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# On Our Theoretical Understanding of Charm Decays*

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## ABSTRACT

A detailed description of charm decays has emerged. I sketch the various concepts involved. Although this description is quite successful in reproducing the data the chapter on heavy flavour decays is far from closed. Relevant questions like on the real strength of weak annihilation, Penguin operators, etc. are still unanswered. I try to identify important directions in future work, both on the experimental and theoretical side.

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## 1. Enlightened History

To recount history the way it actually happened is at times more confusing than illuminating. Charm decays seem to be such a case. Therefore, I will describe history the way it should have happened.

Treating charm decays in complete analogy to muon decays - which is the so-called spectator picture - one finds for the charm lifetime (ignoring all QCD corrections)

$$\tau(\text{charm}) \simeq \frac{1}{5} \left(\frac{m_\mu}{m_c}\right)^5 \tau(\mu) \sim 7 \times 10^{-13} \text{ sec} \tag{1}$$

for $m_c = 1.5$ GeV. Comparing that with the experimental findings[1] $\tau(D^\circ) \sim 4.4 \times 10^{-13}$ sec, $\tau(D^+) \sim 10^{-12}$ sec one should first note how remarkably close the naive prediction (1) is to these data; for one has to keep in mind that Eq. (1) represents an extrapolation over more than six orders of magnitude! Having said that it is then fair to state that the agreement between the prediction (1) and the data is not perfect since $\tau(D^+)/\tau(D^\circ) \sim 2.2$ experimentally while naively one expects $\tau(D^+) = \tau(D^\circ)$. This shows that hadronic effects are important - yet they are less rampant than for kaons where one has $\tau(K^\pm)/\tau(K_s) \sim 135$.

The MARK III analysis[1] reveals that most non-leptonic $D$ decays lead to two-body final states of the type $D \to PP,\ PV$ where $P[V]$ denotes a pseudoscalar [vector] meson. The remainder could largely be made up by $VV$ final states. (It is true that this dominance of two-body final states had not been anticipated; yet in the noble tradition of "Monday morning quarterbacking" one can argue that one should have guessed it).

At this point one concludes that a detailed understanding of charm decays requires inclusion of ordinary hadronic effects like form-factors, final state interactions etc.; gross features on the other hand can be reproduced from fairly simple quark level calculations up to a factor of two or so as another manifestation of the idea of duality.

## 2. The Art of Theoretical Engineering

Stech and co-workers[2] had developed a comprehensive framework for describing all two-body decay modes of charm (and bottom) states before detailed data existed. Their prescription is based on five ingredients:

(i) the usual effective quark operators for charm decay are employed:

$$\mathcal{L}(\Delta C=1) \propto \frac{c_+ + c_-}{2}\, \bar{s}_L\gamma_\mu c_L \bar{u}_L\gamma_\mu d_L + \frac{c_+ - c_-}{2}\, \bar{u}_L\gamma_\mu c_L \bar{s}_L\gamma_\mu d_L \qquad (2)$$

where the coefficients $c_\pm$ contain the QCD radiative corrections.[1]

(ii) Only quark decay diagrams are retained while $W$ exchange diagrams are ignored.

(iii) The quark currents in (2) are replaced by the corresponding hadronic currents $J_H$ when computing transition amplitudes. This is treated as a trivial procedure for diagrams like Fig. 1(a) where the colors are already properly aligned; for diagrams with the topology of Fig. 1(b) one introduces an a-priori undetermined new parameter $\xi$ since the colors are not automatically matched: (Naively, by just counting degrees of freedom one would have $\xi \sim 1/N_C = 1/3$.) Thus one writes for the transition amplitude

$$\begin{aligned} T(D\to f) \propto\ & a_1\langle f|(\bar{s}_L\gamma_\mu c_L)_H(\bar{u}_L\gamma_\mu d_L)_H|D\rangle \\ &+ a_2\langle f|(\bar{u}_L\gamma_\mu c_L)_H(\bar{s}_L\gamma_\mu d_L)_H|D\rangle \end{aligned} \qquad (3)$$

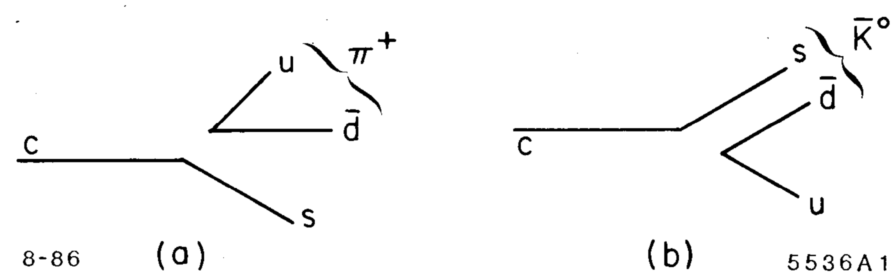


Fig. 1: (a) Quark decay diagram with color alignment. (b) Quark decay diagram without automatic color alignment.

