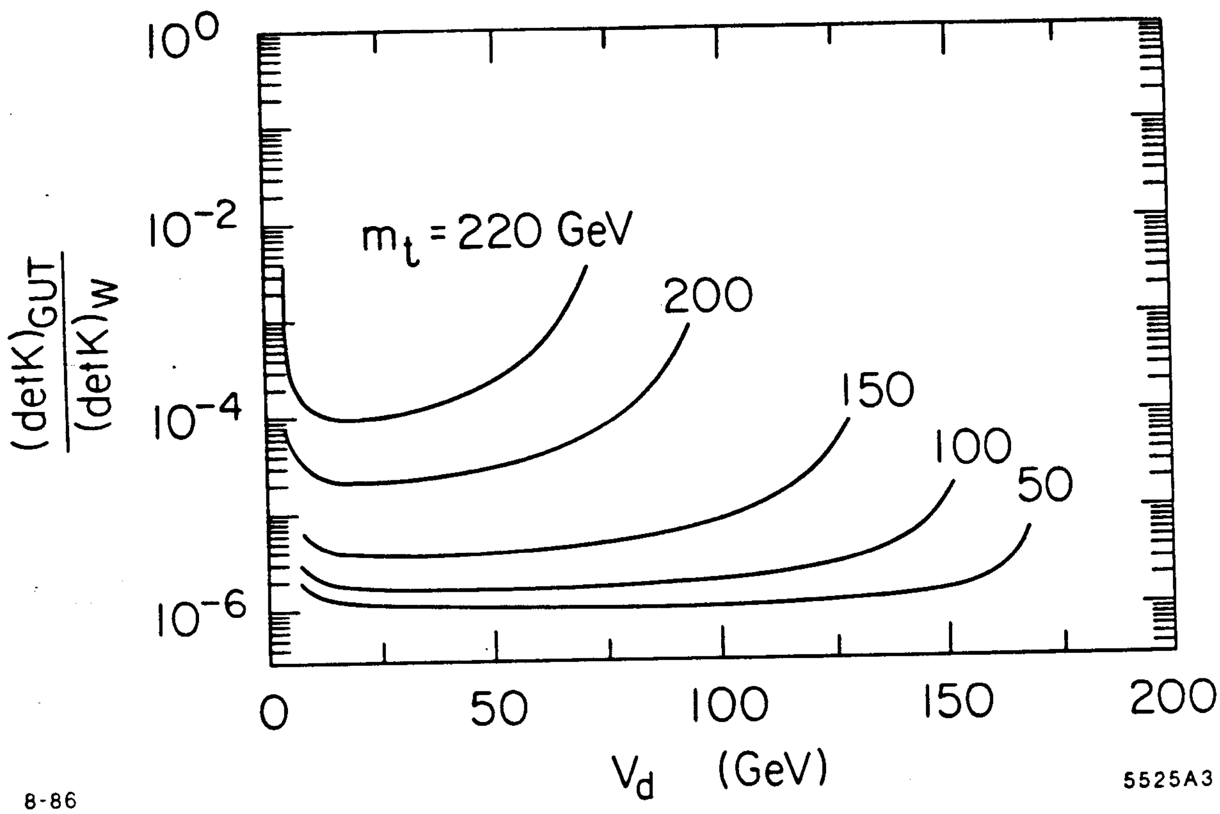
10^0
10^{-2}
10^{-4}
10^{-6}
$\frac{(\det K)_\mu}{(\det K)_{M_W}}$
m_t=
250 GeV
200
150
50
100
0
0.05
0.1
0.15
0.2
8-86
τ
5525A1

Fig. 1



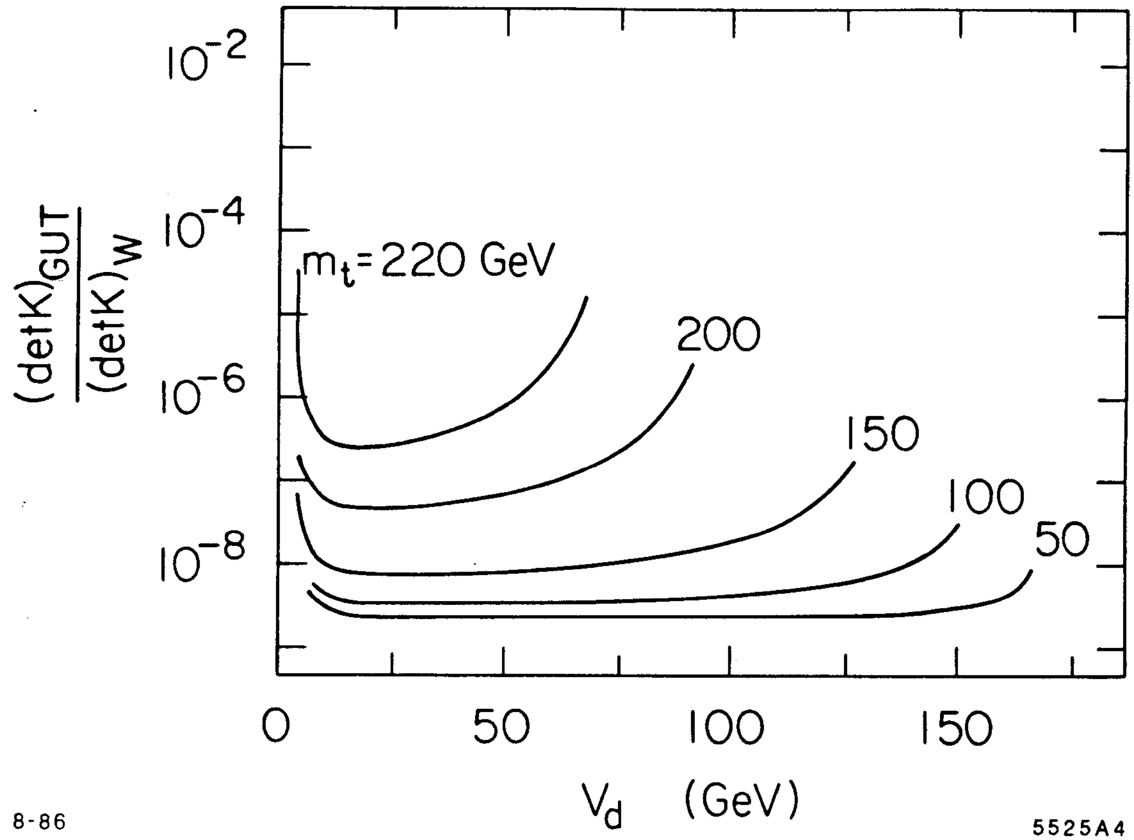
10^{0}
10^{-2}
10^{-4}
10^{-6}
$\frac{(\det K)_{GUT}}{(\det K)_W}$
50
100
150
200
250
m_t (GeV)
8-86
5525A2

Fig. 2



10^0
10^{-2}
10^{-4}
10^{-6}
$\frac{(\det K)_{GUT}}{(\det K)_W}$
m_t = 220 GeV
200
150
100
50
0
50
100
150
200
V_d (GeV)
8-86
5525A3

Fig. 3



10^{-2}
10^{-4}
10^{-6}
10^{-8}
$\frac{(\det K)_{GUT}}{(\det K)_W}$
m_t=220 GeV
200
150
100
50
0
50
100
150
V_d (GeV)
8-86
5525A4

Fig. 4