SLAC – PUB – 4001
June 1986
(T/E)

CP VIOLATION IN B & D DECAYS*

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ABSTRACT

Non-leptonic B decays offer the best opportunity to discover the violation of CP invariance outside the neutral K system. Employing the Standard Model one predicts – with reasonable confidence – CP asymmetries of up to 20% (or even more in some cases). The branching ratios for the individual exclusive modes of interest are not expected to exceed the 10^{-3} level in most cases; the identification of such decays poses non-trivial problems. It is shown that by summing intelligently over appropriate classes of decays one can greatly enhance statistics without jeopardizing the signal. Data that contain 10^6 produced B mesons would allow meaningful searches for CP violation. It is noted that "New Physics" could lead to CP asymmetries in D° decays of order 1% . Due to higher branching ratios one can search for such effects in samples of 10^6 produced D mesons.

CPT invariance forces CP violation to reside solely in complex phases of amplitudes. Therefore a certain process will allow the observation of CP violation only if at least two different amplitudes contribute to it in a coherent fashion. This requirement can be satisfied in two different ways, namely by relying on

A) $B^\circ - \bar{B}^\circ$ mixing or

B) final state interactions.

The paper will be organized as follows: in Sec. I I describe those CP asymmetries that involve $B^\circ - \bar{B}^\circ$ mixing and in Sec. II those that have to invoke final state interactions; in Sec. III I examine the most promising search strategies before making an appeal for building a broader data base. At appropriate places I will remark on an analogous analysis for D decays. Few technical details and even fewer references will be given here: they can be found in Ref. 1.

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* Work supported by the Department of Energy, contract AC003-76SF00515.

† Max-Kade Fellow.

Invited Talk at the International Symposium on Production and Decay of Heavy Flavours, Heidelberg, Germany, May 19-23, 1986.

1. CP Symmetries and $B^\circ - \bar{B}^\circ$ Mixing

The time evolution of a meson that was produced as a B° (bottom) or $\bar{B}^\circ$ meson respectively at time $t = 0$ is given by[2]

$$|B^\circ(t)\rangle = g_+(t)\,|B^\circ\rangle + \frac{q}{p}\,g_-(t)\,|\bar{B}^\circ\rangle \tag{1.1}$$

$$|\bar{B}^\circ(t)\rangle = \frac{p}{q}g_-(t)\,|B^\circ\rangle + g_+(t)\,|\bar{B}^\circ\rangle \tag{1.2}$$

$$\begin{aligned} g_\pm(t) &= \frac{1}{2}\exp\left\{-\frac{\Gamma}{2}t\right\}\ \exp\{\,im_1t\}\ \ (1 \pm \exp\{\,i\Delta mt\}) \\ \Delta m &= m_2 - m_1, \qquad \frac{q}{p} = \frac{1-\epsilon}{1+\epsilon} \end{aligned} \tag{1.3}$$

m_i, $i = 1,2$, are the masses of the two mass eigenstates B_i; $\Gamma = \Gamma_1 = \Gamma_2$ has been set for convenience.[1]

Mixing will manifest itself most clearly via decays of neutral bottom mesons to "wrong sign" leptons.[2,3] Using the definitions of Pais and Treiman

$$r_B = \frac{\Gamma(B^\circ \to \ell^+X)}{\Gamma(B^\circ \to \ell^-X)} \simeq \left|\frac{q}{p}\right|^2 \frac{x^2}{2+x^2}, \quad \bar{r}_B = \frac{\Gamma(\bar{B}^\circ \to \ell^-X)}{\Gamma(\bar{B}^\circ \to \ell^+X)} \simeq \left|\frac{p}{q}\right|^2 \frac{x^2}{2+x^2}, \quad x = \frac{\Delta m}{\Gamma} \tag{1.4}$$

one finds

$$y = \frac{N(B^\circ\bar{B}^\circ \to \ell^\pm\ell^\pm X)}{N(B^\circ\bar{B}^\circ \to \ell^+\ell^- X)} = \begin{cases} r & \text{for } \Upsilon(4s) \to B\bar{B} \quad (1.5) \\ \frac{2r}{1+r^2} & \text{for } e^+e^- \to b\bar{b} \text{ continuum} \quad (1.6) \end{cases}$$

[Equation (1.5) holds also in the presence of photons emitted from the e^+e^- beams.]

