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Perturbative Quantum Chromodynamic Prediction For The Heavy Quark Fragmentation Function*

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ABSTRACT

A perturbative QCD analysis is presented for the inclusive production of pseudoscalar and vector mesons to derive fragmentation function of heavy quarks produced in e^+e^- annihilation. The results with no arbitrary parameters are in reasonable agreement with experimental data for bottom and charm quark fragmentation functions.

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The new generation of high energy e^+e^- colliders such as TRISTAN, SLC, and LEP, is expected to provide a wealth of information about production and hadronization of heavy quarks (c, b, t). The production of heavy particles in these accelerators will be an important testing ground for the perturbative quantum chromodynamics (QCD).[1] One of the interesting properties of heavy quarks which can be studied in the framework of perturbative QCD is the evolution of quarks into hadrons. The mechanism responsible for hadronization, in general, is specified by the fragmentation function $D_Q^H(z, q^2)$ which represents the fragmentation of quark Q into final state hadron H with the momentum fraction $z \equiv \frac{2E}{\sqrt{s}}$, where E is the energy of hadron and s is the square of total e^+e^- c.m. energy. The fragmentation function D_Q^H depends also on the four-momentum transfer q involved in the process. Various phenomenological models, like Lund model,[2] Cascade model,[3] and Peterson et al model,[4] motivated by QCD, have been developed to describe the fragmentation function D_Q^H.

Since these models[2,3,4] are based on phenomenology, they involve parameters to be fixed by the experimental data. In this letter, we will calculate the heavy quark fragmentation function into meson M (D_Q^M) using perturbative QCD. Our results for D_Q^M involves no arbitrary parameters and acquire a simple form in the limit of $\frac{m^2}{s} \ll 1$, where m is the mass of the meson. This limit $\left(\frac{m^2}{s} \ll 1\right)$ is realized when we consider the production of charmed mesons and B mesons (mesons with b quark) at c.m. energy of about 30 GeV. The z-dependence of D_Q^M derived in this calculation takes the form

$$D_Q^M(z) \sim z(1-z)^2\ F(z)$$

which is consistent with the expected behavior of phenomenological model of

Ref. 4. As we will see, the detailed form of $F(z)$ determines where the peak of the fragmentation function appears.

Our approach is based on studying the inclusive production of heavy mesons (mesons with at least one heavy quark), in high energy e^+e^- annihilation. The diagrams which contribute to this process in the leading order of QCD running coupling constant $\alpha_s(q^2)$ are given by Fig. 1. The differential cross section for these reactions can be written as

$$d\sigma = \frac{|\mathcal{M}|^2}{2s} \frac{d^3p}{2E} \frac{d^3q_1}{2E_1} \frac{d^3q_2}{dE_2} \frac{1}{(2\pi)^5} \delta^4(k_1 + k_2 - p - q_1 - q_2) \ , \tag{1}$$

where E_1 and E_2 are the energies of primary and secondary quarks respectively (see Fig. 1).

The invariant amplitude $\mathcal{M}$ can be obtained from the generic form[5]

$$\mathcal{M}(k_i,\ q_i) = \int [dx]\ T_H(k_i,\ q_i,\ x_i)\ \phi_M(x_i,\ q^2) \ , \tag{2}$$

where T_H is the hard scattering amplitude which can be calculated perturbatively from quark-gluon subprocesses and ϕ_M is the probability amplitude to find quarks which are collinear up to the scale q^2 in a mesonic bound state.[1] In Eq. (2), x_i's are the momentum fractions carried by the constituent quarks and $[dx] = dx_1\ dx_2 \delta(1 - x_1 - x_2)$.

For the heavy quark systems, the bound state wavefunction can be reliably determined by non-relativistic considerations. Therefore, we use the quark distribution amplitude ϕ_M given in Ref. 5 which is derived in the same approximation.

For pseudoscalar mesons, ϕ_M is given by[5]

$$\phi_M(x_i,\ q^2) = \frac{f_M}{2\sqrt{3}}\, \delta\left(x_1 - \frac{m_1}{m}\right)\ , \tag{3}$$

where f_M is the pseudoscalar meson decay constant and $m = m_1 + m_2$ with $m_1 \geq m_2$.

