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REPORT OF THE WORKING GROUP FOR POLARIZATION IN THE SSC MAIN RING*

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ABSTRACT

The task of the main ring working group was to study polarized beams in the SSC main ring. Many problems were studied; however, the primary emphasis was in the determination of the acceptable resonance strengths and the number of Siberian Snakes which would be necessary given those resonance strengths. During the workshop all of the members of the working group realized that there was much more work to be done and consequently there have been some changes in the general conclusions during the following months. The present conclusion really reflects a balance between pessimism and optimism. At present we feel that if the resonance strengths in the SSC main ring are kept below about 5, then about 78 Siberian Snakes would be sufficient to maintain polarization. However, since the calculations here indicate a quadratic dependence on the resonance strength, if resonance strengths could be kept below 3, then only about 26 snakes would be needed. These numbers are estimates, and with further calculation we may find a solution which lowers the number of Siberian Snakes. This paper should be viewed as a brief introduction to the problem and as a 'road map' to the many excellent contributions to these proceedings by the members of the working group.

1. INTRODUCTION

The task of the working group on polarization in the SSC main ring was to determine the conditions for which polarization could be maintained from 1 TeV to 20 TeV. This is quite a formidable task since it amounts to an extrapolation by a factor of a thousand from existing polarized proton beams. This extrapolation is made possible by the invention of the Siberian Snake[1] which is a completely different technique for dealing with depolarizing resonances from that presently used at the AGS at BNL. For storage rings up to about 1 TeV it is expected that one could maintain polarization with 2 Siberian Snakes. However, there is

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still much work to be done even for this more modest goal. Perhaps the most important result of this workshop is the study which it has stimulated, both in understanding 2 Siberian Snakes with acceleration and in understanding the use of many snakes to cure the stronger resonances at energies up to 20 TeV. Before beginning a discussion of the workshop it is useful to do some ground work with a brief introduction to the problems of accelerating polarized beams. This introduction will be descriptive. If more detail is desired, we refer the reader to Refs. 2 and 3.

1.1 Depolarizing Resonances

The primary difficulty in accelerating polarizing beams is the presence of depolarizing resonances. These occur when the precession frequency of the spin of a particle is equal to the frequency of the fields it sees on its orbit. The two main types of resonances are imperfection resonances, those driven by errors in quadrupole placement or strength, and intrinsic resonances, those driven by the horizontal magnetic fields that a particle sees on its orbit while performing betatron oscillations.

The resonance condition for these types of resonances can be written

$$\gamma G = k_1 + k_2 \nu_y \tag{1}$$

where k_1 and k_2 are integers, ν_y is the vertical betatron frequency, γ is the energy in units of the rest mass, and G is the anomalous magnetic moment coefficient

$$G = \frac{g-2}{2} \quad . \tag{2}$$

The values of k_1 and k_2 which correspond to the different types of resonances are given by:

1. Intrinsic resonances $\Longrightarrow k_2 = \pm 1$, k_1 is a multiple of the periodicity of the ring.
2. Imperfection resonances
   (a) Vertical closed orbit $\Longrightarrow k_2 = 0$
   (b) Gradient errors $\Longrightarrow k_2 = \pm 1$

Actually, the number of possible resonances is much larger. The general nonlinear resonance condition can be written

$$\gamma G = k_1 + k_2 \nu_y + k_3 \nu_x + k_4 \nu_s \quad , \tag{3}$$

where ν_x and ν_s are the frequencies in the horizontal and longitudinal direction respectively.

From the resonance equations it is evident that as the beam is accelerated many resonances are crossed. In the case of the SSC many thousand resonances are crossed. In addition, these resonances increase in width with increasing energy. Thus, we need Siberian Snakes to cure the problem.

1.2 SIBERIAN SNAKES

A snake is a series of magnets whose net effect is to rotate the spin of a particle about some axis. First consider one pair of snakes separated by a bending angle of π. The first snake rotates the spin of a particle by π about the horizontal axis while the second snake rotates the spin by π about the longitudinal axis. In this case the equilibrium spin direction is up in one half of the ring and down in the other half. In addition, and perhaps more importantly, in an ideal machine the spin tune $\nu_p = 1/2$ independent of energy. For the case of N pairs of equally spaced snakes $\nu_p = N/2$; thus, to keep ν_p far from an integer resonance N must be odd.

