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FIXED POINT STRUCTURE OF QUENCHED, PLANAR QUANTUM ELECTRODYNAMICS*

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Gauge theories exhibiting a hierarchy of fermion mass scales may contain a pseudo–Nambu–Goldstone boson of spontaneously broken scale invariance. The relation between scale and chiral symmetry breaking is studied analytically in quenched, planar quantum electrodynamics in four dimensions. The model possesses a novel nonperturbative ultraviolet fixed point governing its strong coupling phase which requires the mixing of four fermion operators.

In the chiral symmetric limit, QCD-like gauge theories with N flavors of fermions possess an $\mathrm{SU(N)_L \times SU(N)_R}$ chiral symmetry which is spontaneously broken by a dynamical fermion condensate to its diagonal $\mathrm{SU(N)_V}$ subgroup resulting in the appearance of an SU(N) multiplet of pions as Nambu–Goldstone bosons. In addition to these chiral symmetries, the classical formulation of gauge theories also exhibits, in four dimensions in the chiral limit, an exact scale invariance. Here I will discuss various aspects of dynamical symmetry breaking with particular focus on the scale symmetry. A more complete discussion appears in work done in collabortion with W. A. Bardeen and C. N. Leung.[1,2]

In quantum chromodynamics, the scale symmetry is explicitly broken by quantum radiative corrections as reflected by the anomalous nonconservation of the dilatation current:

$$
\begin{aligned}
D_\mu &= \chi^\nu \theta_{\mu\nu} \,, \\
\partial_\mu D^\mu &= \theta^\mu_\mu = \frac{\beta(g)}{g}\,\frac{1}{4}\,G^{\mu\nu}G_{\mu\nu} \,.
\end{aligned}
\tag{1}
$$

When combined with the nonperturbative QCD vacuum structure which gives $\langle G^2_{\mu\nu}\rangle \sim \Lambda^4_{\mathrm{QCD}}$, a large explicit breaking of the anomalous symmetry ensues. That is, the explicit scale symmetry breaking accompanying the rapid running of the QCD coupling dominates at low energies and no vestige of the classical scale symmetry remains. In particular, there is no evidence for a Nambu–Goldstone boson of scale symmetry in conventional QCD.

The above picture need not hold, however, in all gauge models. It may be possible that the spontaneous breaking of the chiral symmetry might also trigger the spontaneous beaking of an approximate scale symmetry. This would be the case if the chiral symmetry breaking occurs at a scale where the explicit scale breaking is small. Such a situation could occur in theories possessing a hierarchy of fermion mass scales. An example is afforded by a model where fermions transforming as higher dimensional representations of the gauge group are present in the theory. Indeed, results from numerical studies in lattice gauge theories[2] indicate that the scale of chiral condensation for these fermions is relatively short compared to the confinement scale.

The chiral condensation scale is roughly characterized by the requirement that the effective fermion coupling $C_2(f)\,\alpha(\mu)$ reach a critical value α_{crit}. Here $\alpha(\mu)$ is the gauge theory running coupling and $C_2(f)$ is the quadratic Casimir invariant of the fermion representation. For a sufficiently large $C_2(f)$, spontaneous chiral symmetry breaking could occur in the asymptotically free region where $\alpha(\mu)$ varies only logarithmically with energy. The explicit breaking of the scale symmetry is then but a small effect at this scale compared to the large spontaneous breaking associated with the chiral condensation and consequently the scale symmetry should be realized in a Nambu–Goldstone fashion resulting in the appearance of a scalar dilaton. Since the coupling is not fixed,

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the dilaton should actually emerge as a pseudo–Nambu–Goldstone boson acquiring a mass of order the scale at which the explicit scale symmetry breaking becomes important, which is roughly the confinement-scale of the gauge theory. The dilaton should couple to heavy states, *e.g.,* W, Z in a manner similar to the physical Higgs boson, but may be distinguished from it due to its Nambu–Goldstone nature. For a discussion of dilaton phenomenology, see Ref. 4.

In order to study the dynamical aspects of chiral and scale symmetry breaking, I consider the simplest approximation to a gauge field theory with a fixed but critical coupling. This corresponds to quenched, planar (ladder) quantum electrodynamics. The quenched approximation excludes fermion loop corrections and consequently guarantees that the perturbative gauge coupling β–function vanishes. It is thus anticipated that the theory should exhibit an exact or spontaneously broken scale symmetry.