1.1 Semi-leptonic Decays

If mixing occurs, i.e. $r \neq 0$, then one can search for CP asymmetries in semi-leptonic B decays:

$$a_{SL} = \frac{N(B^\circ\bar{B}^\circ \to \ell^+\ell^+X) - N(B^\circ\bar{B}^\circ \to \ell^-\ell^-X)}{N(B^\circ\bar{B}^\circ \to \ell^+\ell^+X) + N(B^\circ\bar{B}^\circ \to \ell^-\ell^-X)} = \frac{1 - \left|\frac{p}{q}\right|^4}{1 + \left|\frac{q}{p}\right|^4} \tag{1.7}$$

Unfortunately the prospects for measuring such an asymmetry are very discouraging. In the

Standard Model one predicts[4]

$$a_{SL}(B_d) \sim 10^{-3}, \qquad a_{SL}(B_s) \sim 10^{-4} \tag{1.8}$$

This asymmetry is defined for like-sign dileptons. Thus it can be measured only if sizeable mixing occurs. Since one expects in the Standard Model $r(B_d) \leq 4\%$, $r(B_s) \sim 30 - 100\%$ one concludes that more than 10^{10} produced B mesons were needed. It should be noted that most "New Physics" models allow for

$$a_{SL}(\text{"New Physics"}) \sim 10^{-2} \tag{1.9}$$

Data samples of at least 10^6 produced B_s or 10^7 produced B_d mesons might allow us to search for such asymmetries.

1.2 Non-leptonic Decays

For a final state f that is common to both B° and $\bar{B}^\circ$ decays (a property which is then shared by the CP conjugate channel $\bar{f}$) one can sensibly define a CP asymmetry

$$\begin{aligned} |A_{NL}| &= \left| \frac{\Gamma(B^\circ \to f) - \Gamma(\bar{B}^\circ \to \bar{f})}{\Gamma(B^\circ \to f) + \Gamma(\bar{B}^\circ \to \bar{f})} \right| \simeq \frac{\sqrt{2r(1-r)}}{1+r} \left| Im\, \frac{p}{q}\rho_f \right| \\ \rho_f &\equiv \frac{A(\bar{B}^\circ \to f)}{A(B^\circ \to f)} \end{aligned} \tag{1.10}$$

where $A(B^\circ \to f)$ denotes the amplitude for $B^\circ \to f$.

In deriving (1.10) I have integrated over all decay times from zero to infinity. If one is able to observe the finite decay times of B mesons, then one will deal with significantly increased signals (see Ref. 1 for details) – in addition to the obvious advantages in suppressing the background.

Equation (1.10) can be simplified for two limiting cases:

$$|A_{NL}| \sim \begin{cases} \sqrt{2r} \left| Im \frac{p}{q}\rho_f \right| & \text{for } r \ll 1, \text{ i.e., small mixing} \\ \sqrt{\frac{1-r}{2}} \left| Im \frac{p}{q}\rho_f \right| & \text{for } r \simeq 1, \text{ i.e., maximal mixing} \end{cases}$$

i.e. $|A_{NL}|$ vanishes in both limits, but considerably more slowly than one might expect naively.

1.3 EXAMPLES AND PREDICTIONS

In principle the cleanest decay modes are generated by the quark transitions $b \rightarrow c\bar{c}s$, $c\bar{c}d$. For they can lead to CP eigenstates in the final state; e.g., $B_d \rightarrow \psi K_S$, $\psi K_S \pi^\circ$, $D\bar{D}K_S$, $D\bar{D}$; $B_s \rightarrow F^+F^-$, $\psi\phi$. One can show[1] that in this case $\frac{p}{q}\rho_f$ can be calculated rather reliably in terms of KM angles only:

$$\frac{p}{q}\rho_f = \frac{(U^*_{bt}U_{qt})^2}{\left|U^*_{bt}U_{qt}\right|^2} \times \begin{cases} \dfrac{(U_{bc}U^*_{sc})^2}{\left|U_{bc}U^*_{sc}\right|^2} & \text{for } b\bar{q} \rightarrow c\bar{c}s\bar{q} \\ \dfrac{(U_{bc}U^*_{dc})^2}{\left|U_{bc}U^*_{dc}\right|^2} & \text{for } b\bar{q} \rightarrow c\bar{c}d\bar{q} \end{cases} \tag{1.11}$$

Table I contains examples of such decay modes together with predictions of the CP asymmetry derived from Eq. (1.11) and estimates on their branching ratios:

Table I: B° Decays to CP Pure Final States

Decay Mode	Estimated Branching Ratio	CP Asymmetry
$B_d \rightarrow \psi K_s$	5×10^{-4}	
$\psi K_s \pi^\circ$	10^{-3}	2–20%
$D\bar{D}K_s$	$(0.1 - 2) \times 10^{-2}$	
$D\bar{D}$	5×10^{-3}	
$B_s \rightarrow \psi\phi$	10^{-3}	
F^+F^-	0.03	0.1% – 1%

Two kinds of uncertainties enter the prediction of the asymmetry: our ignorance concerning

(*a*) the KM angle $U(bu)$ and

(*b*) the top mass m_t or more specifically the strength of $B^\circ - \bar{B}^\circ$ mixing.

