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# A MINIMAL RELATIVISTIC MODEL FOR THE THREE NUCLEON SYSTEM*

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## ABSTRACT

A start is made on disentangling relativistic kinematic effects from "three body forces" by presenting a minimal relativistic model in which the *internal* mesonic degree of freedom is treated on the same footing as the nucleonic degrees of freedom. The meson is not allowed to appear asymptotically, specifying the two-nucleon "off shell" amplitudes which can be used to calculate the three nucleon problem. The results are identical to those obtained from the same model starting from three nucleons and one meson. In effect we have discovered a "relativistic potential model" which does not generate "three body forces".

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"Three body forces" are notoriously difficult to define; this conference arrived at no consensus, even though the problem was restricted to the three nucleon system. One formulation of the physics needed was given long ago[1] . An attempt was made in 1972 to pose the problem more generally[2] , using these words:

"What we mean by 'three body forces' requires a prior understanding, either tacit or explicit, of what we mean by the separate words 'three' 'body', 'forces', what we mean by conjoining them, or alternatively what we mean by the undecomposed phrase. The latter usage is more common as qualified, for example, by the addenda '... in nuclear physics', '... in quantum mechanics', or '... in elementary particle physics.' In such cases the qualification implies that the discipline named already specifies what bodies and forces are under consideration and what is meant by a three-body system. We will be concerned with all three examples just given, but unfortunately each requires further clarification. If we restrict ourselves to non-relativistic quantum mechanics, we must specify a two-body 'potential', which is supplied in atomic physics to order $e^2$ by the coulomb force, but in other cases is phenomenological. Once we allow non-local forces in nuclear physics, only a local potential can be related to two-body experiments, and off-shell effects crucial to the understanding of the three-nucleon problem remain highly ambiguous. If we try to remove these ambiguities by turning to the theory of elementary particles in its conventional second quantized form we necessarily introduce an infinite number of particles, and in trying to extract from these a useful desciption of the three-nucleon system we encounter raging controversy as to which *ad hoc* prescription provides the 'best' approximation."

I leave it to the reader of these proceedings find among these papers those which meet any of the previously suggested criteria. The approach used here is to strip down the "meson exchange" problem to its minimal kinematic elements, keeping exact 3-momentum conservation and probability flux conservation inviolable, while preserving the relativistic connection between free-particle energy and momentum. The claim of this paper is that by so doing one can arrive at a minimal relativistic model, formally equivalent to the nonrelativistic scattering theory with pair-"potentials" and no "three body forces". I claim that it is possible to remove relativistic *kinematic* effects from the problem, – effects which other authors invoke in *ambiguous* ways as "three body forces". *If* consensus could be achieved at this minimal level, then what remains could reasonably be attributed to the "three body force effects" due to specific internal degrees of freedom (eg. $\pi - \pi, N - \Delta, 6$ *quark*, ...). Of course further refinements in our "finite particle number" approach will be needed before *those* effects can be sorted out.

Although the problem we attack can be cast in manifestly covariant form, anti-particles included, and "crossing symmetry" discussed[3] using the framework invoked here, such an approach would bring in *genuine* three body forces if pursued very far. We use, with a significant change in the "two-particle" input, the relativistic but not "manifestly covariant" formalism developed by Lindesay.[4] The critical idea is[5] that nucleon and meson can form a "bound state" with the same mass and quantum numbers as the nucleon. Of course this is not new. Long ago Fermi and Yang suggested[6] that the pion be thought of as an s-wave "bound state" of a nucleon-antinucleon pair (an idea being exploited by Pastrana in the extension of our model[3]). But it is hard to make this consistent with a Hamiltonian or Lagrangian theory.

Fortunately the "zero range scattering theory" developed in a non-relativistic context[7] allows scattering amplitudes to be inserted in Faddeev equations without specifying their relation to the non-invariant concept of "potential energy distribution". Then the idea reduces to the *kinematic* requirement that the "elementary" (or input) two-particle amplitude for meson-nucleon scattering have a pole when the invariant four-momentum of this pair is equal to the nucleon mass. As has been noted many times[8] , the use of Faddeev dynamics guarantees unitarity without ever producing the self-energy infinities caused by the quantum field theory formalism. Clearly our general philosophical framework is that of S-Matrix theory, although we part company from the usual approaches to that theory by restricting ourselves to finite particle sectors. The second critical physical input is that 3-momentum be conserved in each elementary scattering. All particles are "on-shell"; only the energy of the system as a whole is allowed to fluctuate within the limits provided by the uncertainty principle. Again this is hardly new; Wick used this idea long ago[9] to provide physical insight into Yukawa's[10] meson theory. Putting this together with the requirement that observable probabilities be conserved specifies a minimal theory, as we now show.