with

$$a_1 = \frac{1}{2}\,(c_+ + c_-) \;+\; \frac{\xi}{2}\,(c_+ - c_-)$$
$$a_2 = \frac{1}{2}\,(c_+ - c_-) \;+\; \frac{\xi}{2}\,(c_+ + c_-) \tag{4}$$

(iv) For two-body final states, i.e. $f = PP,\ PV,\ VV$, one employs a factorization ansatz

$$\langle f|J_H \cdot J_H|D\rangle \simeq \langle P \text{ or } V|J_H|0\rangle\langle P \text{ or } V|J_H|D\rangle \tag{5}$$

These simple matrix elements are given in terms of decay constants $f_\pi$, $f_K$, $f_\rho$, ... and nearest neighbour pole terms.

(v) Final state interactions (phase shifts, absorption etc.) are included as best as possible.

— There are basically just two free parameters to be fitted, namely $a_1$, $a_2$ (although in practice one has some freedom in parameterizing the relevant final state interactions). Keeping this in mind I find the success of the Bauer-Stech fit to some twenty $D$ decay modes[1] very remarkable. They obtain[2]

$$a_1 \simeq 1.3 \pm 0.1\,; \quad a_2 \simeq -0.5 \pm 0.1 \tag{6}$$

Using values for $c_\pm$ (see Eq. (2)) as obtained from a perturbative QCD calculation[1] – a procedure that seems reasonable although its correctness has not been proven beyond a reasonable doubt – one translates Eq. (6) into

$$\xi \sim 0 \tag{7}$$

Adding up all the two-body modes one finds in this description

$$\frac{\tau(D^+ \to PP,\ PV,\ VV)}{\tau(D^\circ \to PP,\ PV,\ VV)} \sim 2-3 \tag{8}$$

which is easily understood. $D^\circ$ (and $F^+$) decays receive incoherent contributions from the $a_1\langle f|J_H \cdot J_H|D^\circ\rangle$ and $a_2\langle f|J_H \cdot J_H|D^\circ\rangle$ terms, whereas for $D^+$ decays they contribute coherently to the same channel; the resulting interference has to be destructive since $a_1 \cdot a_2 < 0$ – as expected from QCD. This slows $D^+$ decays down while not affecting $D^\circ$ and $F^+$ decays. That something like this affects charm decays in a significant way was first suggested in Ref. 3 where a simple quark model illustration was given. For me it was hard to see how the necessary coherence could be maintained in genuine multi-body decays like $D^+ \to \overline{K}{}^\circ\pi^+\pi^\circ$, where $\pi^+\pi^\circ$ do not form a $\rho$ meson. The MARK III findings on the relative insignificance of such modes have erased this criticism.

## 3. Evidence for $W$ Exchange?

There are other decay mechanisms that can produce a lifetime difference $\tau(D^+) > \tau(D^\circ)$, namely $W$ exchange and weak annihilation diagrams (both hereafter referred to as $WA$) which are depicted in Fig. 2. On the Cabibbo allowed level they can contribute to $D^\circ$, $F^+$ and $\Lambda_c^+$ but not to $D^+$ decays.

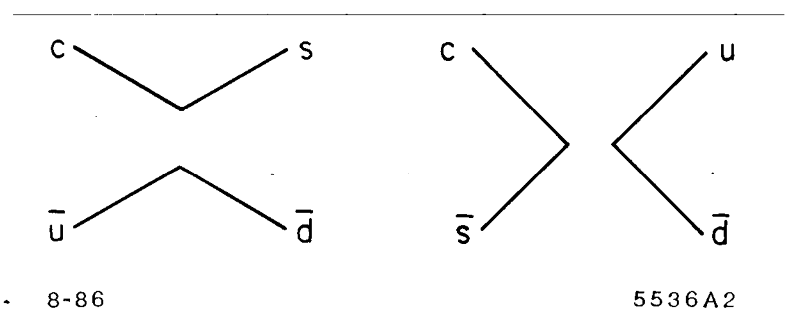


Fig. 2 : $W$ exchange and weak annihilation diagram.

