SLAC – PUB – 4000
June 1986
(T/E)

Mixing and CP Violation in B and D Decays – Future Searches at Hadron Machines*

I. Bigi†

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

The standard Model predicts sizeable if not even maximal $B_s - \bar{B}_s$ mixing; $B_d - \bar{B}_d$ mixing is not expected to exceed the one percent level unless top quarks are very heavy ($m_t \gtrsim 150$ GeV). B decays also offer the best opportunity to discover CP violation outside the neutral K system. Employing the standard model one predicts - with reasonable confidence - CP symmetries of up to 20% (or even more in some cases). The branching ratios for the individual exclusive modes of interest are not expected to exceed the 10^{-3} level in most cases; the identification of such decays poses non-trivial problems. It is shown that by summing intelligently over appropriate classes of decays one can greatly enhance statistics without jeopardizing the signal. Very similar searches can be performed for D decays.

1. Introduction

Particle-antiparticle mixing and CP violation are subtle, yet highly intriguing phenomena which so far have been observed only for neutral kaons. Searches have been performed for mixing in the transitions of neutral charm and bottom mesons. Non-trivial upper bounds have been obtained for $D^\circ - \bar{D}^\circ$ and $B_d - \bar{B}_d$ mixing while the UA1 Group has reported some intriguing evidence for sizeable, if not even maximal $B_s - \bar{B}_s$ mixing.

Using the expressions

$$r_M \equiv \frac{\Gamma(M^\circ \to \text{"wrong sign" } \ell + x)}{\Gamma(M^\circ \to \text{"correct sign" } \ell + x)} = \frac{x^2}{2 + x^2}\,, \quad x = \frac{\Delta m_M}{\Gamma_M} \tag{1.1}$$

one can express the experimental findings as shown in Table I:

⋆ Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.
† Max-Kade Fellow.

Invited talk presented at the Workshop on Physics Simulation
at High Energies, Madison, Wisconsin, May 5-16, 1986.

Table I Experimental Findings

	$r \longrightarrow$	$x \longrightarrow$	$\frac{\Delta m}{m}$
$D^\circ - \bar{D}^\circ$	$\leq .01^{(1)}$	≤ 0.14	$\leq 10^{-13}$
$B_d - \bar{B}_d$	$\leq 0.12\ ARGUS^{(2)}$ $\leq 0.18\ CLEO^{(3)}$	≤ 0.52	$\leq 6.3 \times 10^{-14}$
$B_s - \bar{B}_s$	$\sim 0.3 - 1\ UAI^{(4)}$	≥ 1	$\geq 1.2 \times 10^{(-13)}$

For clarity it should be kept in mind that the numbers given above were derived assuming

$$\tau(B_u) \simeq \tau(B_d) \simeq \tau(B_s) \simeq 10^{-12} \text{ sec} \tag{1.2}$$

Most authors would not quarrel with (1.2) though sizeable lifetime ratios are not ruled out experimentally:[3]

$$0.48 \lesssim \tau(B_u)/\tau(B_d) \leq 1.9$$

The predictions of the Standard Model with three families are given in Table II. For $B^\circ - \bar{B}^\circ$ mixing I list predictions for three different values of m_t since the size of the top quark mass has not been established. Furthermore I take Suzuki's caveat[(5)] into account for the B_d case whereas I ignore it in the B_s case for reasons explained in Ref. 6.

Table II
Theoretical Expectations in the Standard Model with Three Families

	$x \longrightarrow$	r	
$D^\circ - \bar{D}^\circ$	≤ 0.045	≤ 0.001	
$B_d : m_t =$ 40 GeV	0.03F	$\frac{10^{-3}F^2}{2+10^{-3}F^2}$	
170 GeV	0.36F	$\frac{0.13F^2}{2+0.13F^2}$	$F = \left(\frac{U(bu)}{0.01}\right)^2 \frac{(Bf_B)^2}{(100MeV)^2}$
300 GeV	0.85 F	$\frac{0.72F^2}{2+0.72F^2}$	
$B_s : m_t =$ 40 GeV	1-4.5	0.33-0.91	
170 GeV	11-50	≥ 0.98	
300 GeV	27-120	≥ 0.99	

Comparing the entries in Table I and II one can draw three conclusions.

(i) The Standard Model with three families allows for sizeable, if not even near maximal $B_s - \bar{B}_s$ mixing for top quarks, as light as 40 GeV.

(ii) The ARGUS and CLEO limits on $B_d - \bar{B}_d$ mixing suggest $m_t \lesssim 300$ GeV unless $|U(bu)| \ll 0.01$ (here I have used the approximate equality $|U(bu)| \simeq |U(td)|$ that holds in a three family ansatz).

