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W-boson Pair Production Cross Section and Measurement of W-mass.*

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ABSTRACT

The W-pair production cross section in e^+e^- collision is investigated as a measuring tool for the W-mass. Against the recent argument, the cross section itself is not useful for determining the parameter of the electroweak theory. Also the leading logarithmic correction to the cross section is shown to be small.

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Recently the W-boson pair production has been investigated as one of the main subjects for future high energy accelerators, particularly for LEP2 [1]. Among others, the three gauge vertices (WWZ and WWγ), W-mass and W-decay processes are to be studied through this pair production process rather exclusively.

In this letter, we concentrate on the W-boson mass measurement via the total W-pair production cross section. Naively the production cross section will increase very rapidly at the threshold and this behavior was considered to give a good measurement of M_W. However, recent papers [1,2] claim that the finite width of W will wipe out the rapid increase of the cross section. Then they consider the total cross section itself as a measure for M_W. Ref.1 estimates the change of the cross section (at slightly above the threshold) with respect to the change of M_W as

$$\frac{\Delta\sigma}{\Delta M_W} = 2.7 \text{ pb/GeV} , \tag{1}$$

and Ref.2 gives almost the same results for wide range of the center of mass energy. They both conclude that one can measure W-mass well by using the above rather large dependence of the production cross section on M_W. Note that Ref.1 uses a cut-off for the invariant mass of the W-decay products, $M_W \pm$ 10 GeV, while Ref.2 integrates out the cross section over whole range of the final states. Their results are consistent with each other taking account of the above difference. We will use the same cut-off as that of Ref.1 in the following numerical estimate.

The above results, particularly Eq.(1), are confusing and misleading. The cross section is not a direct measurement of any particle mass. The dependence of cross section on a particle mass is an implicit one. Hence one should carefully treat all the relevant parameters simultaneously.

The standard electroweak theory contains basic three parameters (apart from fermion masses and the Higgs mass which are of little importance here, assuming that they are not extremely heavy). We have two experimental data with high precision, the muon life time τ_μ and the fine structure constant α. The present experimental status of the determination of gauge boson masses, M_Z and M_W, is still far away from precision measurement. Then effectively one parameter remains undetermined which corresponds to the Weinberg angle $\sin^2\theta_W$ typically, or anything else. Due to the remaining one parameter, the theory predicts only the relation between M_Z and M_W, that is, M_W is a function of M_Z or vice versa. Hence, as long as we are working in the standard theory, any change of M_W enforces the corresponding change of M_Z and of everything else. Thus the dependence of some physical quantities on M_W should be discussed with proper account of the above restriction.

We write the total cross section σ at some s as

$$\sigma = \sigma(e, M_Z, M_W) \ , \tag{2}$$

where we have adopted three physical quantities, e, M_Z, M_W, as three basic parameters of the theory. These masses, M_Z and M_W, are the on-shell masses, and e is the on-shell photon coupling constant [3]. A complete and explicit formulas for the above function is found in Ref.2 which includes the finite width effect of decaying W. What is estimated in the previous papers [1,2] is nothing but a partial differentiation of σ with respect to M_W,

$$R_1 = \frac{\partial\sigma(e, M_Z, M_W)}{\partial M_W} \ . \tag{3}$$

The two inputs, τ_μ and α, give two constraints, a fixed value for e and a correlation between M_Z and M_W. The above differentiation contradicts the correlation. What should be estimated for discussing the power of determining the remaining one parameter is the total derivative,

$$R_2 = \frac{d\sigma(e, M_Z(M_W), M_W)}{dM_W} = R_1 + \frac{\partial M_Z}{\partial M_W}\frac{\partial\sigma(e, M_Z, M_W)}{\partial M_Z}, \tag{4}$$

where M_Z is regarded as a function of M_W. This interrelation (in the lowest order) is written down as

$$M_{\rm W} = M_{\rm W}^{(0)}(M_{\rm Z}) = M_{\rm Z}\sqrt{\frac{1}{2}\left(1+\sqrt{1-\frac{2\sqrt{2}\pi\alpha}{M_{\rm Z}^2 G_F}}\right)}\ , \tag{5}$$

where the inverse function is shown for later use, and G_F is the Fermi coupling constant determined by the muon life time τ_μ.

