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Beam Optical Design and Studies of the SLC Arcs*

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Abstract

The first- and second-order optics for the Stanford Linear Collider (SLC) Arcs are designed to preserve the small transverse beam emittance while transporting electrons and positrons of ±0.5% energy spread from the linac to the Interaction Point. This paper describes the repetitive and special sections lattices, the orbit correction scheme to compensate field and alignment errors, and the results of the studies of the Arc optical properties. The effects on the beam of quantum emission, chromatic distortions and higher-order aberrations are discussed. Utilization of various computer programs for Arcs studies is described briefly. Relevance of this work to future linear colliders is pointed out.

Introduction

The technology of electron and positron storage rings, which has been successfully developed from table size devices with a low energy range of 2×100 MeV up to LEP, the 27 km circumference machine at CERN with the energy 2×70 GeV, may have reached its practical limit. At this time, a further increase in both energy and luminosity in such rings appears to be technically and economically unfeasible. Progress in high energy electron beam collision techniques is now being pursued by the development of a new technology, namely, linear colliders.[1]

This paper is being written during the initial commissioning of the world's first linear collider – the SLAC Linear Collider (SLC). The status and development of this machine is presented at this Conference by G. Fischer.[2] Here we describe the design and the results of theoretical studies of the SLC Arcs. The South (positron) and the North (electron) transport lines (Arcs) make possible the collisions of electron and positron bunches accelerated by a single linac.

Preserving the small emittance needed to achieve the design luminosity demands a careful design and assembly of the Arcs. A special group, the Beam Dynamics Task Force (BDTF), was assigned to specify this design and establish the tolerances for assembly errors. This paper is a condensed version of a full report to be published later describing the work of the BDTF.[3]

The results obtained, the mathematical tools developed, and the conclusions derived during the Arcs studies have potential applications to other projects. For example, they may be relevant to the future development of any wide band beam transport line for dense small cross section beams and to future linear colliders where curved transport lines may be used to provide multiple collision points.

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Optical Design of the Arcs

This design must minimize the synchrotron radiation emittance growth due to ***quantum fluctuations*** and minimize the apparent emittance growth due to ***chromatic and geometric aberrations*** of the lattice over a wide momentum band pass. The Arcs design should also have the flexibility to follow the site terrain and include ***Special sections*** for matching purposes. These considerations led to the application of the principle of the second-order achromat and the choice of short FODO cells made up of combined function magnets with a large packing factor.

Quantum Fluctuations Limiting the increase in the beam emittance due to quantum fluctuations requires the maximum possible bending radius. The available SLAC site allows a maximum Arc radius of approximately 280 m.

The magnitude of the energy loss to synchrotron radiation can be derived as:

$$U = E_o[1 - (1 + 3\alpha\phi)^{-1/3}] \qquad (1)$$

where E_o is the particle energy at the beginning of the beam line, ϕ the total deflection angle of the beam line and $\alpha = 8.85 \times 10^{-5}\ E_o^3\ (\mathrm{GeV}^3)/(2\pi\rho(m))$, where $\rho(m)$ is the bending radius. At 50 GeV the total energy loss of a particle is 1.2 GeV or 2.4%. Since the average energy of the emitted photons is only 300 keV, each particle emits an average of 4000 photons while traversing the 460 magnets of the Arc.

To minimize the quantum excitation of the beam emittance, a FODO lattice has been chosen which utilizes combined function magnets.[4]

For a FODO lattice the increase of the emittance for a beam line with a total deflection angle ϕ can be expressed as:[5]

$$\Delta\epsilon(\text{rad-m}) = 2.1 \times 10^{-11}\ \phi\ E_o^5\ \frac{1}{\rho}\ \langle \mathcal{H}/\rho \rangle\ \frac{\ell_c}{2\ell_b}\ , \qquad (2)$$

where ℓ_c is the cell length and ℓ_b is the magnet length. The ratio $\ell_c/2\ell_b$ is the reciprocal of the FODO cell packing factor and is chosen to be near the minimum value of one.

