SLAC – PUB – 4032
July 1986
T

CONFORMAL INVARIANCE ON CALABI-YAU SPACES*

DENNIS NEMESCHANSKY

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

The possibility of superstring compactification on Calabi-Yau manifolds is analyzed. Despite the apparent non-zero β function at four loop order, it is possible to construct a conformally invariant sigma model on a Calabi-Yau manifold. The background metric is not Ricci flat, but is related to the Ricci flat metric through a (non-local) field redefinition.

Invited talk Presented at the
Symposium on Topological and Geometrical Methods in Field Theory
Espoo, Finland, June 8-14, 1986
and
23rd International Conference on High Energy Physics
Berkeley, California, July 16-23, 1986

* Work supported by the Department of Energy, contract DE–AC03–76SF00515.

During the last few months it has become clear that $N = 2$ supersymmetric models on Calabi-Yau manifolds with a Ricci flat Kahler metric have a nonvanishing β-function at the four loop level.[1,2] This destroys the expectation that such models have a vanishing β-function to all orders in perturbation theory. Despite the four loop contribution, I will show that it is always possible to choose a Kahler metric on a Calabi-Yau space such that the β-function vanishes to all orders in perturbation theory. This talk summarizes a recent paper with A. Sen.[3]

These theories are of current interest since they provide us with possible solutions of the classical string equations.[4] This is based on the equivalence between the equations of motion of the massless fields and conformal invariance of the two dimensional sigma-model.[5] In this talk I will only consider the equation of motion for the graviton. I will set the antisymmetric tensor field to zero and I will assume that the dilaton field is constant. It is a trivial generalization to include other background fields. My talk is divided into two parts. First I will show how the contribution to the β-function arises at four loops. In the second part of the talk I will sketch how to choose the new Kahler potential.

The classical equation of motion for the string can be derived from the effective action for the massless fields.[2] To find this effective Lagrangian one has to consider tree level string scattering amplitudes. The three graviton scattering is that of general relativity with no corrections coming from the string theory. The situation changes for four gravitons.[2] In terms of an ordinary Feynman diagram the four graviton scattering can be represented in the form shown in fig. 1. The sum is over all string states. Since we are constructing the effective action for the massless fields we have to subtract off the massless poles. This then leaves us with intermediate states with masses of the order of the Planck scale. For energies much below the Planck scale, the propagator $(p^2 + m^2)^{-1}$ of the intermediate states in fig. 1 can be expanded in powers of the momenta p^2, giving rise to effective four graviton couplings. In order to reproduce the S-matrix elements of the string scattering amplitudes, one has to add to

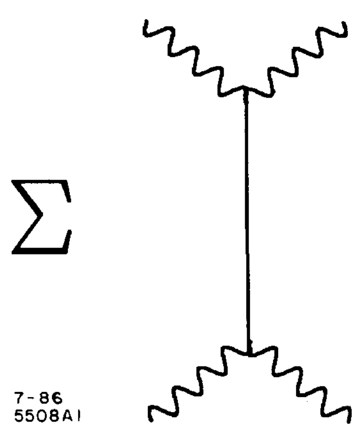

Fig. 1. Four graviton scattering.

the effective Lagrangian new local four point vertices order by order in α'. This procedure can then be repeated for five and higher graviton couplings to yield, in principle, the effective Lagrangian to all orders.

The effective Lagrangian is not unique since any local field redefinition of the fields will not affect the S-matrix. However, the equations of motion derived from the effective Lagrangian do not change under this local field redefinition.

As mentioned earlier, there are no additional three graviton interactions coming from strings to Einstein's general relativity. Therefore we may conclude that in the generic case of Ricci flat metric the β-function is zero at one, two and three loop order. However, since the four graviton scattering was modified by the exchange of heavy states the four loop β-function does not vanish[2,6] for the generic Ricci flat manifold. This result agrees with an explicit calculation of the four loop β-function for an $N = 2$ supersymmetric sigma model.[1]

Having shown how the four loop β-function arises I will spend the rest of the talk proving that despite the apparent nonzero contribution to the four loop β-function the theory can be made to have a vanishing β-function to all orders in perturbation on a Calabi-Yau space.[3]

Let me start the argument with the observation that the vanishing at the β-function takes different forms under the redefinition of the coupling constant of the two dimensional sigma model. Remember that the metric G_{ij} is nothing else than the coupling constant of the theory. Under the replacement $G_{ij} \to G_{ij} + T_{ij}$, the β-function is given by the Ricci tensor calculated from the new metric $G_{ij} + T_{ij}$. To linear order in T this is given by

$$R_{ij}(G) + \tfrac{1}{2}\left(T_{ij;m}{}^{m} + T^{m}{}_{m;ij} - T_{im;j}{}^{m} - T_{jm;}{}^{m}\right) \tag{1}$$

For a specific example $T_{ij} = R_{imnp}R_j{}^{mnp}$ the second term in eq. (1) gives a three loop contribution. Changing the metric in the sigma model corresponds to adding finite counter terms. Therefore the vanishing of the β-function depends on how we subtract the ultraviolet divergences in the theory.

