SLAC – PUB – 4018
September 1986
(T/E)

HEAVY QUARK PRODUCTION IN QUANTUM CHROMODYNAMICS*

STANLEY J. BRODSKY

Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94305

For very heavy quark masses, the inclusive hadronic production of hadron pairs containing heavy quarks is predicted to be governed by QCD fusion subprocesses. For intermediate mass scales other QCD mechanisms can be important including higher-twist intrinsic contributions and low relative velocity enhancements, possibly accounting for the anomalies observed in charm hadroproduction, such as the nuclear number dependence, the longitudinal momentum distributions, and beam flavor dependence. We also discuss scaling laws for exclusive processes involving heavy quarks and diffractive excitation into heavy quark systems.

1. INTRODUCTION

There are a number of reasons why heavy quark hadroproduction plays a critical role in particle physics phenomenology:

(1) For large quark mass or large jet transverse momentum compared to the QCD scale $\Lambda_{\overline{MS}}$ the perturbative predictions are unambiguous and are governed by definite factorization theorems[1]. Heavy quark production cross sections are maximally sensitive to the gluon distribution of the proton.

(2) An understanding of heavy quark production is necessary to project the rate for new particle production such as weak bosons, Higgs particles, supersymmetric hadrons, etc.

(3) The heavy quark events must be understood in order to unravel flavor mixing parameters, and to correctly project backgrounds to rare processes.

(4) The muon content of high energy photon and nucleon induced cosmic ray showers depends in detail on the properties of charm photoproduction and hadroproduction[2].

(5) QCD predicts a number of novel features for the hadroproduction of heavy quarks, such as forward-backward asymmetries[3] in $p\bar{p}$ collisions, and exclusive channnel dominance near threshold[4].

(6) There are intriguing anomalies in the data for charm hadroproduction which are difficult to explain from standard perturbative QCD. These include flatter distributions in x_F than apparently predicted by primary "fusion" subprocesses, the dependence of the cross section on the nuclear number in fixed target experiments is significantly less than additive, the production cross section for the charmed-strange baryon $A^+(csu)$ in hyperon beams appears anomalously large, and there are indications that the charmed sea distribution in the proton may be larger than predicted by standard evolution. Here we shall argue that the charm mass scale is not sufficient to be in the QCD perturbative domain, and that the above anomalies give new insights into physics at the interface between perturbative and non-perturbative QCD.

1. FACTORIZATION THEOREMS

As shown this past year by Collins, Soper, and Sterman[1], the analysis of QCD factorization for massive lepton pairs[5] can be carried over to the production of heavy quarks in the regime $p_T^2 \geq m_Q^2 \gg \Lambda^2_{\overline{MS}}$. Furthermore factorization holds for the total heavy quark pair production cross section, at least to leading order in $\alpha_s(m_Q^2)$ since the dominant integration region comes from p_T^2 of order m_Q^2. The origins[6] of factorization for the total cross section can be seen in the original all orders QED analyses of the Bethe Heitler cross section for lepton pair production in a strong Coulomb field. As shown by Bethe and Maximon[7], the leading logarithmic contribution, which involves soft momentum transfer to the target, only occurs in Born approximation. All contributions higher order in $Z\alpha$ involve impact distances $b_\perp$ of order m_Q^{-1}. In QCD the leading gluon exchange contribution is represented by the gluon structure function of the target. Since the multi-gluon exchange amplitudes all involve small impact parameter, they are higher order in $\alpha_s(m_Q^2)$.

The leading order analysis of CSS then indicates that the fusion $gg \to Q\overline{Q}$ and $q\bar{q} \to Q\overline{Q}$ subprocesses and the factorization formula

$$\frac{d\sigma}{dx_a\,dx_b} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_B\, G_{a/A}(x_a,Q) \times G_{b/B}(x_b,Q)\,\sigma_{ab\to cd}(x_a x_b s)$$

is correct to leading order in μ^2/m_Q^2 and $\alpha_s(m_Q^2)$. The analysis of CSS also shows that diffraction is already included in the above analysis to leading order and should not be added separately.

*Work supported by the Department of Energy, contract DE–AC03–76SF00515.

Talk presented at the 23rd International Conference on High Energy Physics, Berkeley, California, July 16-23, 1986

The dominant short-distance subprocesses are the $gg \to \overline{Q}\overline{Q}$ and $q\bar{q} \to Q\overline{Q}$ fusion reactions. In addition, integrating over each collinear region of two to three subprocesses such as $gg \to gQ\overline{Q}$ yields logarithmically dominant two to two subprocesses contributions such as $gQ \to gQ$ and $gg \to gg$ with subsequent splitting[8] of a final gluon into the produced heavy quark pair. (See Fig. 1.) The actual $Q\overline{Q}$ production can occur either through the fusion subprocesses, or the heavy components of jet fragmentation, or incident hadron evolution. All such processes have to be taken into account in the leading twist cross section.

