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Classical Derivation Of Black Hole Entropy*

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ABSTRACT

The proportionality between black hole entropy and area is derived from classical thermodynamics. The relationship between the classical and quantum formulas is shown to be similar to that of black body radiation. Classical thermodynamics is shown to imply certain characteristics for classical waves which are normally thought to be quantum mechanical in origin.

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1. Introduction

Historically the formulas for the entropy and temperature of many systems were first derived using classical thermodynamics. Later the formulas were refined using quantum mechanics. For example, black body radiation obeys the classical Stefan-Boltzmann Law[1]

$$E/V = \sigma T^4, \tag{1.1}$$

$$S/V = \frac{4}{3}\sigma T^3, \tag{1.2}$$

where E is energy, T is temperature, V is volume and S is entropy. σ is an integration constant which, when evaluated quantum mechanically, is found to be

$$\sigma = \frac{\pi^2}{15\hbar^3}, \tag{1.3}$$

$(G = c = k_B = 1)$.

For black holes on the other hand, the corresponding Bekenstein-Hawking[2,3] formulas

$$T_{BH} = \frac{1}{8\pi\varsigma} g_H, \tag{1.4}$$

$$S_{BH} = \varsigma A, \tag{1.5}$$

$$\varsigma = \frac{1}{4\hbar}, \tag{1.6}$$

were first derived quantum mechanically. Here

$$A = 4\pi(r_+^2 + a^2) = (area), \tag{1.7}$$

$$g_H = \frac{2\pi(r_+ - r_-)}{A} = (surface\ gravity), \tag{1.8}$$

$$r_{\pm} = M \pm \sqrt{M^2 - a^2 - Q^2}, \tag{1.9}$$

$$a = J/M, \tag{1.10}$$

and M, J, and Q are the mass, angular momentum, and charge of the black hole. (It is sometimes said that Bekenstein's original argument was "classical." However, this argument makes essential use of energy quantization and $\hbar$ appears in it explicitly, so it is not classical in the ordinary sense.)

The fact that (1.4) and (1.5) have never been derived from classical thermodynamics makes them appear different from (1.1) and (1.2) . This has perhaps contributed to the idea that gravity has a somehow deeper relationship to thermodynamics than do other branches of physics. The classical derivation of (1.4) and (1.5) is straightforward. It relies on slight variations of thought experiments which have become standard in the literature. One must just insure that at each step, only classical reasoning is in fact employed. Since the derivation is based on the axioms of thermodynamics, the conclusions will be valid in the "classical regime," a term which will be made precise in the next section.

2. Derivation

I adopt a 19th century viewpoint: No quantum mechanics, no statistical mechanics. Just as the classical derivation of (1.1) and (1.2) required the assumption that a black body's temperature and entropy are definite functions of its energy, volume, and pressure, I will assume that a black hole's entropy is a function of its macroscopic parameters. Taken together, various "no hair" theorems strongly imply that isolated black holes evolve asymptotically toward a limit which is completely described by 3 parameters, which may be taken to be mass, angular momentum and charge.[4] Thus

$$S_{BH} = S_{BH}(M, J, Q). \tag{2.1}$$

By means of Penrose processes,[5,6] one can alter a black hole's parameters arbitrarily, provided that the area does not decrease. These processes involve interactions with point particles and hence no exchange of entropy with the environment. Thus, two black holes with the same area must have the same entropy, since otherwise one could violate the second law by Penrose processes. This implies that the entropy of a black hole is a function only of its area:

$$S_{BH} = f(A). \tag{2.2}$$

Differentiating (2.2) and substituting (1.7) through (1.10) gives

$$dS = f'(A)dA = \frac{8\pi f'(A)}{g_H}(dM - \Omega dJ - \Phi dQ) \tag{2.3}$$

where

$$\Omega = \frac{4\pi a}{A}, \tag{2.4}$$

$$\Phi = \frac{4\pi r_+ Q}{A}, \tag{2.5}$$

and f' is the derivitive of f with respect to its argument.