Originally they had been discarded as insignificantly small; for in analogy to $\pi \to \ell\nu$ decays one finds these contributions to be helicity suppressed by $(m_s/m_c)^2$ where $m_s[m_c]$ denotes the strange [charm] quark mass. Prodded by early experimental data on charm lifetimes theorists found however ways of avoiding this helicity suppression; in the presence of gluons the $(c\bar{q})$ pair can find itself in a spin one configuration; its annihilation into a pair of light fermions will then not be impeded by small mass ratios.

There exists another effect limiting the strength of $WA$: weak interactions are local on the distance scales that are relevant for charm (and bottom) decays. Thus $WA$ requires the $c$ and $\bar{q}$ pair to come together in space for which the probability is given by the $c\bar{q}$ wavefunction at zero separation. Unfortunately we do not know at present how to calculate the wavefunction directly from the theory in a reliable fashion. Thus no firm prediction can be made, only educated guesses.

In one of these guesses it was suggested[4] to treat gluon bremsstrahlung off the initial quark lines by a perturbative approach which leads to

$$\Gamma_{WA,\ \text{perturb.}}(D^\circ) \propto G_F^2 \left(\frac{f_D}{m_u}\right)^2 m_D^5 \tag{9}$$

To obtain $\Gamma_{WA,\ \text{perturb.}} \simeq \Gamma_{\text{spect.}}$ – i.e. to blame $\tau_{D^+} \sim 2\ \tau_{D^\circ}$ solely on $WA$ – one has to require[5]

$$\frac{f_D}{m_u} \sim 2.2 \tag{10}$$

Using constituent masses, i.e. $m_u \sim$ 330 MeV, the requirement of Eq. (10) translates into $f_D \sim$ 720 MeV. This is much larger than most guestimates: using QCD sum rules, potential models or bag models one finds $f_D$ ranging between 100 and 230 MeV. Alternatively one can equate the measured $D^* - D$ mass difference with the quantity that is obtained in an one-gluon-exchange ansatz

$$m(D^*) - m(D) \simeq \frac{8\pi}{27} \frac{M_D}{m_c m_u} \alpha_s f_D^2 \tag{11}$$

which leads to $\sqrt{\alpha_s}\ f_D \sim$ 200 MeV.

The MARK III group has just now derived an upper bound on $f_D$ by searching for $D^+ \to \mu^+\nu$ :[1]

$$f_D \leq 340 \text{ MeV} \tag{12}$$

Hence one concludes that $WA$ implemented via perturbative gluon bremsstrahlung can at best introduce a 25% difference in $\tau(D^+)$ and $\tau(D^\circ)$.

There appears an easy way to avoid such a conclusion: if one uses smaller mass values for the up quark, say $m_u \sim 100$ MeV, one can increase the strength of $WA$ at will. However, I harbor grave doubts that Eq. (9) represents a bona fide perturbative calculation: the problem is not that one integrates down to $m_u$ but that almost the whole contribution originates from the regime $m_u$ to $\sim 2\ m_u$. Therefore it does not make much sense to me to lower $m_u$ even further. The emergence of gluons should then be treated as a non-perturbative phenomenon by putting them into the hadronic wavefunction.[6] Such an ansatz leads to

$$\Gamma_{WA,\ \text{non-pert.}}(D^\circ) \propto G_F^2 \tilde{f}_D^2\ m_D^3 \tag{13}$$

where $\tilde{f}_D$ contains the wavefunction describing the ($c\bar{q}+$ gluon) configuration. The quantity $\tilde{f}_D$ is often identified with $f_D$ measured in $D \to \ell\nu$ decays, yet such a procedure is in general <u>not</u> correct. For the $c$ and $\bar{u}$ quarks annihilate in the presence of gluon fields; therefore it will <u>not</u> lead to a purely leptonic final state and bounds on $D \to \ell\nu$ per se do not teach us something on the $c\bar{q}$ + glue wavefunction.

The preceding discussion shows that $WA$ could produce a 25% lifetime difference "perturbatively" or considerably more non-perturbatively. Considering this vagueness in the prediction it is then suggestive to analyze whether special decay modes could clearly reveal the significance of $WA$. Two decay modes have been enlisted to testify in favor of $WA$: semi-leptonic $D^\circ$ decays and the $D^\circ \to \overline{K}^\circ\phi$ mode.