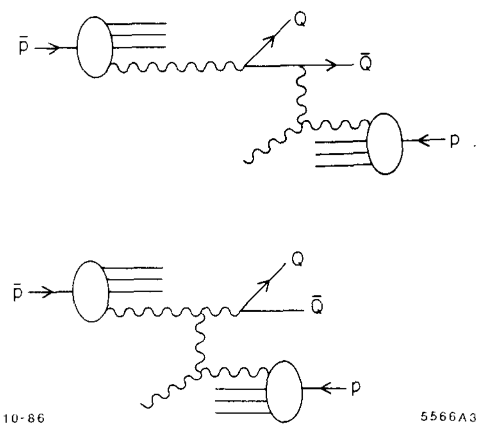

Figure 1. Decomposition of $2 \to 3$ processes to a sum of $2 \to 2$ heavy quark subprocesses.

Although the two-to-three tree subrocesses have been evaluated[3], the virtual corrections to the two-to-two amplitudes have not been calculated. In view of the large color couplings of incident gluons, one can expect a large "K"- factor correction to the Born results. In addition, the integrated heavy quark cross section depends sensitively on the effective quark mass. Thus at this time there is no absolutely normalized QCD prediction for heavy quark production, either at high transverse momentum or for the total cross section. Although the physical arguments are convincing, a complete proof that factorization gives the leading power law contribution to the cross section is highly non-trivial and has only been outlined. As first shown for the Drell-Yan process, one must satisfy a target length condition[9] in order that inelastic initial state interactions do not ruin factorization: The active quark or gluon energy must be large compared to a scale proportional to the length of the target: $x_a s > M_N^2 L_A \mu^2$ where μ^2 is a typical hadron scale and L_A is the length of the target in its rest frame. One also requires that[1] the transverse momenta p_T^c and p_T^d of the produced heavy particles are of order m_Q (the natural scale), the rapidity difference $y_c - y_d$ is finite, and the production energy is well above threshold, $s \gg 4m_Q^2$. In the perturbative analysis of the non-Abelian theory there are subtleties in higher orders concerned with initial-state elastic interactions and their possible effect on color averaging[9]. A convincing demonstration of this aspect has not yet been given, except in the case where the subprocess amplitude corresponds to annihilation into a color singlet, as in massive lepton pair production.[5]

2. ANOMALOUS FEATURES OF CHARM HADROPRODUCTION

Do the existing date for heavy quark production agree with the leading order QCD predictions? The recent measurements of $b\bar{b}$ jets at large p_T by the UA1 collaboration reported at this meeting appear to agree with the scaling behavior of the predicted cross sections, with a rate within a factor of 2 or 3 of estimates for the leading order contributions. As noted above, there is at this time no absolutely normalized QCD prediction.

Whether the data for charm hadroproduction agree with the leading order QCD predictions is problematic. The leading fusion contributions predict cross sections which are essentially additive in the nucleon number of a nuclear target. The FNAL measurements of Ref. 10 however show an A-dependence characteristic of shadowing and diffraction. (See Fig. 2.)

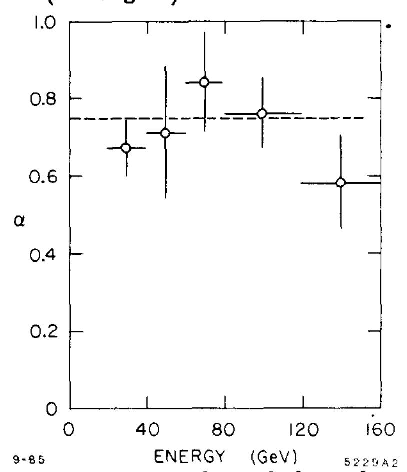

Figure 2. Nuclear number A^α dependence of the charm cross section as a function of neutrino energy. (From Ref. 10).

The $pp \to \Lambda^c X$ data[11] from the ISR gave the first indications that charm production may be much flatter in longitudinal momentum than expected from the very central gluon fusion subrocesses. This appears to be confirmed by Serpurkhov data[12] for $40 GeV$ neutron Carbon collisions: $dN/dx_F(nN \to \Lambda_c X) \sim (1 - x_F)^{1.5\pm0.5}$ for $x_F \geq 0.5$ Recent data for high energy pion, and kaon beams measured by the ACCMOR[13] and LEBC-EHS[14] collaborations at the SPS are shown in Figs. 3-5. The data again shows sizeable contributions at large x_F, although the statistics are not large. There is also some evidence for a *leading particle effect*; *i.e.*, a correlation of the produced charmed hadron with the meson beam quantum numbers. This effect is not predicted by the leading order QCD predictions. The new data from the LEBC $800 GeV/c$ proton beam experiment at FNAL reported at this meeting, however, do not show a leading particle effect and there is only moderate growth in magnitude of the D production cross section.