This model has been the subject of numerous investigations by various authors over the years.[5,8] In the model, the Schwinger–Dyson equation for the fermion self-energy is given by a sum of the rainbow graphs. At weak coupling, $\alpha < \alpha_c = \pi/3$, there exist no spontaneous chiral (or scale) symmetry breaking solutions. If an ultraviolet cutoff Λ is introduced, there are no solutions to the massless equation at fixed Λ and solutions appearing as $\Lambda \to \infty$ do not correspond to spontaneous symmetry breaking but rather reflect the anomalous dimension of the fermion mass operator $\bar{\psi}\psi$ so that

$$d_{\bar{\psi}\psi} \;=\; 2 + \sqrt{1 - \frac{\alpha}{\alpha_c}} \,. \tag{2}$$

On the other hand, at strong coupling, $\alpha > \alpha_c$, the massless equation was shown to possess a nontrivial solution leading to the generation of the fermion mass scale

$$\Sigma(0) \simeq \Lambda \;\exp\left\{\delta + 1\right\}\; \exp\left\{\frac{-\pi}{\sqrt{\frac{\alpha}{\alpha_c} - 1}}\right\}, \tag{3}$$

where $\delta \simeq 0.55$ is a parameter of the asymptotic solution for the self-energy function. The dependence of the fermion mass scale diverging with the cutoff appears to be disasterous for this solution as all the dynamics associated with the spontaneous chiral symmetry breaking occurs at the cutoff Λ. Similar conclusions were also reached in numerical studies.[9]

There is, however, an alternate interpretation of this solution[10] in which the critical coupling α_c is viewed as a fixed point of the strong coupling phase with the gauge coupling α approaching the critical value as

$$\frac{\alpha}{\alpha_c} \;=\; 1 + \frac{\pi^2}{\ell n^2\left(\frac{\Lambda}{\kappa}\right)} \,, \quad \Lambda \to \infty \,, \tag{4}$$

where κ is an infrared scale. This fixed point interpretation leads to a finite fermion mass scale $\Sigma(0) \;\to\; e^{\delta+1}\kappa$ as $\Lambda \;\to\; \infty$. Moreover, a massless pseudoscalar bound state appears as a solution to the Bethe–Salpeter equation reflecting the Nambu–Goldstone realization of the chiral symmetry. However, the solution remains incomplete as it leaves unclear the origin of the running of the gauge coupling and moreover does not properly reflect the scale symmetry as there is no massless scalar bound state solution to the Bethe–Salpeter equation corresponding to the dilaton.

It was attempting to clarify these issues that led to the discovery of the novel fixed point structure of the model.[2] The origin of this structure is the generation of four fermion operators which necessarily mix with the gauge interactions at the fixed point. The mixing results from the large anomalous dimensions generated by the gauge coupling at the fixed point. We have already observed that the mass operator $\bar{\psi}\psi$ has dimension $d_{\bar{\psi}\psi} \;=\; 2 + [1 - (\alpha/\alpha_c)]^{1/2}$ which is three at zero coupling but approaches two at the critical coupling. In the ladder approximation under consideration, the four fermion operator $(\bar{\psi}\psi)^2$ has just twice the mass operator dimension so that

$$d_{(\bar{\psi}\psi)^2} \;=\; 4 + 2\sqrt{1 - \frac{\alpha}{\alpha_c}} \,, \tag{5}$$

which approaches four as $\alpha \to \alpha_c$. Since the four fermion operators are dimension four at the critical gauge coupling, they are relevant operators which must be included in the analysis of the fixed point structure.

We are thus led to study the scale invariant fixed point structure using the chirally invariant effective fermion Lagrangian

$$\begin{aligned}\mathcal{L}_f &= \bar{\psi}\left[i\gamma\partial - e\gamma A - \mu_0\right]\psi \\ &\quad + \frac{G_0}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\right] ,\end{aligned} \tag{6}$$

where μ_0 is a bare fermion mass included to provide explicit breaking. Consistent with the planar approximation for the gauge interactions, only planar diagrams involving the four fermion interactions are to be retained.

The vacuum structure of the modified theory can be deduced using the same methods as employed in the pure gauge case. The Schwinger–Dyson equation in ladder approximation takes the form

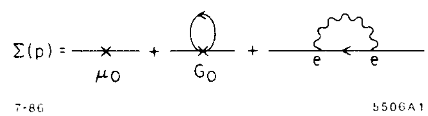

Fig. 1. The Schwinger–Dyson equation.

where the full propagator is to be used in the diagrams. This equation involves an effective bare mass parameter m_0 which includes terms generated by the induced interactions so that $m_0 = \mu_0 - G_0\langle\bar{\psi}\psi\rangle_0$. The fermion bilinear vacuum expectation value must be computed self-consistently including all the QED radiative ladder corrections, so that even in the chiral limit, $\mu_0 = 0$, the effective bare mass will not vanish, $m_0 \neq 0$. This modification leads to a new gap equation and fermion mass scale given by

$$\begin{aligned}\mu &= \frac{\tilde{A}}{2}\exp\{2\delta\}\ \Lambda^2\ \exp\left\{\frac{-2\theta}{\sqrt{\frac{\alpha}{\alpha_c}-1}}\right\} \\ &\quad \left[\frac{(1-G)}{\sqrt{\frac{\alpha}{\alpha_c}-1}}\sin\theta + (1+G)\cos\theta\right]\end{aligned} \tag{7}$$

$$\Sigma(0) = \exp\{\delta\}\ \Lambda\ \exp\left\{\frac{-\theta}{\sqrt{\frac{\alpha}{\alpha_c}-1}}\right\} , \tag{8}$$

