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ANOMALOUS DIMENSIONS OF MULTIQUARK BOUND STATES*

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ABSTRACT

The evolution of six-quark color-singlet state distribution amplitudes is formulated as an application of perturbative quantum chromodynamics to nuclear wave functions. We present a general method of solving the evolution equation for multiquark bound states and predict the asymptotic Q^2 slope for the deuteron charge form factor as a result.

Because of the asymptotic freedom property of quantum chromodynamics (QCD), a perturbative analysis of strong interaction processes should be rigorous when the momentum transfer q is much larger than the QCD scale parameter Λ_{QCD} and the value of the strong coupling constant α_s becomes small. Applying light-cone perturbation theory,[1] amplitudes for exclusive processes are given by factorized forms. For example, hadronic form factors at asymptotic high $Q^2 \equiv -q^2$ are generically given by $\int \Phi(x) T_H(x,y) \Phi(y) [dx][dy]$, with the longitudinal momentum fractions x_i of the quarks in the hadron. Therefore, detailed analyses for exclusive processes require knowledge of the valence-quark distribution amplitude $\Phi_H(x_i, Q)$ of hadrons.[1,2] Since the QCD theory requires new degree of freedoms to form multiquark systems which do not exist in the conventional nuclear theory (hidden-color states), it is important to construct a basis to calculate the multiquark bound state wavefunctions. The multiquark bound state wavefunction should be completely antisymmetric in the total color(C), spin(S), iso-spin(T), and orbit(O) quantum space. As an explicit example, a specific representation for the asymptotic six quark systems with the Young symmetry $f_T = (33), f_{CS} = (222)_C \times (6)_S$, and $f_O = (6)$ ($T = 0$, $S = S_Z = 3$, and S–wave) is given by[3]

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$$
\begin{aligned}
\Phi_d(x_i,q) = \frac{a_0}{48\sqrt{5}}\Big[& -\epsilon_{ijk}\epsilon_{lmn}\left(\epsilon_{ad}\epsilon_{be}\epsilon_{cf} + \epsilon_{ae}\epsilon_{bd}\epsilon_{cf} + \epsilon_{ad}\epsilon_{bf}\epsilon_{ce} + \epsilon_{af}\epsilon_{bd}\epsilon_{ce}\right) \\
& + \epsilon_{ijl}\epsilon_{kmn}\left(\epsilon_{ac}\epsilon_{be}\epsilon_{df} + \epsilon_{ac}\epsilon_{bf}\epsilon_{de} + \epsilon_{ae}\epsilon_{bc}\epsilon_{df} + \epsilon_{af}\epsilon_{bc}\epsilon_{de}\right) \\
& - \left(\epsilon_{ijm}\epsilon_{kln} + \epsilon_{ijn}\epsilon_{klm}\right)\left(\epsilon_{ac}\epsilon_{bd}\epsilon_{ef} + \epsilon_{ad}\epsilon_{bc}\epsilon_{ef}\right) \\
& + \left(\epsilon_{ikm}\epsilon_{jln} + \epsilon_{ikn}\epsilon_{jlm} + \epsilon_{jkm}\epsilon_{iln} + \epsilon_{jkn}\epsilon_{ilm}\right)\epsilon_{ab}\epsilon_{cd}\epsilon_{ef} \\
& - \left(\epsilon_{ikl}\epsilon_{jmn} + \epsilon_{jkl}\epsilon_{imn}\right)\left(\epsilon_{ab}\epsilon_{ce}\epsilon_{df} + \epsilon_{ab}\epsilon_{cf}\epsilon_{de}\right)\Big] \\
& \times a_i^{\dagger}(1) b_j^{\dagger}(2) c_k^{\dagger}(3) d_l^{\dagger}(4) e_m^{\dagger}(5) f_n^{\dagger}(6) \\
& \times x_1 x_2 x_3 x_4 x_5 x_6 \left(\ell n \frac{Q^2}{\Lambda^2}\right)^{-\gamma_0} \quad , \qquad (1)
\end{aligned}
$$

where the indices $i, j, \cdots, n$ and $a, b, \cdots, f$ are the color (r, y, b) and isospin (u, d) indices, respectively. The ϵ_{ijk}'s and ϵ_{ab}'s are the completely antisymmetric Cartesian tensors of $SU(3)_C$ and $SU(2)_T$. The coefficient a_0 represents the six quark amplitude at the origin and can be calculated after the complete six-quark wavefunction is given. However, the variation of the amplitude at the asymptotic high Q^2 region is mainly determined by the leading anomalous dimension γ_0. Anomalous dimensions of exclusive hadron amplitudes are given by solving the QCD evolution equation.