Although the two-nucleon one meson system described by four-vectors has twelve degrees of freedom, our mass shell requirement $(\vec{k})^2 = \vec{k}\cdot\vec{k} = \epsilon_m^2 - \underset{\sim}{k}\cdot\underset{\sim}{k} = m^2$ reduces these to 9, and total 3-momentum conservation to 6. We restrict the Faddeev treatment (which would include the kinematic equivalent of particle "creation" and "destruction") by assuming that we start and end with a "bound pair" plus a free particle, and hence need only consider the residues of the double poles in the Faddeev amplitudes. Under these circumstances, 3-momentum conservation fixes the scattering plane in the *external* (and then laboratory) frame and reduces the *dynamical* (internal) degrees of freedom to 3. The remaining 3 simply allow the result of solving our dynamical equations to be related to external, and via the total 3-momentum to laboratory, coordinates. In general there will be nine "elastic and rearrangement" amplitudes (for example if we have a nucleon and an anti-nucleon, there will be a pole at the mass of the meson), but our "confined quantum" assumption[11,12] reduces these to four. Finally, the $\delta$-function on spectator momentum reduces the 3 degrees of freedom to two dynamical degrees of freedom for each Faddeev channel ( of course care must be exercised because the Faddeev description is "overcomplete"); we take these to be the magnitude of the momentum and the scattering angle, as in nonrelativistic potential scattering, or a single vector variable $\underset{\sim}{p}$ with the understanding that the azimuthal angle (or magnetic quantum number) is an "ignorable coordinate".

There is a further non-trivial kinematic fact which simplifies our result. We use the Goldberger-Watson[13] propagator $R_0^{-1}(z) = \epsilon_1 + \epsilon_2 + \epsilon_\mu - z$ where $\epsilon_i = \sqrt{p_i^2 + m_i^2}$, $i \in 1,2$ and $\epsilon_\mu = \sqrt{q^2 + \mu^2}$. Since we are in the zero momentum frame, this is related to the invariant $S = (\vec{k}_1 + \vec{k}_2 + \vec{k}_\mu)^2 = (\epsilon_1 + \epsilon_2 + \epsilon_\mu)^2$ by

$R_0(z) = (\sqrt{S} - z)^{-1}$ Here $\underset{\sim}{p}_1, \underset{\sim}{p}_2, \underset{\sim}{q}$ refer to the "internal" coordinates where all three particles are "free". But the "external" coordinates refer to a particle of mass $m_a$ and "bound state" of mass $\mu_a$, with the invariant $s_a = (\epsilon_a + \epsilon_{\mu_a})^2$ or $\epsilon_a = \frac{1}{2}\sqrt{s_a} + \frac{m_a^2 - \mu_a^2}{2\sqrt{s_a}}$ because $p_a^2 = \epsilon_a^2 - m_a^2 = \epsilon_{\mu_a}^2 - \mu_a^2$. The model requires the driving terms to have a pole at $S_{i\mu} = (\vec{k}_i + \vec{k}_\mu)^2 = m_i^2 = (\epsilon_i + \epsilon_\mu)^2 - p_j^2$ where we have used the fact that $\underset{\sim}{p}_1 + \underset{\sim}{p}_2 + \underset{\sim}{q} = 0$. Hence (for equal mass nucleons) $S_{i\mu} - m_i^2 = (\sqrt{S} - \epsilon_j)^2 - \epsilon_j^2 = \sqrt{S}(\sqrt{S} - 2\epsilon_j)$, and the pole also occurs at $S = 4\epsilon_j^2$. Finally, we note that on shell, $S = s_i = s_j = 4\epsilon_j^2$ and $p^2 = (p^0)^2$, so the pole also occurs when the two momenta are equal. This allows us to write the driving terms as

$$\frac{g^2 \delta^3(p - p_0)}{p^2 - (p^0)^2 - i\eta}$$

The final result for the nucleon-nucleon amplitude in this (scalar) model is then that

$$T(\underset{\sim}{p}, \underset{\sim}{p}') = K_{11}(\underset{\sim}{p}, \underset{\sim}{p}') + K_{12}(\underset{\sim}{p}, -\underset{\sim}{p}') + K_{21}(-\underset{\sim}{p}, \underset{\sim}{p}') + K_{12}(-\underset{\sim}{p}, -\underset{\sim}{p}')$$