To see the actual numerical values, we take a typical s, $\sqrt{s} = 200$ GeV, and take $M_{\rm Z}$= 93.00 GeV for the center value. The corresponding $M_{\rm W}^{(0)}$(93.00 GeV) is 83.09 Gev, where we do not include some specific QED corrections to the muon decay amplitude which are usually put in the lowest order quantities. (Note that such inclusion of the particular part of the one loop corrections does not make sense absolutely in the renormalization theory of the electroweak theory.) The numerical results are

$$R_1 = 2.3 \text{ pb/GeV}\ , \tag{6}$$

$$R_2 = 0.24 \text{ pb/GeV}\ . \tag{7}$$

As is readily seen, the correct dependence on $M_{\rm W}$ is extremely reduced due to the second term contribution in Eq.(4).

Presumably, the above argument will become impractical, though it is physically correct. At the time when we will be able to observe W-pair production, we certainly know a precise value of $M_{\rm Z}$, say with 0.1 % error, via the e^+e^- experiment around Z-pole. The standard model predicts a definite $M_{\rm W}$ and everything, and the above estimate may lose practical significance. The deviation of the observed cross section away from the predicted one should indicate that there must be some new ingredients beyond the standard model (and/or the higher order effects).

So far we have discussed the issue in the tree approximation (and the finite width effect). It cannot be overestimated to successfully confirm the higher order effects in the electroweak theory. We just mention that the establishment of QED is based on the excellent agreement between the renormalization theory results and the experiments. We proceed to investigate the higher order corrections to this process [4]. No complete results of one-loop corrected amplitude including the finite width effect have been obtained yet. However, one may expect that the leading logarithmic part should dominate in the electroweak loop corrections. In general, the leading logarithmic corrections are rather easily evaluated [5,6,7]. Furthermore, in our scheme of renormalization, for any physical quantities of the weak scale ($\sim$ 100 GeV) processes, the logarithmic terms are able to be simply assembled to the all orders. The corrected amplitude is a function of the effective QED coupling constant at the weak scale and $M_{\rm Z}$[8].

The evaluation goes as follows. At the tree level, amplitude is written as $\sigma^0(e, M_{\rm Z}, M_{\rm W})$. Practically, σ^0 is regarded as a one-parameter function of $M_{\rm Z}$, provided that τ_μ and α are fixed:

$$\sigma^0 = \sigma^0(\sqrt{4\pi\alpha(0)}, M_{\rm Z}, M_{\rm W}^{(0)}(M_{\rm Z})) = \tilde{\sigma}^0(M_{\rm Z}) \ , \tag{8}$$

where $\alpha(0)^{-1} = 137.036$, and the function $M_{\rm W}^{(0)}$ is in Eq.(5). Of course we are supposing that $M_{\rm Z}$ becomes the third best input parameter at the time when W bosons are pair produced. Accordingly, our definition for the higher order correction is that calculated under fixed τ_μ ,α and $M_{\rm Z}$. Then the corrected amplitude with the all order logarithmic terms is expressed as[8],

$$\sigma^l = \sigma^0(\sqrt{4\pi\alpha(M)}, M_{\rm Z}, M_{\rm W}^{(l)}(M_{\rm Z})) = \tilde{\sigma}^l(M_{\rm Z}) \ , \tag{9}$$

where a constant $\alpha(M)$ is the effective QED coupling constant at the weak scale, and $M_{\rm W}^{(l)}(M_{\rm Z})$ is a logarithmically corrected interrelation between $M_{\rm W}$and $M_{\rm Z}$[5]. The corrected relation is defined implicitly by

$$\alpha(0)M_{\rm W}^{(l)2}(M_{\rm Z}^2 - M_{\rm W}^{(l)2}) = \alpha(M)M_{\rm W}^{(0)2}(M_{\rm Z}^2 - M_{\rm W}^{(0)2}) \ . \tag{10}$$