What we now have to show for the $N = 2$ supersymmetric sigma model is that we can modify the metric so that β-function vanishes. This corresponds to modifying the Kahler potential. The ultraviolet divergent terms can be summarized in a single β-function β_K

$$\beta_K = c\mathrm{Tr}\, G + \Delta\beta_K \tag{2}$$

where the first term is the one loop contribution and the second term includes all the higher loop contributions. Unlike the one loop β-function the higher loop β-function $\Delta\beta_K$ is a globally defined scalar on the Calabi-Yau manifold.[3]

It is very easy to show that for a Ricci flat metric $\widetilde{G}_{ij}$ there exists a Kahler potential K such that the β-function vanishes.[3] To find K one has to solve the differential equation

$$c\mathrm{Tr}\, \ell n G + \Delta\beta_K = c\mathrm{Tr} \ell n \widetilde{G} \ . \tag{3}$$

The most convenient way to solve this equation is to iterate it powers of α'. To lowest order in α' eq. (3) takes the form $\Box(K - \tilde{K}) = \Delta\beta_{\tilde{K}}$. The solution of eq. (3) gives a metric in each coordinate patch. In order for the solution to be an admissible metric one must show that the metric is globally well defined. This means that when we calculate G_{ij} in two different coordinate patches G_{ij} must transform like a tensor. Since $\Delta\beta_K$ is a globally well defined scalar it can be shown that G_{ij} also has the correct transformation property to define a metric on the whole manifold.[3]

Thus I have shown to you that given a Calabi-Yau manifold we can always construct to all orders in perturbation theory a conformally invariant sigma model. The metric is not Ricci flat but it is related to it by a nonlocal (in space-time) field redefinition. The new metric is a globally defined tensor on the manifold and hence a valid choice of the metric.

There is one more question that I would like to address. Does the procedure I have outlined converge? For a string theory we cannot stop at any order of perturbation theory because this would ruin the conformal invariance of the sigma model. Today I have shown that the theory is conformally invariant to all orders in perturbation theory. Therefore a violation of conformal invariance can at most be an exponential. But from the work of Dine, Seiberg, Wen and Witten[7] on the effective four dimensional theory we know that a (2,2) supersymmetric sigma model is conformally invariant even when the nonperturbative effects are included. Hence for the case I have studied this means that the procedure converges.[8] They also considered the (2,0) supersymmetric sigma model and they showed that conformal invariance is violated by nonperturbative effects. Therefore, if one repeated the arguments I have presented here for the (2,0) case, the series could not be summed.

Let me conclude by discussing the implication of our results for the superstring theory. Our results tells us that given a Calabi-Yau manifold we can always find a background vacuum expectation value of th metric which satisfies the equation of motion of the string theory. The background metric is obtained by solving eq. (3).

For some purposes (e.g. the study of the four dimensional effective field theory obtained after compactification) we do not need to know what the metric that solves the equation of motion looks like in terms of the Ricci flat metric. We take the Calabi-Yau metric as the background metric and add finite local counterterms in each order of perturbation theory in order to have a vanishing β-function. The result is a two dimensional conformally invariant field theory. We may then calculate the particle spectrum and the interaction in the effective four dimensional theory by identifying operators of conformal dimension (1,1) as vertex operator and calculate their correlation functions in the two dimensional field theory obtained this way. It is in this scheme that Witten's[9] general argument showing the vanishing of the β-function on Ricci flat Kahler manifolds works. This argument is based on a study at the effective four dimensional field theory and does not specify the renormalization scheme in which his proof should work.

REFERENCES

[1] M. T. Grisaru, A. van de Ven and D. Zanon, preprints HUTP-86/A020, HUTP-86/A026, HUTP-86/A027 (1986).

[2] D. Gross and E. Witten, Princeton preprint (1986).

[3] D. Nemeschansky and A. Sen, preprint SLAC-PUB-3925.

[4] P. Candelas G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258 (1985) 46.

[5] C. Callan, E. Martinec, D. Friedan and M. Perry, Nucl. Phys. B262 (1985) 593. A. Sen, Phys. Rev. D32 (1985) 2102, Phys. Rev. Lett. 55 (1985) 1846.

[6] M. Freeman and C. Pope, preprint Imperial/TP/85-86/17.

[7] M. Dine, N. Seiberg, X. Wen and E. Witten, Princeton preprint (1986).

[8] N. Seiberg, private communications.

[9] E. Witten, Princeton preprint (1985).