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Perturbative QCD Analysis of Exclusive Pair Production of Higher Generation Nucleons*

CHUENG-RYONG JI[(a)]

Department of Physics, Stanford University
and Stanford Linear Accelerator Center, Stanford, CA 94305

and

ALAN F. SILL[(b)]

Department of Physics, The American University, Washington, D.C. 20016
and Stanford Linear Accelerator Center, Stanford, CA 94305

Abstract

Results are presented of a perturbative QCD analysis of heavy baryon form factors to leading order in α_s including all higher twist quark mass effects. A dip in the total cross section for the e^+e^- pair production of some higher generation nucleons is predicted. The results are consistent with the definition of charge form factors at zero momentum transfer and reproduce the well known anomalous magnetic moments of nucleons. These considerations resolve a sign ambiguity in previous leading twist analyses of the hard scattering contribution to the proton magnetic form factor.

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(a) Present address: Department of Physics, Brooklyn College, Brooklyn, NY 11210

(b) Present address: Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627

Because of the asymptotic freedom property of quantum chromodynamics[1] (QCD), a perturbative analysis of strong interaction processes should be rigorous when the four-momentum transfer q is much larger than the QCD scale parameter Λ_{QCD} and the value of the strong coupling constant α_s becomes small. From various perturbative QCD analyses[2] it seems now confirmed that Λ_{QCD} is not large but of order 100 MeV. Thus, perturbative QCD should be applicable to experimental data for values of $Q^2 \equiv -q^2$ greater in magnitude than a few $(\mathrm{GeV}/c)^2$ or so. Detailed analyses for exclusive processes require knowledge of the valence-quark distribution amplitudes $\Phi_H(x_i, \widetilde{Q}^2)$ of the hadrons[3,4]. Unfortunately the distribution amplitudes are not yet well known[5–7] for particles made of light quarks such as ordinary pions and nucleons. On the other hand, the wave-functions for bound states of heavy quarks clearly must be determined by nonrelativistic considerations. In heavy particles higher twist effects proportional to the quark masses become important. Perturbative QCD predictions for heavy systems should thus provide reliable tests of the theory once higher twist quark mass effects are incorporated.

Some striking predictions have recently been obtained for the exclusive pair production of higher-generation mesons[8] by including higher twist effects in the leading order perturbative QCD calculations. Specifically, the existence of a zero in the form factor and e^+e^- annihilation cross section for zero-helicity meson pair production was demonstrated. This zero is due to destructive interference between the spin-dependent (transverse) and spin-independent (Coulomb exchange) couplings of the gluon in QCD. In that paper, it was also observed that the form factors of heavy baryons might have a similar behavior. The calculation of the baryon form factors is intrinsically more complex than the meson cases, so it has not yet been demonstrated whether such zeroes would be observable or under what conditions they would occur. In this letter, we analyze the Dirac and the Pauli form factors F_1 and F_2 for higher generation nucleons such as UUD and UDD (U=c,t and D=s,b). The results of this analysis can be directly applied to the prediction of heavy baryon pair production in e^+e^- and $\gamma\gamma$ interactions at present and future accelerators such as SLC, LEP and TRISTAN.

A remarkable feature of the higher twist effects in heavy baryon pair production processes is the prediction of a strong dip in the total cross section for certain particles in the physical region near threshold. Depending on the mass ratio between the quarks in the baryon, we find that the Pauli form factor F_2 is comparable in size with the Dirac form factor F_1, and destructive interference similar to that observed in the meson case[8] can occur. Such a dip cannot appear in the leading twist analysis. The angular dependence of the production rate also exhibits dramatic effects.

Another remarkable fact is that with a simple ansatz to account for the effects of binding energy in the hadrons, our formulae can be made consistent with the definition of the charge form factor at zero momentum transfer and reproduce the well known anomalous magnetic moments of nucleons. These considerations fix the sign of leading twist contributions to the hard scattering amplitude T_H. Therefore we can eliminate the sign ambiguity which exists in previous analyses[3,6,9] of the ordinary nucleon form factors.