We take a typical value, $\alpha(M)^{-1} = 128$, given by the standard contents of light fermions. For corrected $M_{\rm W}$, we rather use an estimate of Ref.5 which includes the leading logarithmic terms and O(α) terms as well,

$$M_{\mathrm{W}}^{(l)}(93.00\mathrm{GeV}) = 82.14\mathrm{GeV}\ . \tag{11}$$

This is 0.95 GeV less than the previous tree relation $M_{\mathrm{W}}^{(0)}(93.00\mathrm{GeV}) = 83.09$ GeV. Finally we get ($\sqrt{s} = 200$GeV, more complete results will be found in Ref.8),

$$\tilde{\sigma}^{(l)}(93.00\mathrm{GeV}) = 17.1\ \mathrm{pb}\ , \tag{12}$$

which should be compared with the tree prediction for the same M_{Z},

$$\tilde{\sigma}^{(0)}(93.00\mathrm{GeV}) = 16.9\ \mathrm{pb}\ . \tag{13}$$

Thus the difference of these two is as small as 1%. It is rather disappointing to see how small the difference is. The leading logarithmic terms for this process are almost cancelled out among themselves. Intuitively, the running of the coupling constant increases the cross section, while the change of M_{W}(-0.95GeV) decreases it. Note that such type of cancellation is not a general feature for the weak scale processes.

We should comment here on the heavy particle effects to the process. For example, unknown heavy fermions may affect the corrected relation between M_{W} and M_{Z} to a large extent. In other words, under fixed M_{Z}, $M_{\mathrm{W}}$is changed due to heavy fermions. Those heavy particles ($m_h > M_{\mathrm{W}}$) do not change $\alpha(M)$. Then one may think of the validity of R_1 in Eq.(3), that is, the change of σ and M_{W} both due to heavy fermions may be related through R_1 (applied to Eq.(9)), and hence R_1 may tell the power of measuring M_{W} by the cross section. However this argument is still wrong. The dominant heavy particle effects to the process is not logarithmic. Therefore one has to take account of the direct correction terms of m_h^2/M_{W}^2 to the cross section itself in addition to the logarithmic part in Eq.(9).

In conclusion, an efficient M_W measurement cannot be done only by looking at the W-pair production cross section. In order to get a precise measurement of M_W, one has to study the subsequent W-decay processes. Also the leading logarithmic corrections to the production process due to the electroweak loop effects is so small that it seems practically impossible to confirm the loop effects directly via investigating the cross section.

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REFERENCES

1. G. Barbiellini et al., in Physics at LEP ;edited by J. Ellis and R. Peccei, CERN 86-02.
2. T. Muta, R. Najima and S. Wakaizumi, Mod. Phys. Lett. A1 (1986) 203.
3. K-I. Aoki, Z. Hioki, R. Kawabe, M. Konuma and T. Muta, Prog. Theor. Phys. Suppl. 73 (1982) 1.
4. M. Lemoine and M. Veltman, Nucl. Phys. B164 (1980) 445 ; R. Phillipe, Phys. Rev. D26 (1982) 1588
5. Z. Hioki, Prog. Theor. Phys. 71 (1984) 663.
6. M. Consoli, S. Lo Presti and L. Maiani, Nucl. Phys. B223 (1983) 474 .
7. F. Antonelli and L. Maiani, Nucl. Phys. B186 (1981) 269; S. Belluci, M. Lusignoli and L. Maiani, Nucl. Phys. B189 (1981) 329; S. Dawson, J.S. Hagelin and L. Hall, Phys Rev. D23 (1981) 2666.
8. K-I. Aoki in preparation.