The lattice dependent quantity $\langle \mathcal{H}/\rho \rangle = \langle (\gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2)/\rho \rangle$ reaches a minimum[6] for a betatron phase advance of about 135° and rises steeply for greater values. To avoid this steep rise and for considerations discussed below the Arc FODO cell phase shift is chosen to be 108° for which:

$$\langle \mathcal{H}/\rho \rangle \approx 1.65 \left(\frac{\ell_c}{2\rho}\right)^3 \qquad (3)$$

To keep the emittance growth small, short cells should be chosen. There is, however, a practical limit. The dipole field is determined by the design energy and the bending radius. The maximum gradient for this field is in turn determined by the

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required aperture of about 8 mm. With these restrictions the half cell length must be at least 2.5 m. In Fig. 1 the increase in the beam emittance as a function of the beam energy is shown for different values of the magnetic field gradient. A gradient of at least 7.0 kGauss/cm must be chosen if the emittance growth is not to exceed 50% of the initial emittance.

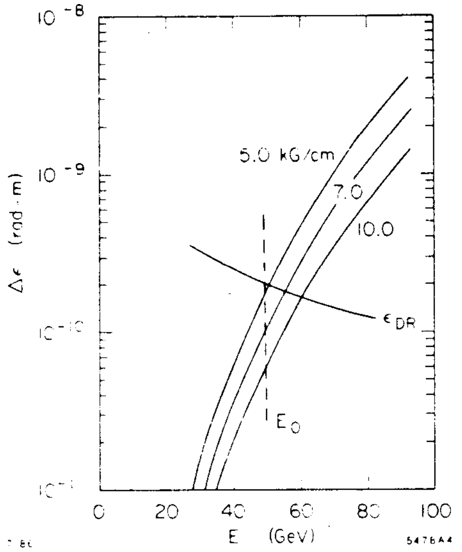

Fig. 1. Beam emittance growth as a function of the beam energy for the gradient values 5.0, 7.0 and 10.0 kGauss/cm. Also is shown the curve for ideal damping ring emittance scaled to the linac energy. The dashed line represents the SLC energy design value.

For the collider Arcs with $\phi \approx 230°$, the calculated growth in the beam emittance from Eq. (2) is $\Delta\epsilon = 1.2 \times 10^{-10}$ m-rad and compares well with the result from computer simulations of $\Delta\epsilon = 1.3 \times 10^{-10}$ m-rad which is 37% of the damping ring equilibrium emittance scaled to 50 GeV. Because of strong energy dependence a fast rising emittance growth is expected at higher energies. Most of the emittance growth in the collider Arcs occurs in the horizontal plane. To a much smaller extent (30%) the emittance is also increased in the vertical plane due to coupling. The effect of radiation antidamping on the emittance is small.[8]

The change in the beam energy spread due to all radiation effects is much smaller than the expected energy spread of the beam from the linac.

Optical Aberrations Using the principles that define a second-order achromat[7] in the optical design of the Arc lattice ensures that in the Arcs both geometric and chromatic aberrations are eliminated up to second order for a perfectly constructed machine. To make use of these principles the Arc cells are grouped in blocks of odd multiples of 180° in betatron phase. For the betatron phase advance in both planes of 108° per cell, ten such cells form a second-order achromat. To increase the energy acceptance of the lattice a sextupole term is included in the magnetic field by shaping the pole profile. The cross section of these magnets can be found in the paper presented by G. Fischer[2] at this Conference.

Since the transfer matrix for an achromat is the identity matrix, input matching conditions are preserved at the output independently of rotation about the input beam axis. This allows deflecting the beam in the vertical plane to follow the terrain without the need for special matching sections for this purpose. Figure 2 presents vertical profiles of the SLC Arcs. In Table 1, the design parameters for the Arc lattice are compiled.