Applying light-cone perturbation theory[3], we write the amplitude for exclusive processes in the factorized form $\int \Phi(x) T_H(x,y) \Phi(y) [dx][dy]$. The variables x_i represent the longitudinal momentum fractions of the quarks in the baryon in the light cone frame. The baryon wave function should be completely antisymmetric in the total color, spin, flavor, and orbit quantum space. Since the color singlet representation ϵ_{ijk} is completely antisymmetric and the flavors cannot mix under gluon exchange, we can use the following effective representation[6,10] for the nucleonic quark distribution amplitude:

$$\begin{aligned}\Phi_N^{\uparrow}(x_1,x_2,x_3,\tilde{Q}^2) = \frac{f_N}{8\sqrt{3}}\Big\{ & U_\uparrow U_\downarrow D_\uparrow \phi_N(x_1,x_2,x_3,\tilde{Q}^2) \\ & + U_\downarrow U_\uparrow D_\uparrow \phi_N(x_2,x_1,x_3,\tilde{Q}^2) \\ & - U_\uparrow U_\uparrow D_\downarrow \left[\phi_N(x_1,x_3,x_2,\tilde{Q}^2) + \phi_N(x_2,x_3,x_1,\tilde{Q}^2)\right]\Big\}\end{aligned} \tag{1}$$

for the proton-like state. U should be interchanged with D for the neutron-like state, with an overall change of sign. The dimensional constant f_N is set by the value of the nucleonic wave function at the origin. The normalization of ϕ_N is fixed by

$$\int [\mathrm{d}x]\phi_N(x,\tilde{Q}^2) = 1 \tag{2}$$

where $[\mathrm{d}x] = \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3\delta(1-\sum_i x_i)$. For heavy quark systems the ground state distribution amplitude is given by the nonrelativistic form[8]

$$\phi_N(x,Q^2) = \phi_{NR}(x) = \prod_{i-1}^{3} \delta(x_i - \frac{m_i}{M}) \tag{3}$$

where m_i is the mass of the i'th quark. $M = 2m_U + m_D$ for the proton-like baryons and $M = 2m_D + m_U$ for the neutron-like baryons. The dynamical dependence of the production is controlled by the hard scattering amplitude T_H which can be expanded perturbatively in terms of $\alpha_s(Q^2)$. In leading order in α_s, two gluons are required to connect the three quarks.

The hadronic vertex of the electromagnetic interaction for the spin 1/2 baryons is determined by two form factors:

$$\Gamma^\mu(Q^2) = \gamma^\mu F_1(Q^2) + \frac{[\not{q},\gamma^\mu]}{4M} F_2(Q^2) \quad . \tag{4}$$

We have calculated F_1 and F_2 to leading order in α_s including all higher twist terms proportional to the quark masses. These higher twist terms come from helicity non-conservation at the individual quark-gluon vertices due to the transverse coupling of the gluon. In heavy quark systems, other higher twist terms such as intrinsic transverse momentum effects should be negligible. In the convenient light-cone reference frame given by $q^+ \equiv q^0 + q^3 = 0$, the form factors F_1 and F_2 correspond in the timelike region ($q^2 > 0$) to total helicity 0 and ± 1 in the final state. In the spacelike region ($q^2 < 0$), F_1 and F_2 correspond to total helicity conserving and total helicity flip interactions, respectively. The constants in Eqn. 4 have been chosen to yield the conventional definitions for the Sachs form factors[11] G_E and G_M such that

$$G_M = F_1 + F_2 \qquad \text{and} \qquad G_E = F_1 - \tau F_2 \tag{5}$$

where $\tau = Q^2/(4M^2) = -q^2/(4M^2)$. The cross section for production of nucleon-like pairs in e^+e^- annihilation is given by

$$\frac{d\sigma}{d\Omega}(e^+e^- \to N\bar{N}) = \frac{\alpha^2}{16M^2\tau^2}\sqrt{1+\frac{1}{\tau}}\left\{G_E^2 \sin^2\theta^* - \tau G_M^2[1+\cos^2\theta^*]\right\} \tag{6}$$

where α is the fine structure constant and θ^* is the c.m. angle for produced baryon pairs. Our results F_1 and F_2 for the proton-like case are summarized[12,13] by the following:

$$\begin{aligned} F_{1,2}(\tau) = \frac{8\pi^2 f_N^2 \alpha_s^2}{27\tau^4 M^4}\Bigg\{ & \frac{e_U}{r^2(1-r)^2(1+r)^4}[A_{1,2}(r)\tau^2 + B_{1,2}(r)\tau + C_{1,2}(r)] \\ & + \frac{e_D}{(1-r)^6(1+r)^2}[D_{1,2}(r)\tau^2 + E_{1,2}(r)\tau + H_{1,2}(r)]\Bigg\} \end{aligned} \tag{7}$$

where $r = m_D/M$ and e_U and e_D are the quark charges. For the neutron-like case, e_U and e_D should be exchanged and r becomes m_U/M. The expressions $A_i, ..., H_i$ are functions of r and are given in Table 1. The leading twist contributions A_1 and D_1 to the Dirac form factor F_1 are in agreement with the results of previous analyses[3,6,9] with the overall sign as given by Ref. 6. In the case of the Pauli form factor, $A_2 = D_2 = 0$ since there is no leading twist contribution. Furthermore Eqn. 7 retains the general property of heavy-hadron pair production noted in Ref. 8 that the dominant contribution to the production amplitude is given by diagrams in which the coupling of the virtual photon is to the heavier quark pair. This can be easily seen by inspection of denominators in Eqn. 7. At low momentum transfer, $F_1^{P,N}(\tau \to 0)$ can be made consistent with the definitions of charge form factors in a fairly natural way. Specifically we found at $r = 1/3$,

$$\tau^4 F_1(\tau \to 0) \propto \begin{cases} (2e_U + e_D) &= 1 \quad (\text{proton}-\text{like case}) \\ (2e_D + e_U) &= 0 \quad (\text{neutron}-\text{like case}) \end{cases} \tag{8}$$

Motivated by the expected form of nonperturbative binding energy corrections[8,14] we replace τ^4 by $(\tau+\epsilon)^4$ and set $F_1(0)$ to the total charge of the system. Equation 7 for F_1 and F_2 then becomes

$$F_{1,2}(\tau) = \frac{1}{3}\left(\frac{8\epsilon^2}{27(\tau+\epsilon)^2}\right)^2 \left\{ \frac{e_U}{r^2(1-r)^2(1+r)^4}[A_{1,2}(r)\tau^2 + B_{1,2}(r)\tau + C_{1,2}(r)] + \frac{e_D}{(1-r)^6(1+r)^2}[D_{1,2}(r)\tau^2 + E_{1,2}(r)\tau + H_{1,2}(r)] \right\} \tag{9}$$

where $\epsilon^2 \equiv 9\pi f_N \alpha_s/(2\sqrt{2}M^2)$. The remarkable aspect of this formula is that $F_2(0)$ can be calculated with no further assumptions and compared with the well-known anomalous magnetic moments of nucleons. At $r = 1/3$, we find

$$\begin{aligned} F_2^p(0) &= 1.83 \\ F_2^n(0) &= -1.89 \quad , \end{aligned} \tag{10}$$

within 2% of the accepted experimental values $\kappa_p = 1.79$ and $\kappa_n = -1.91$.

Table 1– Functions $A_i, ..., H_i$ in Eqn. 7 which control the mass ratio and Q^2 dependence of the heavy baryon form factors F_1 and F_2, as discussed in the text.

	F_1	F_2
A	$2(21r^2 - 18r + 1)$	0
B	$\frac{164r^5+89r^4+212r^3+54r^2+32r+9}{4r(1-r)}$	$\frac{-2(21r^5+21r^4-6r^3+8r^2-3r-1)}{r(1-r)}$
C	$\frac{3(9r^4-r^3+4r^2-r+1)}{r}$	$\frac{(107r^5+23r^4+46r^3+6r^2+7r+3)}{4r(1-r)}$
D	$21r^2 - 30r + 13$	0
E	$-\frac{50r^3+165r^2-172r+1}{2}$	$21r^3 + 39r^2 - 33r - 19$
H	$\frac{3(9r^2+26r+1)r}{2}$	$-\frac{29r^3+93r^2+19r+3}{2}$

Predictions for various heavy nucleons using this formula are shown in Fig. 1. The results for the cross sections are given as ratios to the production cross sections for $\mu^+\mu^-$ pairs. The basic unknown is ϵ, which we can estimate from vector meson dominance arguments[15] and the observed dipole form of ordinary baryon form factors[16]. In Fig. 1 we set $\epsilon = 1/4$ as a typical value and use the approximate quark masses ($m_s \approx 0.5,\ m_c \approx 1.5,\ m_b \approx 5,\ m_t \approx 40$) GeV/$c^2$. The overall magnitude of the cross section will be affected by different choices of ϵ, but the basic features will remain the same[13]. As we can see in Fig. 1, there is a strong dip in the total cross section for certain values around $r = 0.6$ both for charged and neutral cases. Even if the total cross section turns out not to be large enough to measure this dip adequately, the slope of the total cross section just above threshold can provide evidence for these higher twist mass effects.