By substituting Eq. (3) into Eq. (2), the invariant amplitude $\mathcal{M}$ becomes

$$\mathcal{M}(k_i, q_i) = \frac{f_M}{2\sqrt{3}}\, T_H(k_i,\ q_i, r) \tag{4}$$

where $r \equiv m_1/m$. From diagrams of Fig. 1, the hard scattering amplitude T_H can be written as

$$\begin{aligned}
T_H(k_i, q_i, r) = {} & i\, \frac{16\pi^2 C_F \alpha\alpha_s}{s^2}\, \bar{v}(k_2)\gamma^\mu u(k_1) \\
& \times \left[\frac{e_1}{(1-r)(\sqrt{s} - 2E_1)} \left\{ \frac{\bar{u}(q_2)\gamma_\alpha\gamma_s(\not{p} + m)\gamma^\alpha(\not{p} + \not{q}_2 + rm)\gamma_\mu v(q_1)}{\sqrt{s} - 2E_1} \right.\right. \\
& \qquad \left. + \frac{\bar{u}(q_2)\gamma_\alpha\gamma_s(\not{p} + m)\gamma_\mu(-\{\not{q}_1 + \not{q}_2 + (1-r)\not{p}\} + rm)\gamma^\alpha v(q_1)}{\sqrt{s} - 2rE} \right\} \\
& + \frac{e_2}{r(\sqrt{s} - 2E_2)} \left\{ \frac{\bar{u}(q_2)\gamma_\mu\{-(\not{p} + \not{q}_1) + (1-r)m\}\gamma_\alpha\gamma_s(\not{p} + m)\gamma^\alpha v(q_1)}{\sqrt{s} - 2E_2} \right. \\
& \qquad \left.\left. + \frac{\bar{u}(q_2)\gamma_\alpha\{\not{q}_1 + \not{q}_2 + r\not{p} + (1-r)m\}\gamma_\mu\gamma_s(\not{p} + m)\gamma^\alpha v(q_1)}{\sqrt{s} - 2(1-r)E} \right\} \right]
\end{aligned} \tag{5}$$

where e_1 and e_2 are quark charges, α is the fine structure constant and the color factor $C_F = 4/3$, and $\gamma_s = \gamma_5/\sqrt{2}$.

By performing the phase space integration in Eq. (1), the differential cross section is given by

$$d\sigma = \frac{|\mathcal{M}|^2}{64(2\pi)^5}\, dzdy\; d\cos\theta\; d\phi\; d\cos\theta_{MQ}\; d\phi_{MQ}$$
$$\times\, \delta\left(\cos\theta_{MQ} - \frac{2-2(y+z)+yz+r\tau^2}{\sqrt{z^2-\tau^2}\sqrt{y^2-r^2\tau^2}}\right)\, \theta(2-y-z)\ , \tag{6}$$

where $y = \frac{2E_1}{\sqrt{s}}$, $\tau^2 = \frac{4m^2}{s}$ and the angles $(\theta,\ \phi)$ specify the direction of the meson with respect to the beam axis and $(\theta_{MQ},\ \phi_{MQ})$ determine the direction of the primary quark with respect to the meson. After substituting Eqs. (4) and (5) into Eq. (6), the angular integrations can be easily performed. From the δ-function in Eq. (6), the limits of y-integration are determined by the equation $\cos^2\theta_{MQ} = 1$,

$$\left\{4(1-z)+\tau^2\right\}y^2 - 2(2-z)\left\{2(1-z)+r\tau^2\right\}y + \left\{2(1-z)+r\tau^2\right\}^2 + r^2\tau^2(z^2-\tau^2) = 0\ . \tag{7}$$

Since $\tau^2 < 1$, Eqs. (6) and (7) can be expanded in powers of τ^2 and we find the solutions of Eq. (7) to be,

$$y_u = 1 - \frac{(1-rz)^2}{4z(1-z)}\,\tau^2 + \ \ldots,\quad y_\ell = 1 - z + \frac{\{1-(1-r)z\}^2}{4z(1-z)}\,\tau^2 + \ \ldots\ , \tag{8}$$

where $y_\ell \le y \le y_u$.