(i) Semi-leptonic $D^\circ$ decays

The measured semi-leptonic branching ratio of $D^\circ - b_{SL}(D^\circ) \sim 7.5\%$ – is compared with the alleged prediction in the spectator ansatz – $b_{SL}(D^\circ) \sim 14\%$. If the latter were a firm prediction it would clearly establish that $WA$ is essential for describing $D^\circ$ decays. Unfortunately this is not the case. For the prediction quoted above is based on a very simple quark level calculation: $b_{SL}(D) \sim \frac{1}{2+2c_+^2+c_-^2}$, where the naive assignment $\xi = \frac{1}{3}$ was used, which – as discussed above – does not describe the non-leptonic $D$ decays.

Instead a more reasonable ansatz is given by[7]

$$b_{SL}(D^\circ) \sim \frac{1}{2+\frac{3}{2}(c_+^2+c_-^2) + \frac{3\xi}{2}(c_+^2-c_-^2)} \tag{14}$$

which gives $b_{SL}(D^\circ) \sim 10-14\%$ for $\xi = 0$. But the main point is – as stated in the beginning – that these simple quark level computations cannot be trusted to better than a factor of two in $D$ decays. Indeed, if one adds up all the two-body modes in the Bauer-Stech description one finds

$$\frac{BR(D^\circ \to \ell\nu\ P \text{ or } V)}{BR(D^\circ \to PP,\ PV,\ VV)} \sim 8\% \tag{15}$$

Therefore, one cannot cite the measured semi-leptonic branching ratio of $D^\circ$ as firm evidence for $WA$.

(ii) $D^\circ \to \overline{K}{}^\circ\phi$

It had been suggested some time ago[8] that observing $D^\circ \to \overline{K}{}^\circ\phi$ with a branching of not much less than 1% would establish $WA$. Experimentally, a branching ratio of slightly more than 1% was indeed found.[1] However, it has been pointed out recently by Donoghue[9] and by Stech[2] that this evidence is not compelling: it is quite conceivable that in particular the very prominent mode $D^\circ \to K^-\rho^+$ could – via rescattering – generate some rate for $D^\circ \to \overline{K}{}^\circ\phi$ even in the absence of $WA$. In passing it should be noted that these rescattering diagrams do involve $q\bar{q}$ annihilation; however this occurs due to the strong interactions with a typical range of 1 fermi and not the pointlike weak interactions; thus it is clearly distinct from $WA$.

These issues will be clarified considerably once detailed data on $F$ decays are available, in particular $BR(F \to \ell + X)$ and $BR(F^+ \to \pi\rho,\ \omega\pi)$. The latter branching ratio must amount to several percent if $WA$ is significant.

To summarize the status of $WA$ in charm decays:

– A $\simeq$ 20% contribution to $\tau(D^\circ,\ F^+)$, and possibly more to $\tau(\Lambda_c^+)$, is a reasonable though not firm ball park estimate.

– Its strength could actually be considerably larger; however a phenomenological need for its contributions has at present not been established in an unequivocal fashion.

At this point the natural question arises why worry about $WA$ since there is no really compelling evidence for it at present and since it is not based on highly lucid concepts. There are several reasons why one has to be concerned about the strength of $WA$:

(a) Establishing the presence of gluons in the hadronic wavefunction would represent a very nice (though not totally) surprising result.

(b) A comparison of $D^\circ \to K^+K^-$ with $D^\circ \to \pi^+\pi^-$ can give us some information on the strength of penguin transitions; yet the presence of $WA$ can affect the conclusion.

(c) The MARK III analysis of $D^\circ\overline{D}^\circ \to (K^\pm + \pi's)(K^\pm + \pi's)$ transitions has yielded some marginal evidence for $D^\circ - \overline{D}^\circ$ mixing[1] (with a strength well beyond Standard model expectations). Unfortunately the same final state can be produced by doubly Cabibbo suppressed $D$ decays; $WA$ would affect them.

(d) A difference in the lifetimes of bottom mesons - $\tau(B^{\pm}) \neq \tau(B^{\circ})$ - would severely affect any conclusion that one draws on the size of the $KM$ parameter $V(bu)$ and on $B^{\circ} - \overline{B}^{\circ}$ mixing when studying semi-leptonic $B$ decays. Scenarios involving $WA$ can be scaled up to make predictions on $\tau(B^{\pm})/\tau(B^{\circ})$ by using $\tau(D^{+})/\tau(D^{\circ})$ as input. Treating $WA$ perturbatively one finds

$$\frac{\tau(B^{+})}{\tau(B^{\circ})} \simeq 1 + \frac{\tau(D^{+})}{\tau(D^{\circ})} \times \frac{m_c}{m_b} \tag{16a}$$

whereas a different scaling law applies for the non-perturbative treatment

$$\frac{\tau(B^{+})}{\tau(B^{\circ})} \simeq 1 + \frac{\tau(D^{+})}{\tau(D^{\circ})} \times \left(\frac{m_c}{m_b}\right)^2 \tag{16b}$$

In both cases one estimates

$$\frac{\tau(B^{+})}{\tau(B^{\circ})} \lesssim 1.2 \tag{17}$$

Although I consider this a fairly safe prediction, one would prefer to have a measurement of it.