(iii) Experiments both at e^+e^- and hadron machines are reaching the required sensitivity in searching for $B^\circ - \bar{B}^\circ$ mixing.

The preceding remarks are meant to set the stage for the remainder of the paper. In section II I briefly discuss how one can study $B^\circ - \bar{B}^\circ$ mixing in a more detailed way; in section III I introduce the phenomenology of CP violation relevant for B and D decays including predictions obtained in the Standard Model; in section IV I analyze search strategies.

This presentation will be short on technical details and references. They can be found in SLAC-PUB-3949.

2. Detailed Studies of $B^\circ - \bar{B}^\circ$ Mixing

Signals for $B^\circ - \bar{B}^\circ$ mixing coming from like-sign di-leptons yield only a weighted average of the strength of B_d and B_s mixing. Yet it would be highly important to determine $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing separately:

- as discussed before the Standard Model makes quite different predictions for the two cases;
- as will be shown later the strength of $B^\circ - \bar{B}^\circ$ mixing is an important ingredient in CP asymmetries.

Although B production in e^+e^- annihilation might give us the desired information it is still useful to address the potential of measurements possible at hadron machines. One basic distinction should be kept in mind: the most direct signal for mixing is a deviation from an exponential decay law. The observation of BB, DD, DF pairs or of like-sign di-leptons is evidence for mixing only if $(b\bar{b}b\bar{b})$ production can be ignored. This is presumably a safe assumption up to Tevatron energies.

In that case there are quite a few ways to separate $B_s - \bar{B}_s$ from $B_d - \bar{B}_d$ mixing; just two examples:

- one can search for F and D mesons in conjunction with di-leptons. Such a procedure is based on the observation that semi-leptonic B_u and B_d decays are almost saturated by $B \to \ell\nu D^{(*)}$. There is every reason to expect the analogous pattern to hold for B_s decays, i.e.

$$\Gamma(B_s \to \ell\nu X) \simeq \Gamma(B_s \to \ell\nu F^{(*)}) \ .$$

Observing

$$p\bar{p} \to \ell^-\ell^-F^+, \ \ \ell^+\ell^+F^- + X$$

would thus not only confirm the original mixing signal, but also relate it directly to B_s decays.

– When the inclusive rates $B_q \to D,\ \bar{D},\ F,\ \bar{F} + X,\ q = u,\ d,\ s,$ are known then one can translate an observed rate for DD and DF pairs, preferably with a direct lepton, into a measurement of $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing.

3. Phenomenology of CP Violation in B [& D] Decays

CPT invariance forces CP violation to reside solely in complex phases of amplitudes. Therefore, a certain process will allow the observation of a CP asymmetry only if at least two different amplitudes contribute to it in a coherent fashion. This requirement can be satisfied in two different ways, namely by relying on

A) $B^\circ - \bar{B}^\circ$ mixing or

B) final state interactions.

3.1 Mixing and CP Violation

The time evaluation of a meson that was produced as a B° or $\bar{B}^\circ$ meson respectively at time $t = 0$ is given by

$$|B^\circ(t)\rangle = g_+(t)|B^\circ\rangle + \frac{q}{p}\, g_-(t)|\bar{B}^\circ\rangle$$

$$|\bar{B}^\circ(t)\rangle = \frac{p}{q}\, g_-(t)|B^\circ\rangle + g_+(t)|\bar{B}^\circ\rangle$$

$$g_\pm(t) = \frac{1}{2} exp\Big\{-\frac{\Gamma}{2}\, t\Big\} exp\Big\{im_1 t\Big\}\Big(1 \pm exp\Big\{i\Delta m t\Big\}\Big) \tag{3.1}$$

$$\Delta m = m_2 - m_1;\ \ \frac{q}{p} = \frac{1-\epsilon}{1+\epsilon}$$

$m_1,\ m_2$ are the masses of the two mass eigenstates $B_1,\ B_2$; $\Gamma_1 = \Gamma_2 = \Gamma$ has been set for convenience.