If the machine is not perfect, the equilibrium spin direction and the spin tune are shifted and resonances can reappear even for odd N. To distinguish this type of resonance we will call them *snake resonances.* In this case the general resonance condition in Eq. (3) becomes

$$\nu_p = k_1 + k_2\nu_y + k_3\nu_x + k_4\nu_s \quad . \tag{4}$$

To call attention to the nominal value of ν_p, we set

$$\nu_p = N/2 + \delta \quad . \tag{5}$$

Although the driving resonances discussed in Eq. (1) are eliminated by the snakes, their effect is still felt in that they cause a spin tune shift δ.[4] The size of the shift depends upon the size and proximity of the driving resonances.

There are three basic approaches to avoiding the effects of snake resonances. First, we could attempt to avoid them. For example to avoid integer snakes resonances $\nu_p = k_1$, we must require

$$|\delta| < 1/2 \quad . \tag{6}$$

Secondly, we could hope to cross the snake resonances so quickly that they give no depolarization. Finally, we could hope to cross them very slowly which would lead to adiabatic behavior and spin flips.

In the second two cases one would have to use both techniques since the strengths of the resonances vary substantially and the rate of change of δ changes depending upon the proximity of the old spin resonances from Eq. (1). In addition, we do not yet understand passage through snake resonances in detail, although with these proceedings we have improved our understanding substantially. These considerations will lead us in the next sections to attempt to avoid snake resonances, and for simplicity we shall seek to avoid only integer snake resonances.

2. SCALING AND THE NUMBER OF SNAKES

To estimate the number of snakes necessary to 'cure' a spin resonance of a given strength, first consider 2 snakes. For the sake of comparison with Ref. 5 consider 2 snakes with rotation axes +45° and −45° from the longitudinal direction and in the plane of the ring. From Ref. 4, or Ref. 5 and Ref. 6 in these proceedings, the spin tune for this case at $\gamma G = k$ is given by

$$\cos \pi\nu_p = -\sin^2(\pi\epsilon/2)\cos 2\beta \quad , \tag{7}$$

where ϵ is the magnitude of the strength of the driving resonance and β is the negative of the argument of the strength,

$$\epsilon_k = \epsilon e^{-i\beta} \quad . \tag{8}$$

Note that the notation in Ref. 6 and Ref. 5 is slightly different; we use the notation of Ref. 5. For small ϵ the spin tune ν_p is close to 1/2 and quite far from integer resonances. Integer resonances reappear for $\epsilon = 1$ provided that the phase factor $\beta = \pi/2 \bmod(\pi/2)$.

Next consider N pairs of snakes equally spaced around the SSC. Since this case is much more difficult to solve in general, we will consider a simple subset of the resonances. We imagine that we are at an integer driving resonance which is an integer multiple of the number of pairs of snakes ($\gamma G = kN$).

Since the resonance has the same superperiodicity as the number of pairs of snakes, it is only necessary to calculate the spin transfer matrix for $\Delta\theta = 2\pi/N$. The full turn can be constructed by simple matrix multiplication. This is analogous to the case for betatron oscillations in a lattice with superperiods. If we solve for the spin tune in this case, we find

$$\cos(\pi\nu_p/N) = -\sin^2\left(\frac{\pi\epsilon}{2N}\right)\cos 2\beta \quad . \tag{9}$$

If we compare Eq. (9) for $N = 3$ with Eq. (5) in Ref. 5, we find that they are identical for the superperiodic resonances.