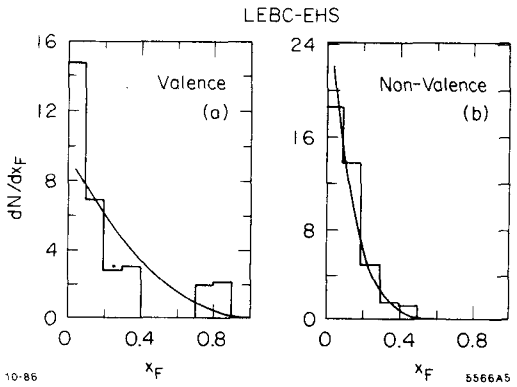

Figure 3. The x_F distribution for $\pi^- p \to DX$ at $360 GeV/c$ measured in the LEBC-EHS experiment (Ref. 14). (a) D mesons containing valence quarks of the pion; (b) nonvalence D mesons. The curves represent fits $(1 - x_F)^n$ with $n = 1.8$ and $n = 7.9$, respectively.

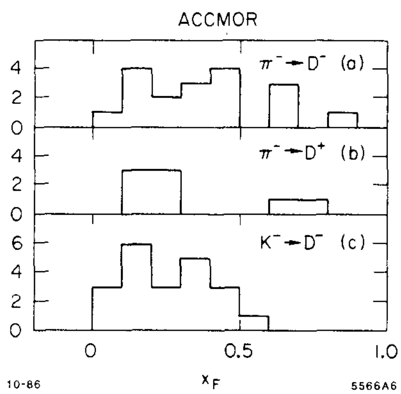

Figure 4. Longitudinal momentum fraction distributions for π and K production of D mesons at $200 GeV/c$. (From Ref. 13). The data are not corrected for acceptance. Production appears to be fairly flat even for charmed hadrons not carrying a valence quark of the incident meson.

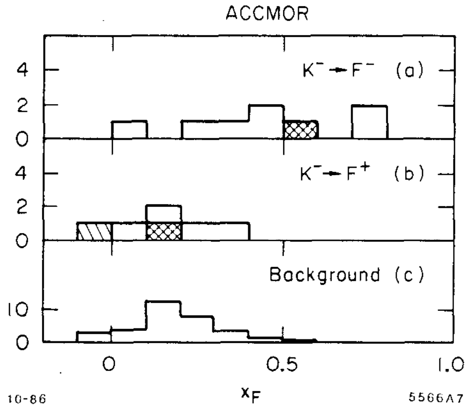

Figure 5. The x_F distribution for F^- production in $200 GeV/c$ $K^- Si$ interactions. (From Ref. 13). The hatched areas represent ambiguous events. A leading particle effect is apparent.

Another intriguing anomaly in charm hadroproduction is seen in the WA–42 experiment[15] at the SPS, which reports copious production of the A^+ (csu) charmed strange baryon in 135 GeV Σ^- collisions on a beryllium target. The A^+ is observed in the $\Lambda K^- \pi^+ \pi^+$ channel with a hard distribution $(1 - x_F)^{1.7 \pm 0.7}$ for $x_F > 0.6$. (See Fig. 6.)

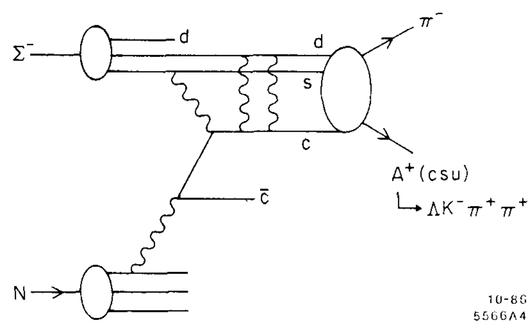

Figure 6. Schematic representation of A^+ production by hyperon beams. The multigluon exchange can represent either intrinsic heavy $c\bar{c}$ contributions to the Σ^- wavefunction (an initial state effect) or prebinding distortion from final state interactions.

The corresponding cross section times branching ratio (taking the above form for all x_F), for forward x_F is 4.7 μb/nucleon assuming A^1 dependence. If the branching ratio for the measured channel is 3% to 5% this implies a total cross section in the 100 to 150 μb range. Even larger cross sections might be expected for the production of charmed-strange (csd) baryons which carry two valence quarks of the Σ^- (sdd). The experimental results suggest the possibility of systematically enhanced production of heavy quark states by hyperon and kaon beams.