Rearranging terms in (2.3) gives

$$dM = \tau dS + \Omega dJ + \Phi dQ, \tag{2.6}$$

where

$$\tau = \frac{g_H}{8\pi f'(A)}. \tag{2.7}$$

I now restrict to the case of a Schwarzschild hole ($J = Q = 0$). This is not necessary, but it simplifies the argument and is sufficient for the purpose at hand because by (2.2) , the entropy of any black hole is equal to that of the Schwarzschild hole with the same area.

It follows immediately from (2.6) that a Schwarzschild hole must radiate every massless field at a temperature τ defined by (2.7) . If it did not radiate or radiated at a temperature above or below τ, or in a "non-thermal" manner, one could arrange to reduce net entropy by placing it in an appropriate bath of thermal radiation, possibly with a system of filters.

Thus τ is indeed the temperature of the hole measured at infinity:

$$T_{BH} = \tau = \frac{g_H}{8\pi f'(A)}. \tag{2.8}$$

By "thermal" radiation I mean only that the flux is that emitted by a black body whose energy distribution is some definite (but from a classical standpoint, unknown) function of temperature and frequency $u(\omega, T)$. This distribution may depend on the type of massless field. Actually, of course, it is not strictly speaking this flux which is observed, but black body flux as observed through the angular momentum barrier filter.[7] Only if the black hole produces such a filtered thermal flux can it be in equilibrium with black body radiation and thus avoid the possibility of declining net entropy.

One may now consider doing an experiment first proposed by Geroch[8] . One adiabatically lowers a perfectly reflecting box filled with electromagnetic radiation at a temperature $T \gg T_{BH}$ to a Schwarzschild radius r, close to the event horizon. One then exchanges radiation with the hole and, again adiabatically, raises up the box. The local temperature at which the exchange takes place must be

$$T_\chi(M) = \frac{T_{BH}(M)}{\chi}, \tag{2.9}$$

where

$$\chi = \sqrt{1 - 2M/r}, \tag{2.10}$$

is the red-shift factor. Otherwise, again, one could arrange to lower the net entropy through the exchange.

Geroch's thought experiment was originally subject to the following *quantum mechanical* objection by Bekenstein[2] : Bekenstein asserted that one cannot lower the box arbitrarily close to the horizon because it must have some finite height. If the height of the box were less than $b \simeq \hbar/T$, the contents would not have a Planckian distribution. This objection was not well-founded. The geometry of the box is irrelevant; one could just as well use a two-dimensional box. All that matters about the distribution is that it is thermal, not necessarily Planckian. However, there is another possible essentially *classical* objection which evidently does not appear in the literature: One might argue that for the box to be nearly perfectly reflecting, it would have to be nearly infinitely massive. The presence of such a massive box would alter the gravitational field. Thus, on *classical* grounds, Geroch's apparatus would not be a *test* apparatus. In Appendix A, I show that this effect can, in principle, be kept very (though perhaps not arbitrarily) small for electromagnetic radiation. On the other hand, it prohibits the adiabatic lowering of boxes filled with gravitational waves. For this reason, I use only boxes with electromagnetic radiation in this derivation.

Now compare the result (2.9) to the gravitational "pull" (geodesic deviation) g_χ felt by a local stationary observer,

$$g_\chi(M) = -\frac{d\chi}{dr} \simeq \frac{g_H(M)}{\chi}. \tag{2.11}$$

Combining equations (2.8) , (2.9) , and (2.11) gives

$$T_\chi(M) = \frac{1}{8\pi f'(A)} g_\chi(M) \tag{2.12}$$

If the temperature measurement $T_\chi(M)$ could be shown to be *local*, both in the sense of *measuring locally* and in the sense of *measuring an effect which is local in origin*, then by the principle of equivalence, $T_\chi(M)$ could depend only on the local "pull" felt by the observer, $g_\chi(M)$, and not on the presence, absence, or size of a nearby black hole. One could then conclude from (2.12) that $8\pi f'(A)$ is a universal constant of nature, or

$$f'(A) = \varsigma = constant. \tag{2.13}$$

In order to understand why $T_\chi(M)$ is indeed a *local* physical quantity even though one often hears that Hawking radiation is a *global* effect, it is necessary to disentangle conflicting notions of the opposition "local/global." "Local" in the equivalence principle sense means "small on the scale of the background curvature." There are three length scales in the problem:

(i) $M =$ (*scale of the background curvature*)

(ii) $\frac{1}{g_\chi(M)} =$ (*acceleration length scale*)

(iii) $\lambda_\chi =$ (*typical thermal radiation wavelength*)

The last two scale as χ while the first does not. This implies that by moving sufficiently close to the hole, one may make the wavelength arbitrarily small compared to the background curvature. Thus the *measurement* is local. The temperature measurement made by an accelerated observer in flat space would also qualify as local, since in this case the background curvature scale is infinite. On the other hand, temperature measurements at corresponding wavelengths in the neighborhood of relativistic stars would not qualify as local. This is because the weak energy condition places restrictions on the equation of state of such stars, which in turn place a definite lower limit on the star-surface red-shift factor, a limit which is of order unity. [With sufficient ingenuity, one may construct "stars" whose surface red-shift factor is arbitrarily small[9] . However, it turns out that thermodynamics conspires to prevent one from using the equivalence principle to establish an ambiant temperature near such "stars". This is discussed in Appendix B.]

[Now that these scales have been defined, it is possible to make precise what is meant by the "classical regime." Unless λ_χ can be made to satisfy

$$M \gg \lambda_\chi \gg \hbar^{\frac{1}{2}}, \tag{2.14}$$

the above thought experiment cannot actually be carried out without quantum gravity becoming a factor. Of course it is impossible to define this scale from *within* classical thermodynamics.]

The *effect* is also *local in origin.* The observer is incapable of seeing global effects (that is, effects arising from the black hole as a whole) because these exist on a scale M which is large compared to his acceleration scale, $1/g_\chi(M)$. Of course, the observer "sees" an event horizon, but he attributes this to his local acceleration and not to any global phenomena. Global effects represent a correction of order χ to this conclusion. That is, it is just this horizon which makes it difficult for him to perceive global effects. Global effects can be observed only by those whose acceleration scale is large compared to the global scale. When it is said that Hawking radiation is a *global* effect, what is meant is that it is an effect of the large scale structure of space-time, in particular, the existence of an event horizon. In the case being considered, the event horizon is close on the scale of the background curvature, so that it is a local effect in the equivalence principle sense even though it is a global effect in the sense of the structure of space-time.

Thus the principle of equivalence is indeed applicable, so that (2.13) is valid. Integrating this equation gives

$$f(A) = \varsigma A + C. \tag{2.15}$$

Since a vacuum (or a black hole with $M = 0$) has vanishing entropy, one finds

$$S_{BH} = \varsigma A, \tag{2.16}$$

$$T_{BH} = \frac{1}{8\pi\varsigma} g_H. \tag{2.17}$$

The constant ς cannot, of course, be evaluated within the framework of classical thermodynamics, but neither is there justification for the claim, sometimes made, that classically it it should be set to infinity. If one lets $\hbar \to 0$ in (1.6) then the entropy of the black hole diverges and its temperature vanishes. The black hole would then be truly *black.* It would absorb but not radiate. Thus, it is said, Hawking radiation is a *quantum mechanical* phenomenon. However, exactly the same thing could be said about black body radiation. If one lets $\hbar \to 0$ in (1.3) and holds the energy constant in (1.1) , then the black body's entropy also diverges, its temperature also vanishes, and it also becomes truly *black.* In fact, the quantum mechanical formula for the entropy of *any* system diverges in the limit $\hbar \to 0$. Classical thermodynamics implicitly assumes that entropies are finite. This assumption is not stated explicitly because it is considered self-evident, but it is necessary for numerous formal manipulations. This means that while classical *mechanics* is consistent with the limit $\hbar \to 0$, classical *thermodynamics* is not. In this sense, thermodynamics "knows something" about quantum mechanics. More will be said about this in the next section.