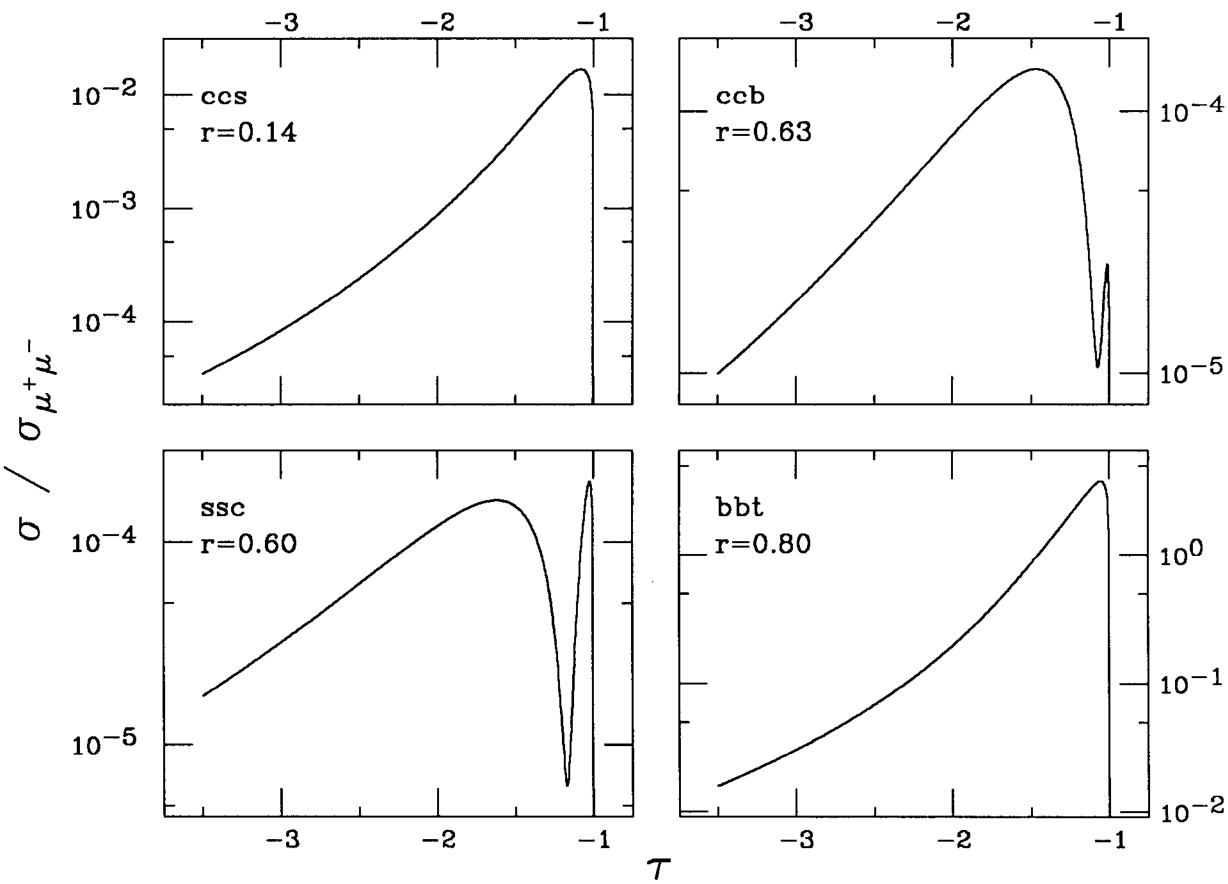

Figure 1. Predicted cross sections for heavy baryon production in e^+e^- annihilation using the formulae given in Eqn. 9 and Table 1. The cross sections are given as ratios to the cross section for production of a $\mu^+\mu^-$ pair as functions of $\tau \equiv Q^2/(4M^2)$ where $\tau = -1$ corresponds to the production threshold. The relative normalizations and structure of the cross sections are dramatically affected by the ratio r of the mass of the unlike quark to the total mass of the baryon. The upper graphs are for charged baryons and the lower graphs are for neutral baryons with the indicated quark content.