3. BREAKDOWN OF FACTORIZATION AND FINAL STATE INTERACTION EFFECTS

Physically, factorization is expected to be valid when initial and final state interactions with the spectator constituents of the incident and outgoing hadrons can be neglected.[6] The basic fusion production process occurs within a *unitary volume* $\Delta b_\perp \Delta z$ where (in the target rest frame) $\Delta b_\perp \sim (p_T^Q)^{-1}$, $\Delta z \sim (m_Q v_Q)^{-1}$. Only gluons with wavelength sufficiently short relative to the unitary volume can resolve the produced pairs and cause significant reinteractions with the spectator hadrons. Thus breakdown of factorization can occur when $k_\perp \sim O(p_\perp^Q)$ or $k_z \sim O(m_Q v_Q)$; *i.e.*, important final state corrections to the QCD factorization formula may occur when the heavy pair is produced at low p_T or at small relative rapidity in the target (or beam) fragmentation regions. Such behavior is well known in QED, which predicts a strong redistribution in the Coulomb field of the target. In addition, higher twist contributions arise from the intrinsic heavy quark Fock components of the hadron wavefunction. We discuss each of these effects below.

Consider the cross section for the photoproduction of a heavy lepton pair in the Coulomb field of a nucleus in QED. For large Z the cross section is strongly distorted at small lepton velocities $v_\pm \ll Z\alpha$ by multiple soft Coulomb interactions[20]

$$d\sigma\ (\gamma Z \to \ell\bar{\ell}X) \ = \ d\sigma_0 \, \frac{\varsigma_+ \varsigma_-}{[(e^{\varsigma_+} - 1)(1 - e^{-\varsigma_-})]} .$$

Here $d\sigma_0$ is the Bethe–Heitler cross section computed in Born approximation, and $\varsigma_+ = 2\pi Z\alpha/v^+$, $\varsigma_- = 2\pi Z\alpha/v^-$. These results are strictly valid for $\varsigma_+ \ll 1$, but ς_- can be unrestricted. The effect of the correction factor is to distort the cross section toward small negative-lepton velocity (relative to the target rest frame). As $v^- \Rightarrow 0$, the enhancement is so strong that even the threshold phase-space suppression factor in σ_0 is cancelled. Conversely, the cross section is exponentially damped when the positive lepton has low velocity.

An analogous effect evidently will also occur in QCD. For example in charm photoproduction $\gamma g \to c\bar{c}$, the charmed quark rapidity will be skewed toward that of the spectator qq system of the nucleon where it can bind to form color singlets. We can estimate[16] this QCD prebinding effect by replacing $\pi Z\alpha \to (4/3)\pi\alpha_s(Q^2)$ in the QED distortion factor, Eq. (4.1). (We take Q^2 to be the relative momentum of the c–quark and the spectator system and limit $|\alpha_s| \le 4$.) Clearly this gives only a very rough estimate of physics controlled by QCD nonperturbative effects. The behavior predicted by this model indicates significant increases in the magnitude of the heavy quark production cross sections and significant skewing of the heavy particle momentum distribution towards large x_F. (See Fig. 7.) It thus seems likely that the distortion of the fusion Born approximation cross section due to prebinding attractive forces in QCD will be significant for the production of heavy quarks at collider energies, enhancing the production of Λ_c *etc.* in the forward low p_T region. Unlike the case of final-state interaction corrections to hard scattering processes, the corrections discussed here coherently enhance the production process and are not limited by unitarity to be of $\mathcal{O}(1)$. If there are strange quarks in the incident-hadron, then the distortion is likely to be magnified, since a strange quark tends to be more nonrelativistic than u or d quarks in a hadron and thus more effective in "capturing" other quarks. This possibility could explain the relatively copious production of the $A^+(csu)$ in the Σ^- fragmentation region, and suggests an important role of hyperon and strange meson beams for charm and heavy particle production experiments.

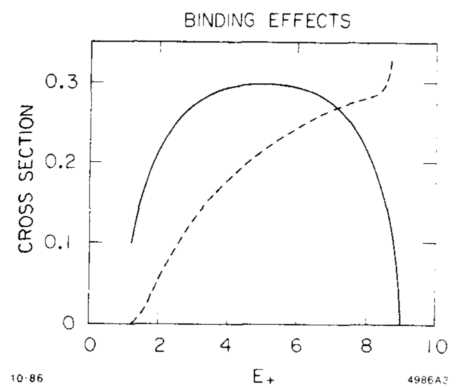

Figure 7. The Bethe–Heitler cross section $\gamma Z \to \ell^+\ell^- Z$ in Born approximation (solid curve) as a function of the positive lepton energy. The dotted curve shows the modified spectrum due to multiple scattering $Z\alpha \to (4/3)\alpha_s(Q^2)$. We have used $\alpha_s(Q^2) = 4\pi/(\beta_0 \ell n(1 + Q^2/\Lambda^2))$, $|\alpha_s| < 4$, where Λ = 200 MeV and Q^2 is the 4-momentum squared of the lepton relative to the target.