It should be noted that the black hole formulas bear a closer resemblance to the black body formulas than they do to those of the ideal gas. In both the black body and black hole the integration constant appears in both the entropy and temperature formulas, and when this constant is evaluated quantum mechanically, it contains an inverse power of $\hbar$. This contrasts with the ideal gas formulas[1]

$$E = \frac{3}{2}NT \tag{2.18}$$

$$S = N[\ln \frac{VT^{3/2}}{N} + C] \tag{2.19}$$

$$C = \frac{3}{2}\ln \frac{m}{2\pi\hbar^2} + \frac{5}{2} \tag{2.20}$$

where the energy-temperature formula does not contain the integration constant and $\hbar$ appears logarithmically.

The similarity between the black hole and black body is evidently due to the fact that both are ultra-relativistic.

3. Quantum "Predictions"

Classical thermodynamics, when combined with classical gravity, predicts certain quantum-like characteristics for classical waves. First, as a black hole radiates into a vacuum, its area decreases. This violates the Hawking area theorem[10] which in turn rests solely on the assumption of the weak energy condition. Thus, there must be frames in which the energy density dips below zero. This must apply separately to gravitational, electromagnetic, and spinor fields.

Second, by considering the interaction of classical spinor (Dirac) waves with a rapidly spinning, uncharged black hole, and inverting an argument originally given by Unruh[11] , one may show that the second law implies some sort of "exclusion principle" for spinor waves.

Unruh originally derived the formula for spontaneous emission of neutrinos from Kerr black holes,

$$\frac{dN}{d\omega dt} = \frac{1}{2\pi}(1 - R(\omega, l, m))\theta(\Omega - \frac{\omega}{m}), \tag{3.1}$$

where ω is the frequency, m is the azimuthal quantum number, R is the barrier reflection coefficient, and N is the number of neutrinos escaping in the (l, m) mode. Even classically, ω/m is the energy per unit angular momentum. It was already known that

$$R(\omega, l, m) < 1, \tag{3.2}$$

even for the low energy neutrinos considered by Unruh. That is, if one shines low energy neutrinos on the Kerr hole, they will be partially absorbed even though, according to (2.6) this tends to reduce the entropy (and area) of the hole. (This contrasts with the case of scalar, vector, and tensor waves which super-radiate in this range). Unruh observed that the spontaneous emission would cause the hole's area to increase, and because phase space was only "falling into" the hole at a rate

$$d(Phase\ Space) = \frac{d\omega dt}{2\pi}, \tag{3.3}$$

the Pauli exclusion principle would prevent one from shining neutrinos on the hole any more rapidly than they are spontaneously emitted. Thus the Hawking area theorem would not be violated by (3.2) .

One may reverse this argument by making the assumptions of this paper (the validity of the second law, nothing about quantum mechanics) the starting point. Consider an experiment where one shines neutrinos characterized by

$$\Delta\omega \equiv m\Omega - \omega > 0, \tag{3.4}$$

on a Kerr hole in the presence of (possibly zero) spontaneous emission. Let the energy absorbed from the incoming waves be

$$\frac{dE_{ab}}{d\omega dt} = (1-R)\alpha_{in}, \tag{3.5}$$

and the energy emitted by the spontaneous emission process be

$$\frac{dE_{em}}{d\omega dt} = (1-R)\alpha_{out}, \tag{3.6}$$

Then the net influx of energy is given by

$$\frac{d\Delta E}{d\omega dt} = (1-R)(\alpha_{in} - \alpha_{out}), \tag{3.7}$$

By equations (2.6) , (3.4) , and (3.7) the rate of entropy gain by the hole will be

$$\frac{dS_{BH}}{d\omega dt} = \frac{(1-R)(\alpha_{out} - \alpha_{in})\Delta\omega}{T_{BH}}. \tag{3.8}$$

This quantity will be non-negative if and only if

$$\alpha_{out} \geq \alpha_{in}. \tag{3.9}$$

From this one may conclude first that α_{out} is positive definite, that is, there is spontaneous emission. And second, it follows that there must be some principle of physics which prevents one from shining neutrinos on black holes with greater than some definite intensity.