(a) Semi-leptonic decays

Using the notation of Pais and Treiman one finds

$$r_B = \frac{\Gamma(B^\circ \to \ell^+ X)}{\Gamma(B^\circ \to \ell^- X)} \simeq \left|\frac{q}{p}\right|^2 \frac{x^2}{2 + x^2}$$

$$\bar{r}_B = \frac{\Gamma(\bar{B}^\circ \to \ell^- X)}{\Gamma(\bar{B}^\circ \to \ell^+ X)} \simeq \left|\frac{p}{q}\right|^2 \frac{x^2}{2 + x^2} \tag{3.2}$$

If mixing occurs, i.e., $r_B,\ \bar{r}_B \neq 0$, then one can search for a CP asymmetry in semi-leptonic B decays:

$$a_{SL} = \frac{r_B - \bar{r}_B}{r_B + \bar{r}_B} \simeq \frac{1 - |\frac{p}{q}|^4}{1 + |\frac{p}{q}|^4} \tag{3.3}$$

Unfortunately the prospects for measuring such an asymmetry are very discouraging. In the Standard Model one predicts[8]

$$a_{SL}(B_d) \sim 10^{-3}, \quad a_{SL}(B_s) \sim 10^{-4} \tag{3.4}$$

This asymmetry can be measured via like-sign di-leptons. Since one expects in the Standard Model $|r(B_d)| \leq 4\%$, $|r(B_s)| \sim 30 - 100\%$ one concludes that more than 10^{10} produced B mesons were needed. It should be noted that most "New Physics" models allow - yet only as a possible scenario - for

$$a_{SL}(\text{“New Physics”}) \sim 10^{-2} \tag{3.5}$$

Data samples of at least 10^6 produced B_s or 10^7 produced B_d mesons might allow us to search for such asymmetries.

(b) Non-leptonic decays

For a final state f that is common to both B° and $\bar{B}^\circ$ decays (a property which is then shared by the CP conjugate channel $\bar{f}$) one can sensibly define a CP asymmetry

$$\begin{aligned} |A_{NL}| &= \left|\frac{\Gamma(B^\circ \to f) - \Gamma(\bar{B}^\circ \to \bar{f})}{\Gamma(B^\circ \to f) + \Gamma(\bar{B}^\circ \to \bar{f})}\right| \simeq \frac{\sqrt{2r(1-r)}}{1+r}\left|Im\ \frac{p}{q}\ \rho_f\right| \\ \rho_f &= \frac{A(\bar{B} \to f)}{A(B^\circ \to f)} \end{aligned} \tag{3.6}$$

where $A(B^\circ \to f)$ denotes the amplitude for $B^\circ \to f$.

In deriving (3.6) I have integrated over all decay times from zero to infinity. If one is able to observe the finite decay times of B mesons, then one will deal with significantly increased signals (see Ref. 6 for details), in addition to the obvious advantages in suppressing the background.

Equation (3.6) can be simplified for two limiting cases:

$$|A_{NL}| \sim \begin{cases} \sqrt{2r}\ |Im\frac{p}{q}\rho_f| & \text{for } r \ll 1, \text{ i.e. small mixing} \\ \sqrt{\frac{1-r}{2}}\ |Im\frac{p}{q}\rho_f| & \text{for } r \simeq 1\ , \text{ i.e. maximal mixing} \end{cases} \tag{3.7}$$

i.e. $|A_{NL}|$ vanishes in both limits, but considerably more slowly than one might expect naively.

In principle the cleanest decay modes are generated by the quark transitions $b \to c\bar{c}s$, $c\bar{c}d$. For they can lead to CP eigenstates in the final state, e.g. $B_d \to \psi K_s$, $\psi K_s \pi^o$, $D\bar{D}K_s$, $D\bar{D}$; $B_s \to F^+F^-, \psi\phi$. One can show[6] that in this case $\frac{p}{q}\,\rho_f$ can be calculated rather reliably in terms of KM angles only:

$$\frac{p}{q}\rho_f = \frac{(U_{bt}^* U_{qt})^2}{|U_{bt}^* U_{qt}|^2} \times \begin{cases} \dfrac{(U_{bc}U_{sc}^*)^2}{|U_{bc}U_{sc}^*|^2} & \text{for } b\bar{q} \to c\bar{c}s\bar{q} \\ \dfrac{(U_{bc}U_{dc}^*)^2}{|U_{bc}U_{dc}^*|^2} & \text{for } b\bar{q} \to c\bar{c}d\bar{q} \end{cases} \tag{3.8}$$

Table III contains examples of such decay modes together with predictions of the CP asymmetry derived from Eq. (3.8) and estimates on their branching ratios:

Table III B° Decays to CP Pure Final States

Decay Mode		Estimated Branching Ratio	CP Asymmetry
$B_d \to$	ψK_s	5×10^{-4}	
	$\psi K_s \pi^\circ$	10^{-3}	
	$D\bar{D}K_s$	$(0.1 - 2) \times 10^{-2}$	2 - 20%
	$D\bar{D}$	5×10^{-3}	
$B_s \to$	$\psi\pi$	10^{-3}	0.1% - 1%
	F^+F^-	0.03	

Two kinds of uncertainties enter the prediction of the asymmetry: our ignorance concerning

(a) the KM angle $U(bu)$ and

(b) the top mass m_t or more specifically the strength of $B^\circ - \bar{B}^\circ$ mixing.