The angular distributions for these cases also show dramatic changes in the region of the dip. This happens because both G_E and G_M can become small at nearly the same value of τ, and control of the angular behavior of the cross section passes from one form factor to the other. Equation 6 can be written as $d\sigma/d\Omega \propto (a + b\cos^2\theta^*)$. We show the ratio $b/a = (G_E^2 + \tau G_M^2)/(G_E^2 - \tau G_M^2)$ versus τ in Fig. 2. Note that b/a is identically zero at threshold, and must approach -1 as $\tau \to \infty$.

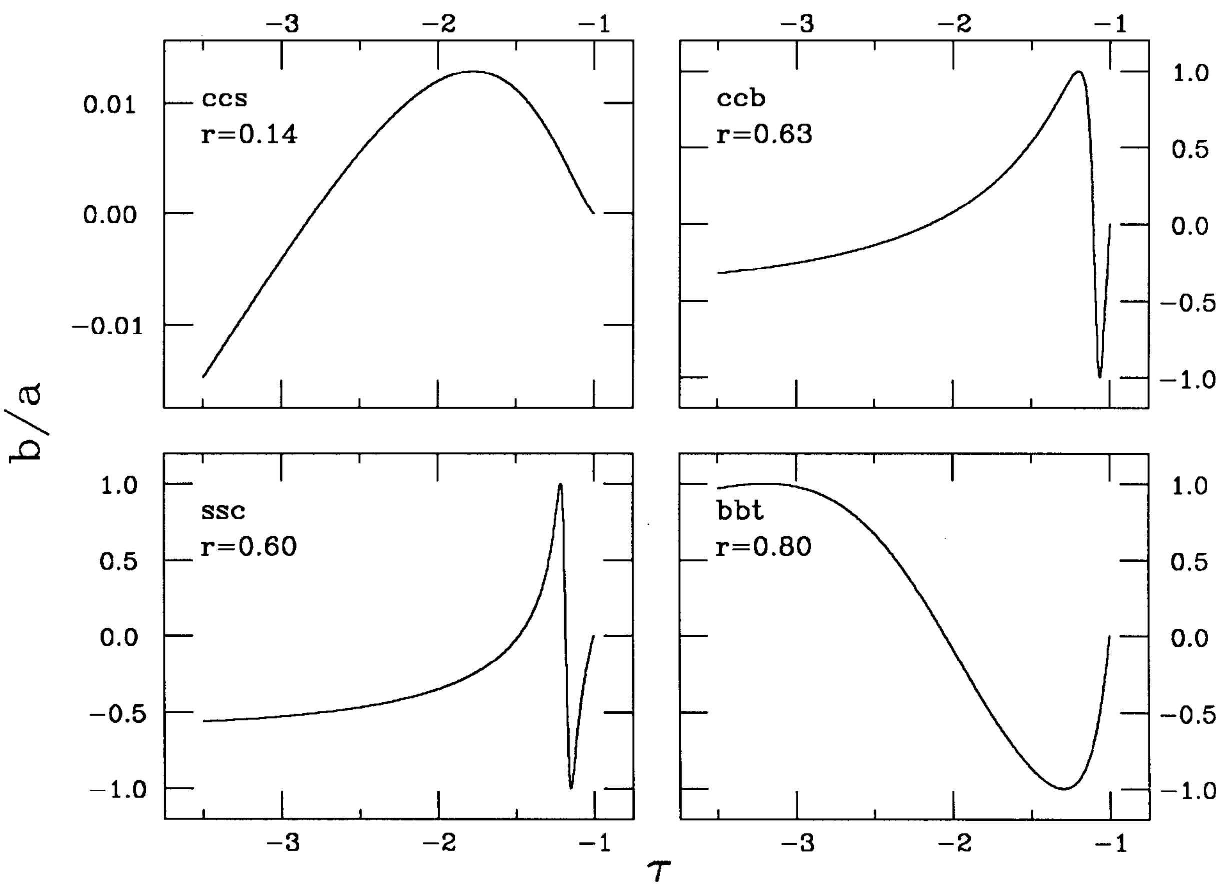

Figure 2. Dependence on τ of the ratio of constants b/a which determine the angular behavior of the production cross section for heavy baryons. This ratio exhibits effects due to interference between the transverse and Coulomb-exchange couplings of the gluons within the baryons. For r near 0.6 the sign changes of b/a occur at values of τ which correspond to the dips in total cross section shown in Fig. 1. Note that the angular distribution is nearly flat for the ccs case.

Since the basic features of these predictions are determined only by the quark mass ratio r without any other tunable parameters, effects discussed above provide clean tests of QCD. There is no way to avoid these phenomena in heavy quark systems, or to postpone them to larger values of momentum transfer. For baryons containing top quarks, our formulae show that the cross section should be large, and these effects will be measurable.

In addition to these points, Eqn. 9 clearly determines the correct sign of the leading twist contributions to the hard scattering kernel. This eliminates the sign ambiguity which exists in previous analyses of the ordinary nucleon form factors[3,6,9]. Since we have derived complete formulae for the kernel including mass effects[12] this analysis can now be applied to ordinary nucleons. For light quark systems, the wave functions will be very different from the simple nonrelativistic form used here and must be determined by nonperturbative treatments such as QCD sum rule analysis[5–7,17] or lattice calculations[18]. Assuming reasonable progress in those areas, a careful analysis including mass effects and different choices of wavefunctions should soon be possible.

The possibility that interference effects in perturbative QCD would produce zeroes in the heavy baryon form factors was first observed in collaboration with Prof. S. Brodsky (see Ref. 8). We are indebted to Prof. Brodsky for many helpful discussions. We would also like to thank Prof. J. D. Walecka for his kind encouragement. This work was supported by the U.S. Department of Energy under contract no. DE-AC03-76SF00515, and by National Science Foundation grants PHY-85-08735 (C-R. J.) and PHY-85-10549 (A.F.S.).