Thus both the existence of negative energy for all waves and some sort of exclusion principle for Dirac waves appear as natural consequences of classical thermodynamics and classical gravity.

4. Conclusion

Black hole thermodynamics can be derived from classical thermodynamics. The relationship of the classical and quantum formulas appears quite similar to the corresponding relationship for black body radiation. From this standpoint, black hole entropy does not appear to be in any way special. If it had turned out that the derivation of black hole thermodynamics depended in an essential way on microscopic physics (quantum mechanics), then it would have been reasonable to hope that one could use these thermodynamic relations to infer something about the *complete* microscopic picture (including quantum gravity). But since, as has been shown, black hole thermodynamics (like all other thermodynamics) is logically independent of microscopic physics, this approach to quantum gravity may prove difficult.

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APPENDIX A: Lowering Boxes

Using the phenomenological Drude model[12] , one may establish a lower bound on the mass of a cavity, μ, in terms of its quality factor, Q,

$$\mu \gg Q\frac{L^2}{a}\frac{(gravitational\ coupling)}{(electromagnetic\ coupling)} = Q\frac{L^2}{a}\frac{m_e m_p}{e^2}, \tag{A1}$$

where a is the shortest dimension of the cavity, L^2 is the area of the remaining dimensions, and m_p is the mass of the proton. This follows immediately from the formulas for conductivity,

$$\sigma = \frac{Ne^2}{m_e(\gamma - i\omega)}, \tag{A2}$$

for skin depth,

$$\delta = (2\pi\sigma\omega)^{-\frac{1}{2}}, \tag{A3}$$

for cavity quality,

$$Q \simeq \frac{(cavity\ volume)}{(skin\ volume)} \tag{A4}$$

and the condition that the conductivity be essentially real,

$$\gamma \gg \omega. \tag{A5}$$

Here N is the density of free electrons, ω is the frequency in the cavity and γ is a phenomenological constant.

Imagine now that this cavity is lowered by means of a "string" to a red-shift factor χ_{min}, so that the ambiant radiation length is

$$\lambda_{min} = \chi_{min}\lambda_{BH}. \tag{A6}$$

The radiation must not leak out during the proper time, $\Delta\tau$, of the adiabatic descent:

$$Q \gg \frac{2\pi}{\lambda_{min}} \Delta\tau. \tag{A7}$$

In certain simple problems what is meant by "adiabatically slowly" can be explicitly calculated[13] . In this problem a cruder but sufficient condition is that the heat, q, generated by moving the cavity through the photon gas be very small compared to the energy exchanged when the cavity is opened. This latter quantity will be very small compared with the energy carried by the cavity when it is at χ_{min}. That is,

$$q \; (\ll)^2 \; \chi_{min} \sigma T^4_{\chi_{min}} L^2 a \frac{\lambda_{min}}{a}, \tag{A8}$$

where both sides are energies as measured at infinity, the factor on the extreme right appears because a thin cavity contains more energy than it displaces at the same temperature, and the exponent on the extreme inequality indicates the number of times it was used in deriving the formula. Taking into account both ascent and descent, one easily finds

$$\frac{dq}{d\tau} \sim \chi \sigma T^4_\chi L^2 v^2, \tag{A9}$$

where v is the velocity relative to a stationary observer. (A8) will clearly be satisfied if

$$\frac{1}{v}\frac{dq}{d\tau} = \frac{dq}{dl} \; (\ll)^2 \; \frac{d}{dl}(\chi^2 \sigma T^4_\chi L^2 \lambda_{BH}) \tag{A10}$$

or

$$v \; (\ll)^2 \; 1, \tag{A11}$$

where l is a proper length coordinate. This implies

$$\Delta\tau \; (\gg)^2 \; \frac{1}{g_H}. \tag{A12}$$

Finally, the "string" mass cannot be regarded as negligible because the weak energy condition demands that its mass per unit length be greater than its tension[14] . A straightforward calculation then shows that the mass-at-infinity of the entire apparatus must be greater than the rest mass of the cavity. This implies

$$\mu \ll M, \tag{A13}$$

since otherwise the test apparatus would disrupt the graviational field of the hole.