The North and the South SLC Arcs are composed of 23 and 22 achromats, respectively. The field of each achromat is matched to the decreasing average beam energy by special trim windings. The short spaces between magnets accommodate beam position monitors, vacuum flanges and bellows.

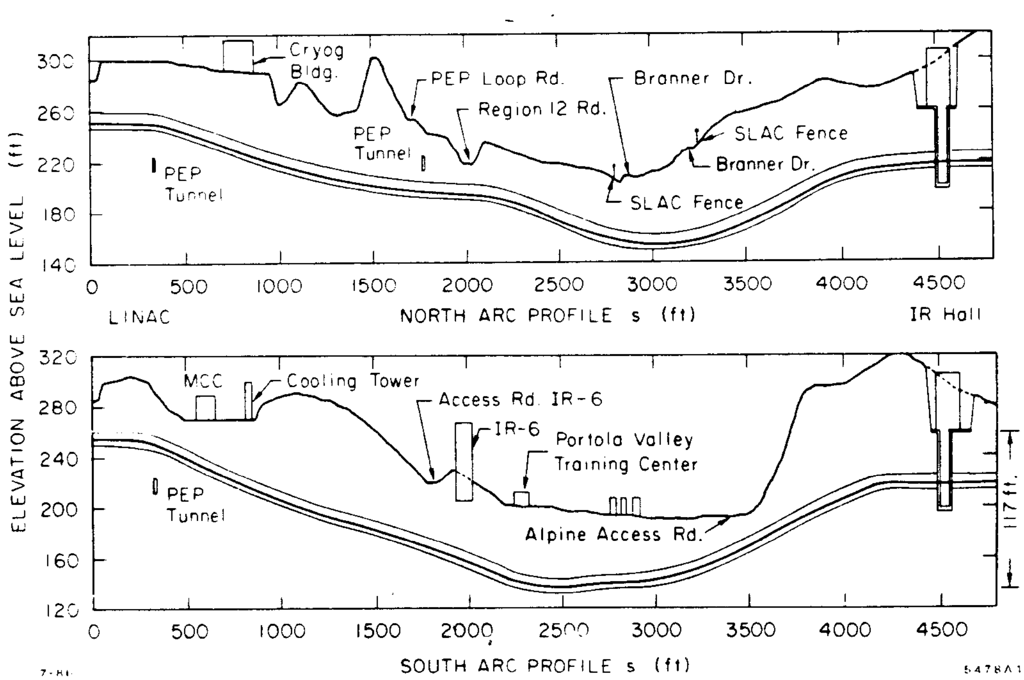

Fig. 2. Vertical profile of the SLC Arcs.

Table 1

Beam Energy	50 GeV
Repetition rate	180 Hz
Number of particle/pulse	5×10^{10}
Emittance	3×10^{-10} m-rad
Bending Radius	279.378 m
Lattice	Combined Function FODO
Cell Length	5.192402 m
Max. value of $\beta_{x,y}$	8.70 m
Min. value of $\beta_{x,y}$	1.14 m
Max. value of η_x	0.047 m
Magnet Length	2.502999 m
Magnet Field	5.96976 kGauss
Magnet Field Gradient	7.01390 kGauss/cm
Second field derivative $\delta^2 B/\delta x^2$	
Focusing magnet	1.62 kGauss/cm^2
Defocusing magnet	-2.71 kGauss/cm^2

Special Sections The special sections included in the optical design of the Arcs are the Linac to Arc Section (LAS), the Reverse Bend (RB) and the Instrument Section (IS). In the LAS the paths of the positrons and electrons emerging from the linac are separated and the envelope of each beam is matched to the corresponding Arc. The RB section is inserted in the lattice of each Arc to invert the sign of the matched dispersion function η at the inflection point. In addition the IS is also inserted in the South Arc to provide needed geometric flexibility and a location for diagnostic instrumentation. The transfer matrix of the RB is the minus identity matrix so the beam envelopes remain matched. Figure 3 presents the planar layout of the SLC Arcs.