After y-integration in Eq. (6) is performed,[6] the general form of the differential cross section becomes,

$$\frac{d\sigma}{dz} = f_M^2\,\frac{\tau^2}{sm^2}\left(\frac{C_1}{\tau^2} + C_2\ell_n\tau^2 + O(\tau^2)\right) \tag{9}$$

where C_i's are dimensionless functions of z and r. In the limit of $\tau^2 \ll 1$, we obtain the differential cross section for the inclusive pseudoscalar meson production which is given by

$$\frac{d\sigma}{dz} = \frac{64\pi\alpha^2\alpha_s^2 f_M^2}{81sm^2}\ z(1-z)^2$$
$$\times\left[\frac{e_1^2}{(1-r)^2}\ \frac{\{1+(1-r)z\}^2}{(1-rz)^4} + \frac{e_2^2}{r^2}\ \frac{(1+rz)^2}{\{1-(1-r)z\}^4}\right] . \tag{10}$$

The fragmentation function[7] $D_Q^M(z)$ is related to inclusive cross section of Eq. (10) by $D_Q^M(z) = \frac{1}{\sigma}\frac{d\sigma}{dz}$ with normalization condition $\int_0^1 D_Q^M(z)dz = 1$. Therefore, $D_Q^M(z)$ for pseudoscalar meson production is given by,

$$D_Q^M(z) = N^{-1}(r)\ z(1-z)^2 f(r,\ z)\ , \tag{11}$$

where $f(r,z)$ is given by the terms in the square bracket of Eq. (10) and $N(r)$ is

$$N(r) = e_1^2\left\{\frac{\ell n(1-r)}{r^6(1-r)^2}\ (10-20r+16r^2-6r^3+r^4)\right.$$
$$\left. + \frac{1}{6r^5(1-r)^3}\ (60-150r+146r^2-69r^3+15r^4)\right\} \tag{12}$$
$$+ e_2^2\Big\{r \leftrightarrow (1-r)\Big\}\ .$$

The fragmentation function for vector meson production can be derived by similar calculations.[8] The result is

$$D_Q^M(z) = N_v^{-1}(r)\ z(1-z)^2$$
$$\times\left[\frac{e_1^2}{(1-r)^2}\ \frac{\{1+(1-r)z\}^2+2z^2}{(1-rz)^4} + \frac{e_2^2}{r^2}\ \frac{(1+rz)^2+2z^2}{\{1-(1-r)z\}^4}\right] \tag{13}$$

where

$$N_v(r) = e_1^2 \left\{ \frac{\ell n(1-r)}{r^6(1-r)^2}(30 - 36r + 18r^2 - 6r^3 + r^4) \right.$$

$$\left. + \frac{1}{2r^5(1-r)^3}\,(60 - 102r + 62r^2 - 23r^3 + 5r^4) \right\} \tag{14}$$

$$+ e_2^2\{r \leftrightarrow (1-r)\}\ .$$

The factor $z(1-z)^2$ in Eqs. (11) and (13) can be understood as a combination of phase space factor and the energy transfer in the hadronization process.[4] By differentiating Eqs. (11) and (13), we find peak value of fragmentation function approximately appears[8] at $z_{\rm peak} \approx \dfrac{1}{\sqrt{5-5r+r^2}}$. For D mesons $z_{peak} \approx 0.8$, while for B mesons $z_{peak} \approx 0.9$. For large z values ($z \approx 1$), the prediction for $D_Q^M(z)$ behaves like $D_Q^M(z) \sim (1-z)^2$, which is consistent with experimental data.[9] Our predictions do not include indirect channels which contribute to experimental data. Also, there could be small corrections due to relativistic effects for the bound state wave function in Eq. (3).

By taking into account all terms in Eq. (9) (i.e. higher twist mass terms), we found an overall effect about 20% – 50% for D and B mesons which depends on the beam energy. As the beam energy becomes smaller, the lower limit of z moves toward $z = 1$ due to production threshold. Therefore, the peak of $D_Q^M(z)$ appears closer to $z = 1$ and larger in magnitude. This effect will be more dramatic for T meson production[10] where the fragmentation function behaves like delta function.