Asymmetries of the type expressed in Eq. (1.10) can occur even when f is not a CP eigenstate;[5] examples are given in Table II:

Table II: B° Decays to non-CP Final States

Decay Mode	Estimated Branching Ratio	CP Asymmetry
$B_d \to D^+\pi^-$	$\frac{1}{2}\%$	$10^{-3} - 0.01$
$D^\circ K_s$	$\mathcal{O}(10^{-3})$	$10^{-3} - 0.01$
$B_s \to F^+K^-$	$\mathcal{O}(10^{-3})$	$0.1 - 0.5$
$D^\circ \phi$	$\mathcal{O}(10^{-3})$	$0.1 - 0.5$

A very detailed discussion of $B_s \to D^\circ\phi$, in particular its time evolution can be found in Ref. 6.

2. CP Asymmetries and Final State Interactions

CP asymmetries can emerge also in the absence of mixing, for which the cleanest scenario is provided by charged B (or D) decays. One finds for the difference between the two CP conjugate widths

$$\Gamma(B^- \to f^-) - \Gamma(B^+ \to f^+) \propto Im\ g_1^* g_2\ \sin(\alpha_1 - \alpha_2) M_1 M_2 \tag{2.1}$$

where M_i, $i = 1, 2$ denotes two different transition amplitudes with the weak couplings g_i and strong phase shifts α_i already factored out.

The asymmetry (2.1) will vanish unless two conditions are satisfied simultaneously:

(*i*) Nontrivial phase shifts $\alpha_1 \neq \alpha_2$ have to be generated from the strong (or electromagnetic) forces. This does not pose a huge problem in principle since the two amplitudes will in general differ in their isospin structure. However in practice it prevents us from making reliable predictions.

(*ii*) The weak couplings g_1 and g_2 have to possess a relative complex phase. In the Standard Model this implies that the transition rates for such decay modes are suppressed by small mixing angles.

There are various ways in which condition (*ii*) can be satisfied:

(α) interplay between two different cascade processes:[1] this can lead to a difference between $\Gamma(B^- \to D^\circ K^- + X \to K_s K^- Y X)$ and $\Gamma(B^+ \to \bar{D}^\circ K^+ + X \to K_s K^+ Y X)$ of up to 1% with a combined branching expected to be of order 10^{-3}.

(β) interplay between quark decay and weak annihilation:[7] this could produce a difference of 10^{-3} up to 10^{-2} between $\Gamma(B^- \to D^{\circ *} D^-)$ and $\Gamma(B^+ \to \bar{D}^{\circ *} D^+)$. The branching ratio for these modes could reach 0.5% .

3. Search Strategies

(A) So far I have discussed the decays of isolated B mesons. Yet in electromagnetic or strong processes one always produces B mesons in conjunction with anti-bottom hadrons. To measure any of the CP asymmetries discussed in Sec. II one has to flavour-tag the decay of the bottom hadron produced in association with the B. This can be achieved most simply (it seems) by observing direct leptons from semi-leptonic bottom decays. Thus the asymmetries in the decay widths get translated into differences between the $\ell^+ f$ and $\ell^- \bar{f}$ correlations; e.g.

$$\tilde{A}_{NL} = \frac{\sigma(B^\circ \bar{B} + B\bar{B}^\circ \to \ell^+ fX) - \sigma(B^\circ \bar{B} + B\bar{B}^\circ \to \ell^- fX)}{\sigma(B^\circ \bar{B} + B\bar{B}^\circ \to \ell^+ fX) + \sigma(B^\circ \bar{B} + B\bar{B}^\circ \to \ell^- fX)} \tag{3.1}$$

The exact relationship between this $\tilde{A}_{NL}$ and A_{NL} as defined in (1.10) can be found in Ref. 1. Suffice it to say that $\tilde{A}_{NL}$ is bound to vanish for the reaction $\Upsilon(4s) \to B^\circ \bar{B}^\circ$ if one integrates over all decay times.