where the renormalized parameters $\mu = \mu_0\Lambda$ and $G = [(G_0\Lambda^2)/\pi^2](\alpha_c/\alpha)$ have been introduced and reflect the anomalous dimensions of the mass and four fermion operators. Here $\tilde{A} \simeq 1.2$ is another parameter of the asymptotic expansion of the fermion self-energy function. There always exists one solution for θ (and hence $\Sigma(0)$) in the region $0 < \theta \leq \pi$ and this corresponds to the ground state solution. Once again, the existence of a non-trivial $\Lambda \to \infty$ limit requires that the gauge coupling approach the critical value $\alpha \to \alpha_c$. Thus the solution is similar to that of Ref. 10 except that θ need not be π. The approach of the gauge coupling to the critical point is now given by

$$\frac{\alpha}{\alpha_c} = 1 + \frac{\theta^2}{\ell n^2\left(\frac{\Lambda}{\kappa}\right)} , \quad \Lambda \to \infty , \tag{9}$$

so that $\Sigma(0) \to e^{\delta}\kappa$. The value of θ depends on the strength of the induced coupoing G. We shall see that the strong coupling phase of the theory corresponds to the ultraviolet stable fixed point with $G \to 1$ and $\alpha \to \alpha_c$.

The search for the fixed point structure can be conducted by examining the fermion-antifermion scattering amplitude (see Fig. 2). The four fermion interactions contribute to the scattering amplitude so that contributions from both the scalar and pseudoscalar channels must be included.

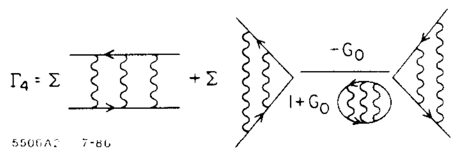

Fig. 2. The fermion-antifermion scattering amplitude.

The additional diagrams are reminiscent of the large N, chirally invariant Gross-Neveu model[11] except that the bubble graphs include all the radiative corrections of planar QED. These radiative corrections effectively make the four fermion interactins renormalizable at the fixed point. It is the presence of these diagrams which is at the origin of the running of the gauge coupling. Although the bubble diagrams are perturbatively quadratically divergent, the large anomalous dimensions allow for a precise determination of their contribution[2] yielding a well-defined four-point function.

The form of the four-point function allows a computation of the asymptotic behavior of the beta functions for both the gauge and four-fermion couplings near the ultraviolet fixed point $[\alpha \to \alpha_c^+,\ G \to 1]$ yielding

$$\beta_\alpha(\alpha, G) = \Lambda\frac{\partial\alpha}{\partial\Lambda} = \frac{-\frac{2\pi}{3}\left(\frac{\alpha}{\alpha_c}-1\right)^{3/2}}{\arctan\left(\frac{2\sqrt{\frac{\alpha}{\alpha_c}-1}}{G-1}\right)}$$

$$\beta_G(\alpha, G) = \Lambda\frac{\partial G}{\partial\Lambda} = \frac{-(G-1)\left(\frac{\alpha}{\alpha_c}-1\right)^{1/2}}{\arctan\left(\frac{2\sqrt{\frac{\alpha}{\alpha_c}-1}}{G-1}\right)} \tag{10}$$

where the angle $\theta = \arctan\{[2\sqrt{(\alpha/\alpha_c)-1}]/(G-1)\}$ is defined in the range $0 < \theta \leq \pi$. These β–functions are clearly nonperturbative and reflect the approach to the ultraviolet stable fixed point of the explicit solution. Moreover, the relevance of the four-fermion interactions is evident from the nontrivial fixed point value of $G = 1$.

The symmetry structure of the solution can also be gleaned from the bound state pole structure of the fermion-antifermion scattering amplitude. The pure ladder graphs do not contain any massless bound states since the four-fermion interactions generate a nonvanishing induced bare mass term, $m_0 \neq 0$, which will appear as an explicit chiral symmetry breaking in these diagrams. Hence, any massless bound state pole must originate from the bubble denominators. Indeed the pseudoscalar denominator at zero momentum vanishes in the chiral limit, clearly displaying the pseudoscalar Nambu-Goldstone boson associated with the spontaneous chiral symmetry breaking. However, the scalar denominator at zero momentum retains a nonvanishing contribution in the chiral limit even at the fixed point. Hence the status of the dilator remains unclear in this approximate treatment of a gauge theory. It is uncertain whether this result reflects a fundamental inconsistency of the quenched, planar approximation or is due to our analysis of the model. We strongly advocate that both the nontrivial mixing of the four-fermion operators and the fixed point structure of our solutions be checked by other methods including lattice calculations.

We anticipate that many of the general features obtained in the ladder model will continue to hold for gauge theories with running couplings possessing widely separated condensate scales. In such cases, provided large anomalous dimensions exist over a wide range of momenta, which is possible due to the slow running of the gauge coupling, the momentum dependence of induced fermion mass terms can be significantly affected. Such behavior may be applicable[12] to the resolution of the flavor changing neutral current problem in extended technicolor models.

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