In Fig. 2, we have compared with experimental data[9] the predictions of Eqs. (11) and (13) for the fragmentation functions of c and b quarks into D and B mesons respectively. Even though the predictions have no arbitrary parameters, the agreement with the experimental data for both b and c quark fragmentation functions are reasonable. On the other hand, for $r \approx 1/2$, as is the case of J/ψ and η_c production, the interference terms proportional to $e_1 e_2$ become non-negligible even though these terms are still suppressed by the factor r^2. We found this contribution is constructive for vector meson and destructive for pseudoscalar meson production. This is reminiscent of the same effect in exclusive processes.[1] To study this case ($r \approx 1/2$), the contributions from some other leading (α_s) order diagrams must be included.[11]

Since $D_Q^M(z, q^2)$ at high c.m. energy (Eqs. (11) and (13)) have no q^2-dependence, we expect the heavy quark fragmentation function to exhibit a scaling behavior. Actually as we can observe from Fig. (2), the present data indicate the scaling behavior to some extent. When better statistics become available, the comparison of the data with the predictions of Eqs. (11) and (13) can serve as an important test of the perturbative QCD.

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6. Due to the gluon propagator in T_H (see Eq. (5)), the integrand of y-integration involves the factor $\frac{1}{(1-y)^2}$. Therefore the leading terms of Eq. (10) in the expansion of τ^2 are determined by the coefficients of τ^2 in y_u and y_ℓ (see Eq. (8)).

7. To the leading order of τ^2, $D_Q^M(z,\ q^2)$ is independent of $q^2 \sim s$.

8. The vector meson production was considered in L. Clavelli, Phys. Rev. D26, 1610 (1982), without deriving analytic form of the fragmentation function. We found a discrepancy with Eq. (2.25) of that reference. According to our calculation, that equation must be

$$\lim_{(s/m^2)\to\infty} R(^3S_1(q\bar{q})) = \frac{A^2}{m^2}\,\alpha_s^2 e_q^2\,\frac{64768}{9}\left(\frac{351}{506} - \ell_n 2\right) .$$

For pseudoscalar mesons with $m_1 = m_2$, we obtain

$$\lim_{(s/m^2)\to\infty} R\left({}^1S_o(q\bar{q})\right) = \frac{f_M^2}{m^2}\alpha_s^2 e_q^2 \frac{13568}{27}\left(\frac{221}{318} - \ell_n 2\right) .$$

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10. In this case, both contributions from γ and Z^o boson must be included. Details of this work will be submitted to Phys. Rev. D.

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FIGURE CAPTIONS

Fig. 1. Inclusive production of mesons in e^+e^- annihilation. Two more diagrams can be obtained from (a) and (b) by exchanging primary and secondary quark pairs.

Fig. 2. Charm (a) and bottom (b) fragmentation function predictions, Eqs. (14) and (16), compared to various experimental data.[9] The solid curves in (a) and (b) are pseudoscalar mesons and the dashed curves are vector mesons.