(e) If $WA$ were a significant contributor to $D$ decays it would still have some impact on $B$ decays. Interference between $WA$ and quark decay would then allow certain $CP$ asymmetries to show up that otherwise were absent.[10]

## 4. "Computational Power to the Masses:" The $1/N$ Approach

The prescription of Stech and co-workers works quite well for $D$ decays. Nevertheless it is fair to say that

(a) its description of the data is not perfect; very recently a new reason for concern has appeared: MARK III has found that only slightly more than 50% of the $D \to \ell\nu K\pi$ transitions come from $D \to \ell\nu K^*$. Not only is this result quite different from theoretical expectations,[2] but it raises – by extrapolation – serious worries about our ability to understand semi-leptonic $B$ decays and extract $|V(bu)/V(bc)|$ from there.

(b) it is not very elegant.

The fit to the data that a priori could have yielded any value for $\xi$ seems to favor $\xi \simeq 0$; first indications suggest that such a value allows to describe two-body $B$ decays as well. It was noted by Buras and coworkers[11] that a consistent application of the $1/N$ approach – $N$ stands for the number of colors – could bring simplicity back to the theoretical description. Its basic rules are indeed simple:

(i) Use the usual transition operators $0_\pm$ with coefficients $c_\pm$ as obtained from perturbative QCD.

(ii) Expand the appropriate matrix element into powers of $1/N$:

$$\langle M_1\ M_2\ |\mathcal{L}_{eff}|\ D\rangle = \sqrt{N}\left\{b_o + \frac{b_1}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)\right\}$$

— In practice only the leading term with coefficient $b_o$ is retained.

(iii) To compute the coefficient $b_o$, one draws all the quark diagrams; the hadrons are described by their valence quarks only.

(iv) Every closed quark loop yields a factor $N$; every hadron introduces a normalization factor $1/\sqrt{N}$; a factor $1/\sqrt{N}$ enters also through quark-gluon couplings.

These rules are easy to apply: only valence quarks have to be considered and final state inter-actions are ignored to leading order in $1/N$ (see rule (iv)). Although this approach certainly increases the transparence and simplicity of the theoretical description and is very user-friendly, it does not provide a fully satisfactory framework:

(a) The non-leading terms in $1/N$ are dropped by fiat; our theoretical understanding is therefore not advanced – unless at least the first non-leading corrections are computed.

(b) Although the overall fit to the data is not bad there are obvious discrepancies, for example $BR(D^\circ \to \overline{K}^\circ \phi)$ and $BR(D^\circ \to K^+K^-)/BR(D^\circ \to \pi^+\pi^-)$ typically come out too small.

This presumably means that final state interactions etc. cannot be ignored, i.e. that non-leading terms are significant for $D$ decays.

## 5. Summary

We do have now a very decent approximate description of charm decays. There is no clearly established need to have $WA$ as the major source of the lifetime differences. The maturity level we have reached is such that we can address fairly subtle issues:

(i) does $WA$ really contribute $\sim 20\%$ to $\tau(D^\circ)$, $\tau(F^+)$? Is it more or is it less?[12]

(ii) Are Penguin operators relevant for Cabibbo suppressed charm decays?[12]

(iii) Do we understand doubly Cabibbo suppressed decays?

Continuing analysis of even more decay modes of $D^{\pm}$, $D^{\circ}$, $F^{\pm}$ and $\Lambda_c$ and of exclusive $B$ decays will help to clarify these issues. Just one simple example: does $\frac{BR(D^+ \rightarrow \pi^{\circ}\pi^+)}{BR(D^+ \rightarrow \overline{K}^{\circ}\pi^+)} = \frac{1}{2}\, tg^2\theta_c$ really hold or not? A violation of this relation would yield very useful information on $SU(3)_{\text{Flavour}}$ violations like $f_\pi/f_K$ etc. On the theoretical side it would represent a major advance if the contributions that are non-leading in $1/N$ could be computed consistently.

A close feed-back between theory and experiment has clearly improved our understanding of heavy flavour decays. There is every reason to believe that this story will repeat itself in the future: final success – after some ups and downs.

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