where

$$K_{ij}(\underset{\sim}{p}_i, \underset{\sim}{p}_j) - V_{ik}(\underset{\sim}{p}_i, \underset{\sim}{p}_j) = \int d^3 p'_k \frac{V_{ik}(\underset{\sim}{p}_i, \underset{\sim}{p}'_k) K_{kj}(\underset{\sim}{p}'_k, \underset{\sim}{p}_j)}{(p'_k)^2 - p_j^2 - i\eta}$$

and

$$V_{ij} = -(1 - \delta_{ij}) \frac{g^2}{\epsilon_\mu^{ij'}(\epsilon_\mu^{ij'} - \epsilon_{\mu_i} + \epsilon'_j)}$$

with $\epsilon_\mu^{ij'} = \sqrt{(\underset{\sim}{p}_i + \underset{\sim}{p}'_j)^2 + \mu^2}$.

If the "bound state" is required to contain exactly one particle and one meson, three particle unitarity fixes a unique constant value for the coupling constant[4]. However, as has been discussed in connection with the "reduced width" (also the residue of a "bound state" pole) in the non-relativistic theory[14] it is possible to treat the residue as a measure of how much of the state is "composite" and how much "elementary"; the density matrix derivation given in the reference is due to Lindesay. In the case at hand, since the $K_{ij}$ satisfy coupled channel Lippmann-Schwinger equations, their unitarity and that of the $T$ constructed from them is immediate, and is independent of the value of $g^2$, making this, as well as the meson mass available as adjustable parameters for use in low energy phenomenology.

In a sense we have not done much as yet. We have a relativistic generalization of the Lippmann-Schwinger equation which reduces to the usual Yukawa "potential" in the nonrelativistic kinematic region. But our potential acts on four coupled amplitudes whose *sum* is the physical amplitude of interest and which describes *space exchange* scattering as a necessary consequence of the model, – an effect that from some non-relativistic points of view would be called "non-local", and requires in some approaches a "velocity dependence" containing arbitrarily large powers of momenta. If we include spin and isospin for the nucleons, it is easy to see that we will have to antisymmetrize rather than symmetrize the $K's$, giving for our scalar meson model the $1 + P_{ex}$ Serber force as our zero$^{th}$ approximation. This reproduces differential cross sections for nucleon-nucleon scattering reasonably well in the 0-100 Mev range, and can allow (in an extended model) a tensor force leading to a deuteron quadrupole moment; it fails to account for the spin-dependent p-p scattering in odd parity states. Having established a reasonable relativistic model, we can take the potential so defined into a three nucleon space and calculate relativistic scattering amplitudes in this space using the Faddeev equations. Because these have no interaction involving three nucleon coordinates directly, these have no more "three body force" than do the corresponding nonrelativistic equations. One advantage of our approach now becomes apparent. We can couple in the electromagnetic field (to lowest order) simply by replacing $\vec{k}_i$ by $\vec{k}_i + \frac{1}{2}(1 + \tau_z^i)e\vec{A}/c)$ *and* $\vec{q}$ by $\vec{q} + I_z e\vec{A}/c$ in our wave functions; note that meson currents and nucleon currents occur on an equal footing.

One might suspect at this point the self-consistency of the model. However, one can start afresh by formulating the three nucleon problem as a three nucleon, one meson problem and writing down the Faddeev-Yakubovsky equations. We do not have space here to give the details, but the fact that the only scatterings which can occur involve the single meson immediately eliminates the 6 (2,2) configurations and 9 of the 12 (3,1) configurations. Hence the model necessarily yields $3 \times 3$ amplitudes whether we first compute the two-nucleon off-shell amplitude (or potential) and then use that in 3-body equations, or start directly in the four particle space. The multiple scattering theory generated is identical in both cases.

A number of interesting applications follow. In effect we have given a relativistic definition of the "potential" due to single meson exchange. Further, since the internal mesonic degree of freedom is completely specified, the interaction of the system with electromagnetic fields, or quanta, can be readily calculated. Since we have completely determined the relativistic kinematic effects *without* producing "three body forces", comparison with treatments containing more internal degrees of freedom (pion-nucleon resonances, pion-pion scattering,anti-nucleons, quarks and gluons,...) should allow an unambiguous separation of *physical* three nucleon forces from the hitherto ambiguous contributions arising from relativistic kinematic effects.

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