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12. We have derived complete formulae for the hard scattering kernel including all higher twist mass effects to leading order in α_s. Due to their length these formulae are not presented in this short letter, but will be published in a suitable form elsewhere to facilitate calculations for similar processes in which the non-relativistic delta functions used here for heavy baryons are not applicable.
13. In detail, the argument of $\alpha_s(Q^2)$ will be different for each of the diagrams contributing to the production process, depending on the momenta of the gluons in the diagram being considered. We have taken the value of α_s to be a constant in order to simplify the form of the result.
14. Note that Eqn. (11) in Ref. 8 for the effect of binding energy corrections is valid only in the equal mass case. For unequal quark masses, general validity is retrieved if we replace the first term in the denominator of that equation by $(q^2 + \gamma_1^2)^2$ where $\gamma_1 = (2M_H/m_2)\gamma$ and $M_H = m_1 + m_2$.
15. The appropriate vector meson would be composed of the quark/antiquark pair to which the photon attaches in each diagram of the production process. Thus the effective mass of the meson is constrained to be some fraction of the mass of the heavy baryon pair. Vector meson dominance would then imply a denominator structure like $(Q^2 + m_V^2)^{-n}$ where $m_V = 2m_q$. In Eqn. 9 we take $n = 4$ and $\epsilon = \langle m_q^2/M^2 \rangle$ with $\epsilon = 1/4$ as a typical value.
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CHUENG-RYONG JI†

Department of Physics, Stanford University
and Stanford Linear Accelerator Center, Stanford, CA 94305

ALAN F. SILL*

Department of Physics, The American University, Washington, D.C. 20016
and Stanford Linear Accelerator Center, Stanford, CA 94305

The table in SLAC-PUB-4009 contains a slight typographical error in the expression for H_1.

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C	$\frac{3(9r^4-r^3+4r^2-r+1)}{r}$	$\frac{(107r^5+23r^4+46r^3+6r^2+7r+3)}{4r(1-r)}$
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† Department of Physics, Brooklyn College, Brooklyn, NY 11210.
* Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627.