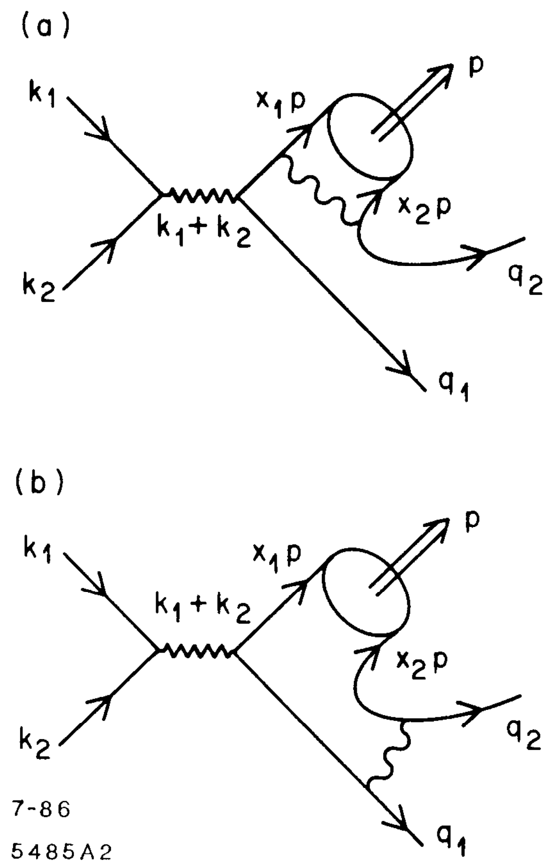
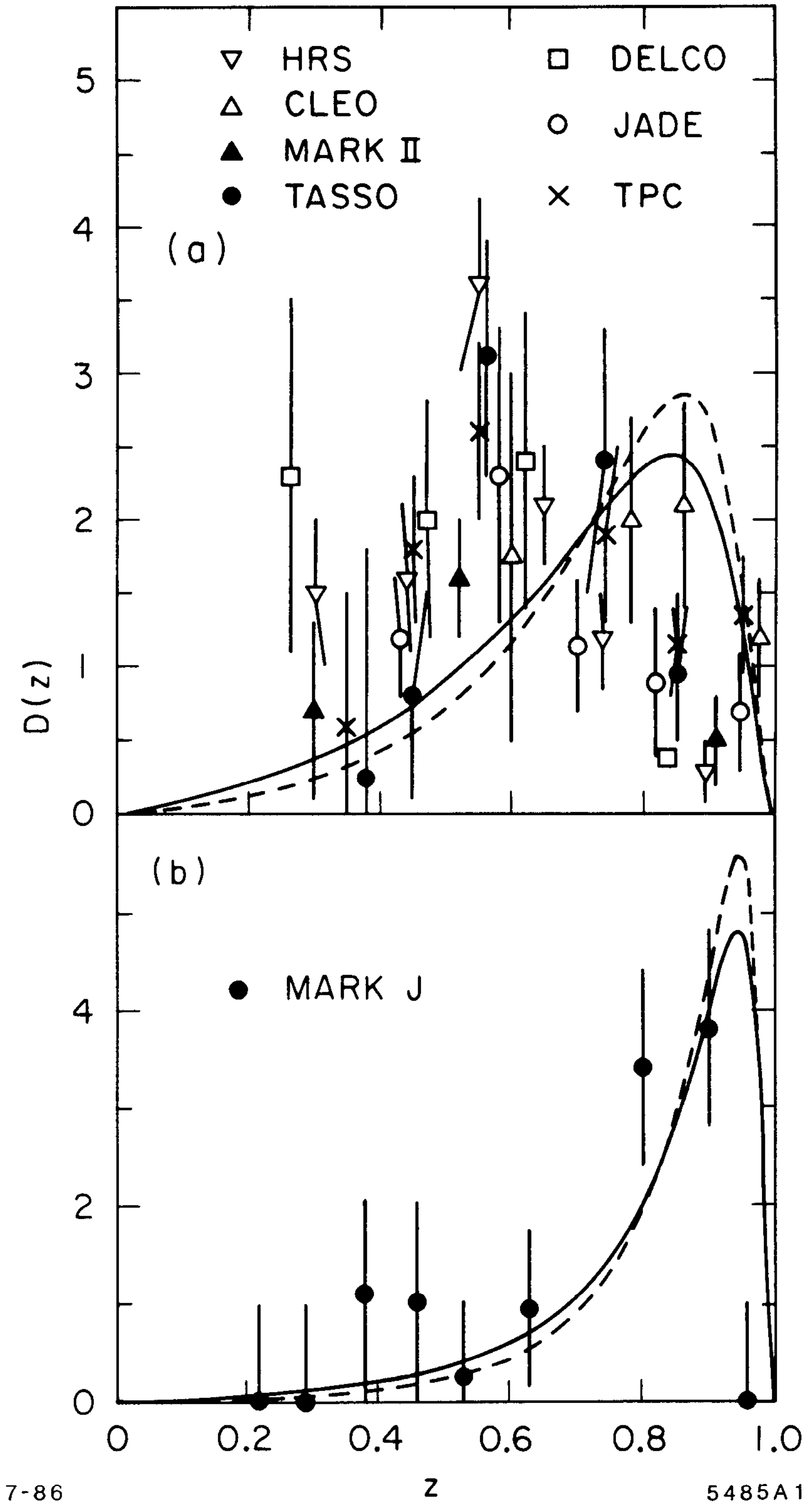


Fig. 1



HRS
CLEO
MARK II
TASSO
DELCO
JADE
TPC
(a)
D(z)
(b)
MARK J
0
0.2
0.4
0.6
0.8
1.0
z
7-86
5485A1

Fig. 2