4. INTRINSIC HEAVY QUARKS

Since the momentum of a charmed hadron tends to follow the momentum of the produced charmed quark (the Bjorken–Suzuki effect), the longitudinal momentum dependence of the charm hadroproduction data suggest that the charm quarks themselves have large momentum fraction in the nucleon. Such a possibility can be checked by measurements of deep inelastic scattering of leptons on the charm constituents of the nucleon. The available high Q^2 data from the EMC collaboration[17] (see Fig. 8), as extracted from $\mu N \to \mu\mu X$ data, seem to indicate an anomalously large $c(x, Q^2)$ distribution at large Q^2 and $x_{B_j} \sim 0.4$ compared to that expected for the photon-gluon fusion diagrams or, equivalently, from QCD evolution.[18] Although the data has low statistics and thus could be misleading, it clearly suggests the existence of mechanisms for charm production other than the standard photon-gluon fusion subprocess.

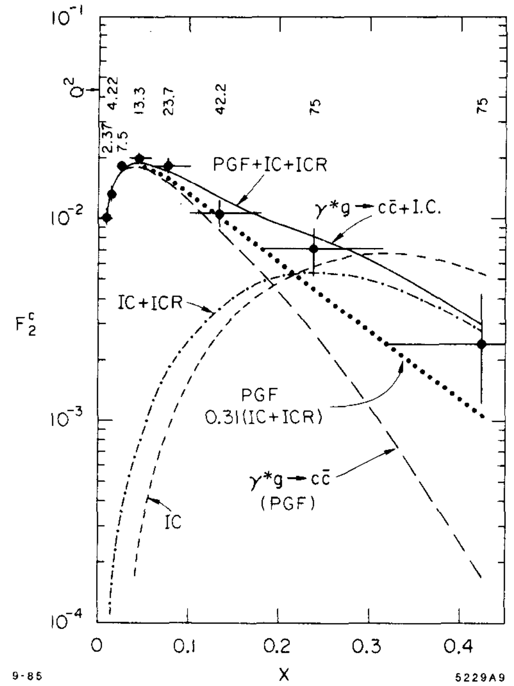

Figure 8. Analysis of the charm structure function of the proton by Hoffmann and Moore,[18] in terms of PGF (photon-gluon fusion $\gamma^* g \to c\bar{c}$, IC (1% intrinsic charm), and ICR (radiated intrinsic charm components). The data are from the EMC collaboration.[17]

Dimension-six contributions to the effective Lagrangian imply the existence of Fock states in the nucleon containing an extra $Q\overline{Q}$ pair.[19] (See Fig. 9.) Eventually lattice gauge theory or the discretized light cone quantization method[20] can be used to determine the heavy particle content of meson and baryon wavefunctions. At this time we can deduce[19,21] the following semiquantitative properties for intrinsic states such as $|uudQ\overline{Q}\rangle$: (1) The probability of such states in the nucleon is nonzero and scales as m_Q^{-2}. (2) The maximal wave function configurations tend to have minimum off-shell energy, corresponding to constituents of equal velocity or rapidity, *i.e.*,

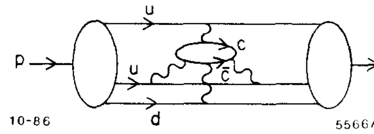

Figure 9. Representation of a heavy quark Fock state in the proton..

$$x_i \equiv \frac{(k^\circ + k^z)_i}{p^\circ + p^z} \propto \sqrt{(k_\perp^2 + m^2)_i} \quad . \qquad (5.4)$$

Thus intrinsic heavy quarks tend to have the largest momentum fraction in the proton wave function, just opposite to the usual configuration expected for sea quarks. (3) The transverse momenta of the heavy quarks are roughly equal and opposite and of order m_Q, whereas the light quarks tend to have soft momenta set by the hadron wave function. (4) The effects are strongly dependent on the features of the valence wave function; the intrinsic heavy quark probability is thus presumably larger in baryons than in mesons, nonadditive in nucleon number in heavy nuclei, and sensitive to the presence of strange quarks. In deep-inelastic scattering on an intrinsic charm quark the heavy quark spectator will be found predominately in the target fragmentation region.

The intrinsic charm structure function will not become fully observable unless the available energy is well above threshold: $W = (q+p)^2 \gg W_{th}^2 = 4m_Q^2$. The correct rescaling variable for deep inelastic muon scattering is roughly $x = x_{B_j} + W_{th}^2/W^2$, not $x = x_{B_j} + m_Q^2/Q^2$ which is appropriate to charge-current single heavy quark excitation.