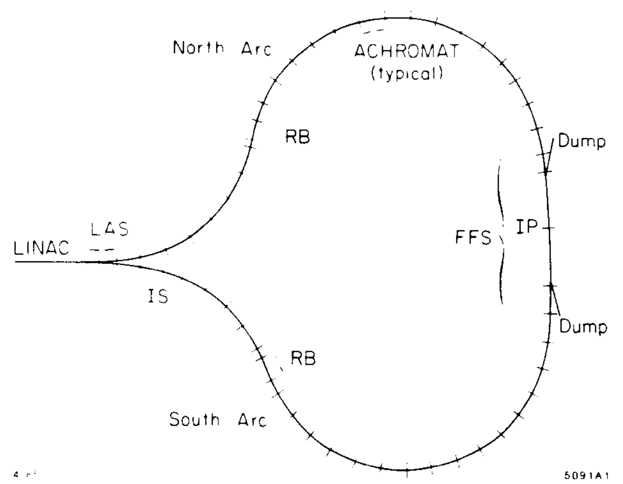

Fig. 3. Layout of Arcs in the horizontal plane.

Orbit Correction

Orbit correction in the SLC Arcs is performed in a local manner, cell by cell along the Arc.

Corrective fields may be introduced either by backleg windings (BLW) or by physically moving the gradient magnets (MM). MM correctors were chosen for the SLC because the BLW significantly changed the dispersion functions of the Arc. Moreover the BLW, which steer in the vertical plane, introduce severe skew quadrupole and sextupole fields.

The adopted MM correction system is nondispersive.[9] *In a system of η-matched second-order achromats, using combined function magnets, the change of the downstream orbit is independent of energy for the linear approximation in the transverse magnet movement.* The η function also does not change in this approximation.

Many different correction patterns were simulated and their convergence properties were studied by the BDTF. The compromise between the increase in the cost and the desired level of correction was achieved by the scheme finally chosen for the Arc orbit correction.[10] In this scheme each focusing magnet is moved horizontally (vertically) to steer the beam through the monitor attached to the next focusing (defocusing) magnet. This scheme utilizes 10 correctors and 10 monitors in the horizontal plane and 10 correctors and 10 monitors in the vertical plane per achromat.

Typical uncorrected orbits deviate from the reference orbit anywhere from 10 to 70 mm which is far outside the vacuum chamber. Maximum excursion of the corrected orbit is of the order of 1 mm. Maximum displacement of the magnets required to correct the orbit is typically 0.5 mm. The rms value of the displacement of a corrected orbit is in the range of 150-200 μm. Subsequent analytical calculations[11] agree with that range.

The resulting orbit correction can also be illustrated by comparing harmonic content of the orbits before and after correction.[12] For the resonance betatron harmonics the reduction factor due to orbit correction is of the order of 100.

Analysis of Linear and Nonlinear Effects

The unavoidable necessity of special sections, which could not be designed to be completely achromatic to second-order, meant that even in a perfectly constructed machine second-order aberrations are present (though minimized by design). Furthermore, the presence of alignment and field errors for the large number of Arc elements causes first-order and subsequent higher-order deviations from this design.

The problem of analyzing the phase space distortions and their projections onto the plane normal to the beam direction, i.e., effective transverse emittance growth is a nonlinear problem for which good analytical methods do not exist. Finding dynamic apertures in circular machines (for large amplitudes) and examination of the transverse emittance (for small amplitudes) in the presence of nonlinear terms are similar problems. Consequently, the methods of solving them (namely, numerical tracking) are also similar.

Computer Programs Used to Study the Optics of the SLC Arcs In general, calculations were completed at least to the third order, and comparisons were made at the interaction point (IP) where the first-order beam size is the smallest to allow easy observation of higher order aberrations. As a cross-check it was required that results obtained by using one code be corroborated by the results obtained by another.