The evolution of the amplitude for simpler hadrons such as quark–antiquark meson[1,4] and three quark baryon[1] systems have already been formulated and solved. While these conventional hadrons have only one color singlet representation, six–quark systems considered here have five independent color singlet representations. The formulation of the evolution equation for totally antisymmetric multiquark states is not trivial even though it is a natural extension of the three-quark case. We have presented a general method for solving the QCD evolution equations which govern relativistic multi–quark wave functions.[5] We have also applied it to a four–quark toy system in SU$(2)_C$ and derived some constraints on the effective force between two baryons.[6] However, since the antisymmetric representation of a multiquark wave function must be constructed explicitly, it is hard in practice to solve the multi-quark evolution equation.

We avoid this problem by exploiting the permutation symmetry of the evolution kernel.[3] Each eigensolution of a multiquark state satisfies a kernel equation of the form

$$
K\,|\phi_A\rangle \;=\; \gamma\,|\phi_A\rangle \;, \qquad (2)
$$

where K, γ and $|\phi_A\rangle$ represent the kernel, the eigenvalue (*e.g.*, anomalous dimensions) and the eigenfunction which is given by a linear combination of the antisymmetric representations respectively. The most important observation in this formulation is that the kernel K is a linear combination of the operators Θ_{fY} in color space each of which has a definite Young symmetry f with Yamanouchi labels Y;

$$K = \sum_{fY} K_{fY} \Theta_{fY} \ . \tag{3}$$

For simplicity (but without loss of generality), let us consider a six-quark case as an example of multiquark systems. The six-quark system has five orthogonal color singlet states $|(222)\alpha\rangle$ with $\alpha = 1, 2, \ldots, 5$ and the evolution kernel becomes 5×5 matrix:

$$K_{\alpha\beta} = \langle (222)_C \alpha | K | (222)_C \beta \rangle . \tag{4}$$

An essential simplification can be obtained by replacing $K_{\alpha\beta}$ with K_{fY} such that

$$\begin{aligned} K_{\alpha\beta} &= \left\langle (222)_C \alpha \left| \sum_{fY} K_{fY} \Theta_{fY} \right| (222)_C \beta \right\rangle \\ &= \sum_{f} \sum_{Y} \langle (222)_C \alpha, \ fY | (222)_C \beta \rangle K_{fY} \end{aligned} \tag{5}$$

where the possible f which gives the non-zero Clebsch-Gordon coefficient[7] is only (6) or (42). For example, the leading anomalous dimension of the deuteron state can be obtained by following procedures. Projecting out a certain state which has common C, T and O representations, we get a set of equations for spin states. Since the kernel of each equation has a definite symmetry and its explicit representation is known, we can determine relative weighting factors among the independent equations by counting the number of spin annihilation terms in the kernel. The only equation which we have to solve explicitly is the equation which has the symmetric kernel $K_{(6)}$. After taking into account the relative weighting factor we get the leading anomalous dimension for a deuteron state,

$$\gamma_0 = \frac{3}{4} \frac{C_F}{\beta} \qquad \text{for} \quad S_Z = 0 \ , \tag{6a}$$

$$= \frac{7}{8} \frac{C_F}{\beta} \qquad \qquad = \pm 1 \ . \tag{6b}$$

Using the result of Eq. (6), one can calculate the asymptotic deuteron form factor $F_d(Q^2)$. The QCD prediction for the asymptotic Q^2–behavior of the deuteron reduced form factor[8] $f_d(Q^2)$ defined by

$$f_d(Q^2) \; = \; \frac{F_d(Q^2)}{F_N^2\left(\frac{Q^2}{4}\right)}$$

is given by

$$f_d(Q^2) \; \sim \; \frac{\alpha_s(Q^2)}{Q^2}\left(\ell n \; \frac{Q^2}{\Lambda^2}\right)^{C_F/2\beta} \quad . \tag{11}$$

The deuteron state which has the leading anomalous dimension is related to the NN, $\Delta\Delta$, and hidden color (CC) physical bases, for both the $(TS) = (01)$ and (10) cases with Young symmetry of $\{33\}$, by the formula[9]

$$\psi_{[6]\{33\}} = \sqrt{\frac{1}{9}}\;\psi_{NN} + \sqrt{\frac{4}{45}}\;\psi_{\Delta\Delta} + \sqrt{\frac{4}{5}}\;\psi_{CC} \quad .$$

The fact that the six–quark state is 80 percent hidden color at small transverse separation implies that the deuteron form factors cannot be described at large Q^2 by meson–nucleon degrees of freedom alone.

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