Combining (A1) , (A7) , (A12) , (A13) , and the condition that the cavity be thin,

$$L \gg \lambda_{min} \gg a, \tag{A14}$$

yields

$$\chi_{min} \ (\gg)^8 \ \frac{m_e m_p}{e^2} \frac{M}{\lambda_{BH}} \simeq 10^{-40} \tag{A15}$$

Evidently the great disparity between the strength of the electromagnetic and gravitational coupling allows one to almost ignore this constraint.

One may estimate the constraints on gravitational wave cavities by dropping the coupling ratio from (A1) . One then arrives at a conjecture, already advanced and partially proven by Smolin[15] , that it is impossible to construct such cavities. (The cavities described by Garfinkle and Wald[9] in an attempt to circumvent Smolin's proof are not relevant to this thought experiment because they will not confine radiation in the presence of a black hole.)

APPENDIX B: Equivalence Principle and Relativistic Stars

Let χ_{min} be the red-shift factor at the surface of a relativistic star and let

$$\beta = \lambda T \tag{B1}$$

characterize the inverse scaling of temperature and typical wavelength. Then by (1.4) and (B1) ,

$$\lambda_{BH} = \frac{8\pi\varsigma\beta}{g_H} \equiv \frac{\xi}{g_H}. \tag{B2}$$

When ξ is evaluated quantum mechanically it is found to be

$$\xi \simeq 8\pi \frac{1}{4\hbar}\frac{2\pi\hbar}{2.8} \simeq 14. \tag{B3}$$

If one were to try to use the equivalence principle to deduce an ambiant temperature near a star, one would have to be able to measure wavelengths

$$\lambda_{min} = \chi_{min}\lambda_{BH} = \frac{\xi}{g_{\chi_{min}}} = \frac{\xi}{g_H}\chi_{min} \tag{B4}$$

locally. Here g_H is defined to be the surface gravity of the black hole whose exterior geometry corresponds to the exterior geometry of the star considered. For static uncharged stars, typical limiting values of χ_{min} are $\frac{1}{3}$ (incompressible fluid) and $\sqrt{4/7}$ (degenerate relativistic fermi gas). Using the lower of these figures and (1.8) and (B4) yields

$$\lambda_{min} = \frac{4}{3}\xi M. \tag{B5}$$

However (with other purposes in mind), Garfinkle and Wald[9] have proposed building a charged spherical shell which would be supported just outside its

Reissner-Nördstrom radius by electrostatic repulsion. Consider their model and set the radius and charge of the shell to

$$R = M(1+\delta) \tag{B6}$$

$$Q = M(1-4\gamma\delta^2) \tag{B7}$$

where δ is a small number and γ is a parameter which is limited by the weak energy condition to

$$-\frac{1}{8} \leq \gamma \leq 1. \tag{B8}$$

γ must also be non-negative for the geometry to be comparable to a black hole. Using these parameters one finds that the surface red-shift is given by

$$\chi_{min} = \delta, \tag{B9}$$

so that it might first appear that by making δ very small one could arrange to bring λ_{min} down to a value small compared to M, and thus within range of the equivalence principle. However, thermodynamics conspires against this attempt. Using (1.8) one finds

$$g_H = \frac{\sqrt{8\gamma}}{M}\delta \tag{B10}$$

so that, combining (B8) , (B9) and (B10) gives

$$\lambda_{min} = \frac{\xi}{\sqrt{8\gamma}}M \geq \frac{\xi}{\sqrt{8}}M. \tag{B11}$$

One might imagine trying to support a "star" near its Kerr-Newman radius by some combination of electromagnetic and rotational forces, but according to (1.4) and (1.8) this would always depress the temperature and thus lengthen the typical wavelength.

This leads me to conjecture that thermodynamics will always conspire to prevent one from employing the equivalence principle to infer a temperature near any stationary, non-collapsed object.

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