The computer program TRANSPORT[13] was used for the basic design of the Arcs optics modules. TRANSPORT is a matrix multiplication program which is now correct to third order. Its primary usefulness stems from its powerful fitting routines and the speed of execution.

Three tracking codes were used to simulate the optics for the Arcs: TURTLE, DIMAD and MURTLE.

TURTLE[14] is a ray tracing program which uses TRANSPORT matrices to represent optical elements. However, instead of calculating the accumulated transfer matrix, as is done by TRANSPORT, TURTLE determines the effect of each element on each ray serially. The matrices for quadrupoles and sextupoles are calculated to second order for the exact momentum of each ray. For combined function magnets the coordinates of an outgoing ray are expressed as a Taylor expansion in terms of the incoming ray coordinates. TURTLE is incapable of correcting orbits or optical effects in the presence of perturbations.

DIMAD[15] also uses TRANSPORT matrix elements and like TURTLE has ray tracing capabilities with the difference being that each element is represented as an expansion correct to second order. In addition many diagnostic aids are available. Arbitrary central rays can be chosen about which linear transfer and sigma matrices can be found. The Twiss parameters can then be expressed as expansions in terms of energy, emittance or initial coordinates. For a perturbed machine DIMAD can correct the orbit and first-order optics.

Important for the Arcs the third- and fourth-order cross-coupling aberrations coming from sextupoles are taken into account by both TURTLE and DIMAD.

MURTLE[16] integrates the equations of motion through an optical system for the 56 rays necessary to determine the transfer matrices up to third order. These matrices are applied to an arbitrary number (usually 10^4) randomly selected input rays. In addition MURTLE calculates the contribution to the final beam size (expressed as a fraction of the first-order beam size of the unperturbed machine) of each higher order term. MURTLE can correct the orbit and first-order optics of a perturbed machine. It is rather fast requiring approximately 1 minute IBM 3081 CPU time to emulate the entire perturbed and corrected SLC Arc and the Final Focus System (FFS). The 3rd order matrices found by MURTLE and by TRANSPORT agree.

The above programs were also used to simulate emittance growth due to quantum fluctuations.

Simulations and Criteria Several figures of merit were used at various times to measure the higher order effects: *a*) the rms transverse sizes of the beam $\sigma_{x,y}$, *b*) the effective transverse emittances $\epsilon_{x,y} = \sigma^2_{x,y}/\beta_{x,y}$, where $\beta_{x,y}$ is the first-order machine amplitude functions and *c*) calculated luminosity, which is defined by $\mathcal{L} = N\ \Sigma\ n_i^2$. Here n_i represents the number of particles in each bin of a scatter plot, and N is a normalizing constant calculated for the assumed beam parameters (number of particles per bunch and frequency of bunches) and the program parameters (bin size, etc.). This calculation implies perfect register of two identical beams.

In the presence of perturbations, the central trajectory and the first-order optics of the overall system deviate significantly from the design. An algorithm to correct the orbit is first applied. Then first-order optical deviations are removed using nine correcting elements placed in each FFS. These optical corrections are almost orthogonal. Four adjustments remove all dispersion in the drift region near the IP, four adjust $\beta_{x,y}$ and $\alpha_{x,y}$ at the IP and a skew quadrupole is used to remove cross plane coupling. After the first-order corrections are applied, the effects upon the beam are examined by monitoring the 'Figure of Merit'.

Loosely defined criteria for establishing acceptable tolerances for construction errors were the following:

- The residual second-order aberrations should not produce a larger effect on the beam than the effect due to the aberrations inherent in the initial design of an unperturbed machine.
- The remaining higher-order aberrations must not produce a larger effect on the beam then do the residual second-order terms.

Application of these criteria had to be tempered by consideration of practicable control of errors.