Asymmetries of the type expressed in Eq. (3.6) can occur even when f is not a CP eigenstate;[9] examples are given in Table IV.

A very detailed discussion if of $B_s \to D^\circ\phi$, in particular its time evolution can be found in Ref. (10).

Table IV B° Decays to non-CP Final States

Decay Mode		Estimated Branching Ratio	CP Asymmetry
$B_d \to$	$D^+\pi^-$	$\frac{1}{2}\%$	10^{-3} - 0.01
	$D^\circ K_S$	$\mathcal{O}(10^{-3})$	10^{-3} - 0.01
$B_s \to$	F^+K^-	$\mathcal{O}(10^{-3})$	0.1 - 0.5
	$D^\circ\phi$	$\mathcal{O}(10^{-3})$	0.1 - 0.5

3.2 *CP* ASYMMETRIES AND FINAL STATE INTERACTIONS

CP asymmetries can emerge also in the absence of mixing, for which the cleanest scenario is provided by charged B (or D) decays. One finds for the difference between the two CP conjugate widths

$$\Gamma(B^- \to f^-) - \Gamma(B^+ \to f^+) \propto Im\; g_1 g_2 \sin(\alpha_1 - \alpha_2) M_1 M_2 \tag{3.9}$$

where M_i, $i = 1$, 2 denote two different transition amplitudes with the weak couplings g_i and strong phase shifts α_i already factored out.

The asymmetry (3.9) will vanish unless two conditions are satisfied simultaneously:

(i) Nontrivial phase shifts $\alpha_1 \neq \alpha_2$ have to be generated from the strong (or electromagnetic) forces. This does not pose a huge problem in principle since the two amplitudes will in general differ in their isospin structure. However, in practice it prevents us from making reliable predictions.

(ii) The weak couplings g_1 and g_2 have to possess a relative complex phase. In the Standard Model this implies that the transition rates for such decay modes are suppressed by small mixing angles.

There are various ways in which condition (ii) can be satisfied:

(α) interplay between two different cascade processes:[6] this can lead to a difference between

$$\Gamma(B^- \to D^\circ K^- + X \to K_s K^- Y X)$$

and

$$\Gamma(B^+ \to D^\circ K^+ + X \to K_s K^+ Y X)$$

of up to 1% with a combined branching expected to be of order 10^{-3}.

(β) interplay between quark decay and weak annihilation:[11] this could produce a difference of 10^{-3} up to 10^{-2} between $\Gamma(B^- \to D^{\circ *} D^-)$ and $\Gamma(B^+ \to \bar{D}^{\circ *} D^+)$. The branching ratio for these modes could reach 0.5%.

3.3 CP VIOLATION IN D DECAYS

The Standard Model predicts, as already stated, very little $D^\circ - \bar{D}^\circ$ mixing and even tinier CP asymmetries. New Physics (e.g. an extended Higgs sector) could lead to $D^\circ - \bar{D}^\circ$ mixing with a strength of up to 1% roughly. Equation (3.7) shows that in such a case a CP asymmetry like A_{NL} can also reach the percent level. The best way to look for such asymmetries presumably is to compare $D^\circ(t) \to K^+K^-$ or $K_s\phi$ or $K_s + \pi's$ vs. $\bar{D}^\circ(t) \to K^+K^-$ or $K_s\phi$ or $K_s + \pi's$.

4. Search Strategies

(A) So far I have discussed the decays of isolated B mesons. Yet in electromagnetic or strong processes one always produces B mesons in conjunction with anti-bottom hadrons. To measure any of the CP asymmetries discussed in section II one has to flavour-tag the decay of the bottom hadron produced in association with the B. This can be achieved most simply (it seems) by observing direct leptons from semi-leptonic bottom decays. Thus the asymmetries in the decay widths get translated into differences between the $\ell^+ f$ and $\ell^- \bar{f}$ correlations; e.g.

$$\tilde{A}_{NL} = \frac{\sigma(B^\circ \bar{B} + B\bar{B}^\circ \to \ell^+ fX) - \sigma(B^\circ \bar{B} + B\bar{B}^\circ \to \ell^- \bar{f}X)}{\sigma(B^\circ \bar{B} + B\bar{B}^\circ \to \ell^+ fX) + \sigma(B^\circ \bar{B} + B\bar{B}^\circ \to \ell^- \bar{f}X)} \tag{4.1}$$

The exact relationship between this $\tilde{A}_{NL}$ and A_{NL} as defined in (3.6) can be found in Ref. 6.