The presence of a hard-valence–like charm distribution in the nucleon can, at least qualitatively, explain some of the anomalous features of the charm hadroproduction data discussed above. The fact that the c and $\bar{c}$ as well as D and $\overline{D}$ distributions are harder than the corresponding strange particle distributions can be attributed to the fact that the skewing of quark distributions to large x only really becomes effective for quarks heavier than the average momentum scale in the nucleon. One can account for leading particle effects and the fairly flat Λ_c ISR and Serpurkhov cross sections if there is coalescence of the intrinsic charm quarks with the u and d spectator quarks of the nucleon. We note that recombination itself cannot explain the comparable distributions observed in the LEBC experiment for proton production of D and $\overline{D}$, unless it is the heavy quarks that carry most of the momentum. Since the intrinsic contribution is associated with higher twist operators it is suppressed by a factor of $1/m_Q^2$ relative to

~~the~~ fusion contributions, and is thus unlikely to be very important for b or t–quark hadroproduction.

The presence of intrinsic charm quarks in the nucleon also has implications for other hard scattering processes involving incident charmed quarks. In general, the charm quark in the nucleon will reflect both extrinsic and intrinsic $(1/m_c^2)$ contributions. Using QCD factorization this implies significant intrinsic charm contributions to hard scattering processes such as $c + g \to c + X$ at $p_T^2 \gg 4m_c^2$, with the intrinsic contribution dominating the large x domain. The characteristic signal for such contributions is a $\bar{c}$ spectator jet in the beam fragmentation region. Similarly, heavier quarks and supersymmetric particles of mass $\tilde{m}$ contribute to intrinsic Fock states[25] in the nucleon at order $1/\tilde{m}^2$. The intrinsic $\tilde{q}(x)$ or $\tilde{g}(x)$ distribution is again predicted to maximize at large x. Hard scattering processes such as $\tilde{q}+\bar{q} \to \tilde{\gamma}+\gamma$ can produce purely electromagnetic monojet events. Note that the associated intrinsic supersymmetric partner appears in the beam fragmentation region.

5. DIFFRACTIVE HARD PROCESSES.

The production of heavy quark systems may occur diffractively in hadron collisions, that is without the excitation of the target. Two pictures have been given for this process:

(1) Diffractive excitation.[22] When a beam hadron fluctuates into a Fock state such that all of its constituents are at small relative impact parameter, it interacts minimally because of its small color dipole moment. Since the normal states interact strongly, the small impact valence Fock state materialize as $q\bar{q}$ or qqq jets. (See Fig. 10). In the case of intrinsic heavy quark Fock states $qqqQ\bar{Q}$ with small transverse size, the incoming nucleon can be diffractively excited into a forward produced system containing a heavy quark pair. An analysis of such processes based on the Good and Walker two-component formalism is given in Ref. 22

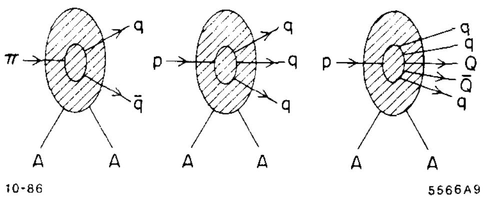

Figure 10. Schematic representation of diffractive excitation in a nuclear target: A high energy hadron can be diffracted into valence quark jets or heavy quark Fock states.

(2) The Pomeron as a gluon source.[23] If one treats the Pomeron as a composite system with gluon constituents, then the gluon-gluon fusion process leads to diffractively-produced heavy quark systems. (See Fig. 11.) The analysis of such processes is given in Ref. 23.

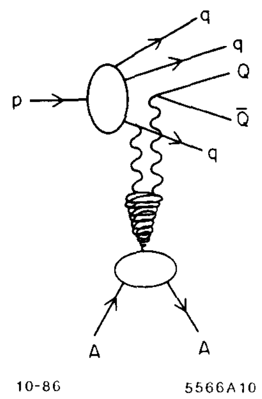

Figure 11. Schematic representation of diffractive excitation via the gluonic component of the Pomeron.

Both pictures of diffractive production lead to similar final states and cross section estimates. In particular the total production rate has a predicted nominal nuclear number dependence $\sigma \sim A^{2/3}$. However, the x_F distribution of the heavy quarks system tends to be harder and the mass of the diffractive system smaller in the intrinsic charm picture.[24]

6. ANOTHER EXPLANATION OF THE LEADING PARTICLE EFFECT

In a contribution to this conference Ganguli and Shankar[25] have proposed a simple explanation for the leading particle effect in $\pi N \to DX$ collisions based on the direct fusion of a light valence quark from the beam hadron with a heavy $\bar{c}$ quark from the target into the produced D meson. However, it should be emphasized that the usual evolution of the target structure function does not become effective until the subprocess scale is large compared to $4m_c^2$. Thus in the GS scheme, the charmed quarks in the hadron have to be regarded as part of the intrinsic heavy quark Fock state. The model fails to explain why no leading particle effect is observed in $\pi N \to D^* X$ or $pp \to DX$.