(B) Tables I and II exhibit a general feature: while the CP asymmetries can reach very large values one estimates that the branching ratios for the corresponding exclusive modes are at best small. In addition one has to identify the final state. A good example for these difficulties is provided by $B_d \to \psi K_s$. It is then very tempting to suggest searching for a difference between the inclusive rates $\Gamma(B_d \to \psi + X)$ and $\Gamma(\bar{B}_d \to \psi + X)$ since the corresponding branching ratio amounts to 1% . However it can be shown that

$$A_{NL}(B \to \psi K_s X) = -A_{NL}(B \to \psi K_L X) \tag{3.2}$$

and thus

$$A_{NL}(B \to \psi + X) \equiv 0 \tag{3.3}$$

The underlying reason is that the sign of the asymmetry in the decays B°, $\bar{B}^\circ \to f$, $\bar{f}$ depends on the CP parity of f. More specifically one finds for the asymmetry when summing over different final states f_i:

$$A_{NL}(B^\circ \to \sum_i f_i) = \sum_i A_{NL}(B^\circ \to f_i) BR(B^\circ \to f_i)(-1)^{CP[f_i]} \tag{3.4}$$

where $(-1)^{CP[f_i]}$ denotes the CP parity of the final state f_i. The following lessons are obtained from (3.4):

- an indiscriminate summation over final states will lead to an at least partial cancellation of the asymmetry;

- if the final state can contain a neutral kaon, one has to identify at least a K_s; otherwise the asymmetry is bound to vanish;
- adding the contributions from different decay modes with the appropriate sign, actually represents a simpler task than it appears at first: one can show that the decays $\overset{(-)}{B^\circ} \to \overset{(-)}{D^\circ} M^\circ \to (K_s N^\circ)_D\, M^\circ$ lead to <u>even</u> CP eigenstates for $N,\ M$ being any neutral member of the pseudoscalar, vector or axial vector nonets:

$$CP\,|(K_s N)_{D^\circ} M\rangle = +\,|(K_s N)_{D^\circ} M\rangle$$

  Thus all these channels contribute with the same sign! Using MARK III branching ratios for $D^\circ \to K_s N^\circ$ when available and theoretical guidance for other $D^\circ \to K_s N^\circ$ modes and for $B^\circ \to D^\circ M$ transitions one arrives at

$$BR(B^\circ \to (K_s N)_{D^\circ} M) \sim \mathcal{O}(1\%\,) \tag{3.5}$$

  with a predicted asymmetry of order 10% .
- One can be even bolder and use the inclusive transition $\overset{(-)}{B}_d \to \overset{(-)}{D} + \ldots \to K_s + \ldots$ to search for a CP asymmetry. Using the same procedure that lead to (3.5) one finds a dilution factor of only 1/2 for the asymmetry.

This problem of cancellations in inclusive transitions also arises when f is not a CP eigenstate.[1]

(C) An analogous procedure can be followed when searching for CP asymmetries in D decays. If the strength of $D^\circ - \bar{D}^\circ$ mixing were between 0.1% and 1% then D° decays could exhibit CP asymmetries of order 1% ; this would be a clear signal for "New Physics". The best channels in this context are: $\overset{(-)}{D^\circ}(t) \to K^+K^-,\ K_s\phi$ (or $K - sK^+K^-$) and $K_s + \pi's$.

4. Summary

The basic phenomenological framework for CP asymmetries in B and D decays has been developed. However a reliable evaluation of which of the many possible searches has the best chance to succeed can be made only after a proper data base has been built, in particular for B decays. However we have to realize already at this time that only dedicated searches offer any prospects for success: assuming a 1% probability for actually identifying the appropriate decays one estimates that 10^6 produced B and D mesons are a typical requirement for a e^+e^- machine. Being able to resolve the finite decay lengths would be of invaluable help in suppressing backgrounds.

Acknowledgements

It is a pleasure to thank the organizers, in particular Profs. K. Schubert and B. Stech, for creating such a fine and stimulating meeting where even the weather was made to cooperate.

References

1. I. I. Bigi and A. I. Sanda, SLAC-PUB-3949.
2. A. Pais and S. B. Treiman, Phys. Rev. D12, 2744 (1975).
3. L. B. Okun *et al.* , Nuovo Cim. Lett. 13, 218 (1975).
4. See for example: A. Buras *et al.*, Nucl. Phys. B238, 529 (1984).
5. R. G. Sachs, preprint EFI-85-22-Chicago.
6. I. Dunietz and J. Rosner, preprint EFI-86-10.
7. J. Bernabeu and C. Jarlskog, Z. Physik C8, 233 (1981); L. L. Chau and H. Y. Cheng, preprint BNL-36915; T. Brown *et al.*, Phys. Lett. 136B, 177 (1984).