Statistical studies involving the simulation of many machines with randomly distributed errors were completed for each type of expected error either with such an error alone or with that error in conjunction with all other errors. This method resulted in error tolerance specifications that were consistent in that each specified error contributed approximately the same magnitude of effect.

Figure 4 can be used to illustrate several points. The data in this figure were calculated using the program MURTLE for 10^4 input rays. The distribution for these rays was gaussian with the exception that the energy spread is truncated at one sigma to closely simulate the expected distribution for the SLC. Energy loss due to synchrotron radiation was included but emittance growth due to quantum emissions was not. The Arc magnets were divided longitudinally into two halves, and these halves were independently misaligned according to standard values given in Table 2. The first half of each magnet was allowed to translate in either the horizontal or vertical plane by the orbit correction algorithm. First order optical corrections were applied that resulted in a round beam ($\sigma_x = \sigma_y$) of a specified size at the IP. The figure of merit is the luminosity calculated for a beam with parameters from Table 1.

Table 2

Transverse alignment (magnets and quads)	100 μm rms
Axial position error (magnets and quads)	1000 μm rms
Roll (magnets and quadrupoles)	1 milliradian rms
Field excitation ΔB	1 part per 10^3 rms
Displacement of Beam Position Monitors	100 μm rms
Reproducibility of MM corrector position	15 μm rms

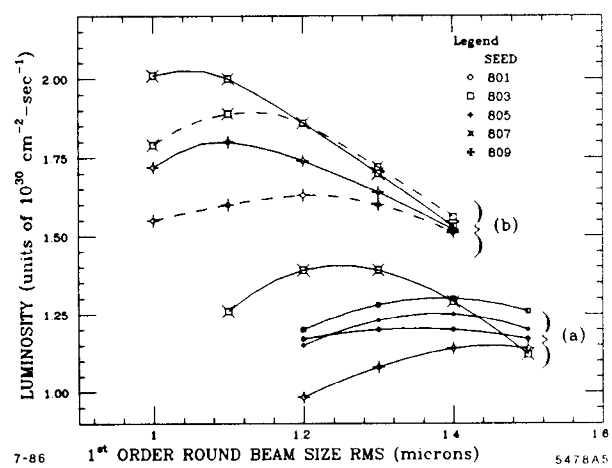

Fig. 4. Luminosity versus first order beam size: (a) for a momentum spread of $\delta = 5 \times 10^{-3}$; and (b) for a momentum spread of $\delta = 2 \times 10^{-3}$.

Figure 4(a) demonstrates the behavior of five different simulated 'machines' (i.e. sets of random numbers defining misalignments and errors in elements of the lattice) as a function of the first-order beam size at the IP. The design value for the beam size is 1.5 μm representing a $\beta^* = 0.75$ cm, a value established for using conventional quadrupoles in the Final Focus System (these parameters will be different for superconducting quadrupoles). The beam energy has a Gaussian distribution truncated at one σ where $\sigma = \pm 0.5\%$. The set generated by the initializing number ('seed') 807 is predominantly linear down to the beam radius ≈ 1.3 μm, where nonlinear aberrations start to deplete the luminosity. On the other extreme the set labeled by the seed 805 has large nonlinear contributions already for the beam radius ≈ 1.5 μm. Figure 4(b) presents similar data for these two seeds for the smaller $\sigma = \pm 0.2\%$ (dashed curves). Here the similarity of these two machines improved due to the suppression of the chromatic effects. Their behavior becomes even more similar (solid lines) when for each first-order beam size the FFS sextupoles are adjusted to match the values which minimize the two largest chromatic matrix elements of the unperturbed system.

Conclusions

The acceptable random errors and tolerances that have been established by the BDTF are summarized in Table 2. The system (Arcs and FFS) is correctable by the trim magnets in the FFS region if the rms value of the alignment errors for the Arcs is in the range of 100 μm or less. Under these conditions the luminosity of the machine can reach values acceptable for particle experiments.

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