7. HEAVY QUARK PRODUCTION IN EXCLUSIVE $p\bar{p}$ REACTIONS

It is an interesting question how to extrapolate from strange baryon pair production to charmed baryon production in *exclusive* $p\bar{p}$ annihilation. If only the subprocess $u\bar{u} \to c\bar{c}$ were important (see Fig. 12) then the cross section would scale simply as[26] $d\sigma/d\Omega \sim \alpha_s^2(4m_c^2)/4m_c^2$. However the recombination of the $c\bar{c}$ quarks with the spectator quarks of the p and $\bar{p}$ requires alignment of the heavy quarks: $p_T^2 \le \mu^2$ where μ^2 is the characteristic momentum scale of light quarks in hadrons.

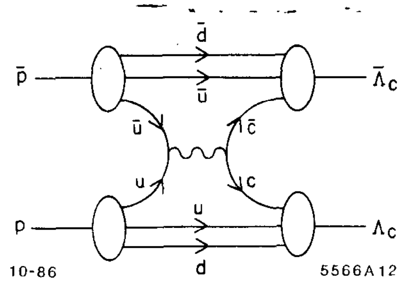

Figure 12. Representation of exclusive charm baryon pair production in $p\bar{p}$ annihilation.

Thus we predict the scaling $\sigma(p\bar{p} \to \Lambda_Q \bar{\Lambda}_Q) \sim \mu^2 \alpha_s^2(4m_Q^2) F(4m_Q^2/s)/m_Q^4$. This implies

$$\Lambda_s \bar{\Lambda}_s : \Lambda_c \Lambda_c : \Lambda_b \bar{\Lambda}_b = 1 : 10^{-2} : 10^{-4}$$

at fixed m_Q^2/s. Thus exclusive pair production of charm hadrons may have significant cross sections not too close to threshold.

The exclusive production of exclusive charmed pairs in e^+e^- annihilation is computed in Ref. 27, where the phenomena of form factor zeroes is discussed. The possibility that $Z^0 \to t\bar{t}$ may be dominated by exclusive channels is discussed in Ref. 4. The cross section for heavy meson pair production in $\gamma\gamma$ collisions at threshold is estimated in Ref. 28.

8. SUMMARY

Although there is little doubt that the leading order perturbative QCD predictions are accurate for very massive heavy quark production, there are interesting and important corrections both at low transverse momentum and in the beam or target fragmentation regions. These are the kinematic regions where intrinsic contributions or coherent effects such as prebinding distortion can occur as the produced quark and spectator fragments coalesce. As reviewed here, the data appear to have anomalies in these regions. It is clearly very important to verify these effects, particularly leading particle effects, enhancements due to hyperon beams, the A-dependence, the importance of diffractive production, and leading particle effects. From the theoretical perspective, the charm production data provide a window to the interface of perturbative and nonperturbative dynamics.

The total inclusive cross section for $pp \to D\bar{D}$ production measured by the LEBC-MPS[14] collaboration at 800 GeV/c ($\sqrt{s}$ = 38.8 GeV) is $59^{+22}_{-15}\mu$b (statistical errors only). This is roughly a factor of 5 below that measured at the ISR at $\sqrt{s}$ = 52.5 GeV and 63 GeV. It will clearly be very important to verify the ISR data, including the production rate and properties for the Λ_c. The energy dependence and diffractive properties are particularly important for extrapolation to new colliders and interpretation of cosmic ray data.

The nominal scaling behavior for the total heavy quark production cross section based on the $gg \to Q\overline{Q}$ process is

$$\sigma_{Q\overline{Q}}(\sqrt{s}) = \frac{m_c^2}{m_Q^2}\,\sigma_{c\bar{c}}\left(\sqrt{s'} = \frac{m_c}{m_Q}\sqrt{s}\right).$$

Prebinding distortion is not expected to significantly modify this behavior. Assuming that most of the observed charm cross section is nonintrinsic, we may use this scaling to extrapolate to the hadroproduction of heavier quarks. For production well above threshold ($s \gg 4m_Q^2$), the ~ 0.5 mb charm cross section reported at the ISR implies $\sigma_{b\bar{b}} \sim 50\ \mu$b at the SP$\overline{\text{P}}$S collider ($\sqrt{s}$ = 540 GeV) and $\sigma_{t\bar{t}} \sim 1/2\ \mu$b for m_t = 40 GeV at the SSC ($\sqrt{s}$ = 40 TeV).

ACKNOWLEDGMENTS

The analysis of the limits of validity of factorization for heavy quark hadroproduction is based on collaborations with J. F. Gunion and D. E. Soper. I also wish to thank G. Bodwin, J. C. Collins, G. P. Lepage, and A. Mueller for helpful conversations.