Suffice it to say here that $\tilde{A}_{NL}$ is bound to vanish for the reaction $\Upsilon(4s) \to B^\circ \bar{B}^\circ$ if one integrates over <u>all</u> decay times.

(B) Table III and IV exhibit a general feature: while the CP asymmetries can reach very large values one estimates that the branching ratios for the corresponding exclusive modes are at best small. In addition one has to identify the final state. A good example for these difficulties is provided by $B_d \to \psi K_s$. It is then very tempting to suggest searching for a difference between the <u>inclusive</u> rates $\Gamma(B_d \to \psi + X)$ and $\Gamma(\bar{B}_d \to \psi + X)$ since the corresponding branching ratio amounts to 1%. However it can be shown that

$$A_{NL}(B \to \psi K_s X) = -A_{NL}(B \to \psi K_L X) \tag{4.2}$$

and thus

$$A_{NL}(B \to \psi + X) \equiv 0 \tag{4.3}$$

The underlying reason is that the sign of the asymmetry in the decays B°, $\bar{B}^\circ \to$

f, $\bar{f}$ depends on the CP parity of f. More specifically one finds for the asymmetry when summing over different final states f_i:

$$A_{NL}\left(B^\circ \to \sum_i f_i\right) = \sum_i A_{NL}(B^\circ \to f_i) BR(B^\circ \to f_i)(-1)^{CP[f_i]} \qquad (4.4)$$

where $(-1)^{CP[f_i]}$ denotes the CP parity of the final state f_i. The following lessons are obtained from (4.4):

- an indiscriminate summation over final states will lead to an at least partial cancellation of the asymmetry;
- if the final state can contain a neutral kaon, one has to identify at least a K_s; otherwise the asymmetry is bound to vanish;
- adding the contributions from different decay modes with the appropriate sign, actually represents a simpler task than it appears at first: one can show the decays $\overset{(-)}{B}{}^\circ \to \overset{(-)}{D}{}^\circ M^\circ \to (K_s N^\circ)_D M^\circ$ lead to even CP eigenstates for N, M being any neutral member of the pseudoscalar, vector or axial vector nonets:

  $$CP|(K_s N)_{D^\circ} M\rangle = +|(K_s N)_{D^\circ} M\rangle$$

  Thus all these channels contribute with the same sign! Using Mark III branching ratios for $D^\circ \to K_S N^\circ$ when available and theoretical guidance for other $D^\circ \to K_S N^\circ$ modes and for $B^\circ \to D^\circ M$ transitions one arrives at

  $$BR(B^\circ \to (K_S N)_{D^\circ} M) \sim \mathcal{O}(1\%) \qquad (4.5)$$

  with a predicted asymmetry of order 10%.
- one can be even bolder and use the inclusive transition $\overset{(-)}{B}_d \to \overset{(-)}{D} + \ldots \to K_S + \ldots$ to search for CP asymmetry. Using the same procedure that lead to (4.5) one finds a dilution factor of only 1/2 for the asymmetry.

This problem of cancellations in inclusive transitions also arises when f is not a CP eigenstate.[6]

(C) An analogous procedure can be followed when searching for CP asymmetries in D decays. If the strength of $D^\circ - \bar{D}^\circ$ mixing were between 0.1% and 1% then D° decays could exhibit CP asymmetries of order 1%; this would be a clear signal for "New Physics". The best channels in this context are: $\bar{D}^\circ(t) \to K^+K^-$, $K_s\phi$(or $K_s K^+K^-$) and $K_s + \pi's$.

5. Summary

The basic phenomenological framework for CP asymmetries in B and D decays has been developed. However, a reliable evaluation of which of the many possible searches has the best chance to succeed can be made only after a proper data base has been built, in particular for B decays. However, we have to realize already at this time that only dedicated searches offer any prospects for success: assuming a 1% probability for actually identifying the appropriate decays one estimates that 10^6 produced B or D mesons are a typical requirement. Being able to resolve the finite decay lengths would be of invaluable help in suppressing backgrounds.

Acknowledgements

It is a pleasure to thank the organizers, in particular Prof. V. Barger, F. Halzen and T. Gottschalk for creating such a fine and stimulating meeting. I also greatly acknowledge useful discussions with F. Paige and A. Sanda.

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