REFERENCES

1. J. C. Collins, D. E. Soper and G. Sterman, Nucl. Phys. **B263**, 37 (1986).

2. T. K. Gaisser and F. Halzen, MAD/PH/287 (May 1986).

3. R. K. Ellis and J. C. Sexton, FERMILAB-PUB-86/111-T (Aug 1986), R. K. Ellis, FERMILAB-CONF-86/35-T (April 1986). Analogous QED/QCD asymmetries are discussed in S. J. Brodsky, C. E. Carlson, and R. Suaya Phys. Rev. **D14**, 2264 (1976).

4. I. Y. Bigi, *et al.*, SLAC-PUB-4021 (Jul 1986) Submitted to Phys. Lett. **B**.

5. G. T. Bodwin, Phys. Rev. **D31**, 2616 (1985). J. C. Collins, D. E. Soper and G. Sterman, Phys. Lett. **134B**, 263 (1984). C. T. Sachrajda, *Proc. of the Workshop on Nuclear Chromodynamics, Santa Barbara, CA, Aug 12-23, 1985*.

6. S. J. Brodsky, D. E. Soper, and J.F. Gunion (in preparation).

7. H. Bethe and L. Maximon, Phys. Rev. **93**, 768 (1954); H. Davies, *et al.*, *ibid.*, 788 (1954).

8. A. H. Mueller and P. Nason, Nucl. Phys. **B266**, 265 (1986). See also R. M. Godbole and D. P. Roy, TIFR/TH/86-27, (Sep 1986).

9. S. J. Brodsky, G. T. Bodwin and G. Peter Lepage, Volendam Multi. Dyn. 1982:841, Fermilab Drell-Yan 1982: 105.

10. M. E. Duffy, *et al.*, Phys. Rev. Lett. **55**, 1816 (1985). A. Bodek *et al.*, Leipzig High Energy 1984 v.1:157.

11. For reviews, see A. Kernan and G. Van Dalen, Phys. Reports **106**, No. 6 (1984); S. Reucroft, *Proc. of the 21st Rencontre de Moriond: Strong Interactions and Gauge Theories, Les Arcs, France (Mar,1986);* and S. J. Brodsky , *Proc. of the 16th Int. Symp. on Multiparticle Dynamics, Kiryat-Anavim, Israel, June, 1985.*

12. A. N. Aleev, *et al.*, JINR-E1-82-759 (Nov 1982).

13. H. Becker, *et al.*, *Cont. the XXIII Int. Conf. Energy Physics, Berkeley, 1986.* M. Bosman, to be published in the proceedings of the 1986 SLAC Summer Institute.

14. LEBC-EHS Collaboration (M. Aguilar-Benitez, *et al.*), Z. Phys. **C31**, 491 (1986); Phys. Lett. **135B** 237 (1984). The LEPC-MPS collaboration data is given in R. Ammar, *et al.*, CERN-EP/86-122 (Sep 1986) Submitted to Phys. Lett. **B**.

15. S. F. Biagi, *et al.*, Z. Phys. **C28**, 175 (1985).

16. S. J. Brodsky, J. F. Gunion, Published in SLAC Summer Inst. 1984:603.

17. M. Arneodo, *et al.*, CERN-EP/86-88 (Jul 1986) Submitted to Z. Phys. **C**. J. J. Aubert, *et al.*, Phys. Lett. **110B** 73 (1982).

18. Erhard Hoffmann and Richard Moore, Z. Phys. **C20**, 71 (1983).

19. S. J. Brodsky, C. Peterson and N. Sakai, Phys. Rev. **D23**, 2745 (1981). S.J. Brodsky *et al.*, Phys. Lett. **93B** 451 (1980).

20. T. Eller, H.-C. Pauli and S. J. Brodsky, SLAC-PUB-3835 (Sep 1986), and references therein.

21. S. J. Brodsky, H. E. Haber and J.F. Gunion, SSC/DPF Workshop 1984:100.

22. G. Bertsch, *et al.*, Phys. Rev. Lett. **47**, 297 (1981).

23. E. L. Berger, *et al.*, ANL-HEP-CP-86-83 (Jul 1986), G. Ingelman and P. E. Schlein, Phys. Lett. **152B**, 256 (1985), and references therein.

24. I wish to thank D. Potter for discussions on experimental tests of heavy quark diffractive excitation.

25. S. N. Ganguli and K. Shankar, *Cont. to the XXIII Int. Conf. Energy Physics, Berkeley,1986.*

26. S. J. Brodsky, SLAC-PUB-3770 (Sep 1985), published in *Proc. of the FNAL Workshop on Low Energy $p\bar{p}$ Collisions (1986).*

27. S. J. Brodsky and C.-R. Ji, Phys. Rev. Lett. **55**, 2257 (1985).

28. S. J. Brodsky, G. Kopp, P. M. Zerwas (in preparation).