It is interesting to use Eq. (9) for scaling the number of snakes necessary to keep the spin tune away from integers. Let us assume that we are willing to accept a small spin tune shift of δ; thus, we set

$$\nu_p = N/2 + \delta \quad . \tag{10}$$

Then for $\epsilon/N \ll 1$ we find

$$\delta \simeq \frac{\pi\epsilon^2}{4N}\cos 2\beta \quad . \tag{11}$$

Therefore, if require a *fixed* tune shift to avoid integer resonances, the number of snakes must increase quadratically with ϵ,

$$N \propto \epsilon^2 \quad . \tag{12}$$

Remember that this is only true for those integer resonances which are multiples of N. Physically it is easy to understand the origin of this difficulty. In the case of perfect superperiodicity, the spin tune shift adds coherently for each superperiod. This leads to a large spin tune shift and integer resonance. The key point here is that the *total* spin tune ν_p must be kept far from an integer to avoid integer snake resonances.

For the case of the other resonances $\gamma G = k$ the snakes work better as is easily seen in Ref. 5. From Figs. 1 and 2 in Ref. 5 the lowest tolerable resonance strength occurs when the driving resonance frequency is a multiple of the number of pairs of snakes, in this case 3ℓ. But at other driving resonance frequencies the tolerable resonance strength is somewhat larger. However, it is easy to verify that the most restrictive case, the superperiodic resonance, in Ref. 5 follows the scaling law in Eq. (12).

3. Results of the Working Group

In this section we summarize the results of the working group. The main task was to determine the acceptable resonance strength for the SSC and, given that strength, to determine the number to snakes needed to preserve the polarization. During the workshop we recognized that much is still unknown about multiple snakes and in particular acceleration through resonances in the presence of snakes. This has led to new work after the workshop much of which is contained in these proceedings and will be briefly discussed in this section.

3.1 Resonance Strengths

The task here was to set a tolerance on the acceptable resonance strength. To do this we can first study one pair of snakes and then use the scaling law from Eq. (11) to generalize to many identical pairs of snakes.

J. Buon presented results of tracking studies with one pair of snakes that indicated depolarization for ϵ as low as 0.25 in the case of intrinsic resonances.[6] However, he also found that much larger imperfection resonances could be crossed with adiabatic behavior. The first results, however, were very sensitive to the vertical betatron tune selected.

S. Y. Lee and S. Tepikian have recently completed new work that explains the source of the snake resonances shown in Eq. (4).[7] In particular they show that the higher order resonances are excited by simple *linear* betatron oscillations. In addition, they find that for one pair of snakes that resonances strengths up to 3 are tolerable provided that snake resonances are avoided.

In the face of these differing results we will take the view that we should avoid *integer* snake resonances. For the case of one pair of snakes this corresponds to a resonance strength $\epsilon = 1$. For the case of many pairs of snakes we will use Eq. (11).

In Ref. 8 the resonance strengths for intrinsic and imperfection resonances are calculated and are presented here in Figs. 1a and 1b. Fig. 1a shows resonances calculated for the nominal $\beta^\star = 1\,\mathrm{m}$ while Fig. 1b shows the resonance strengths for an increased $\beta^\star = 5.66\,\mathrm{m}$. The effect of increasing the $\beta^\star$ is to decrease the resonance strengths by about 30 %. The calculations were preformed with maximum alignment errors of ± 0.1 mm in the arcs and ± 0.05 mm in the insertions. Gradient errors of 0.1 % were also included.

The maximum intrinsic resonance strength calculated ranges from about 8 for $\beta^\star = 1\,\mathrm{m}$ to about 6 for $\beta^\star = 5.66\,\mathrm{m}$. However, the intrinsic resonances were calculated with a normalized emittance $\mathcal{E}_N = 10\pi \times 10^{-6}$ m. The design value for the *rms emittance* of the SSC is

$$\epsilon_N^{rms} = \pi \times 10^{-6}\,\mathrm{m} \quad .$$

It is customary to calculate the intrinsic resonance strengths for an emittance which includes about 95 % of the beam. This would lead us to a value

$$\mathcal{E}_N \simeq 6\,\epsilon_N^{rms} \quad , \tag{13}$$

and therefore

$$\mathcal{E}_N = 6\pi \times 10^{-6}\,\mathrm{m} \quad . \tag{14}$$

Thus, since intrinsic resonance strengths scale like $\sqrt{\mathcal{E}_N}$, we can scale the intrinsic resonance strengths in Figs. 1a and 1b by a factor of about 80 %. This

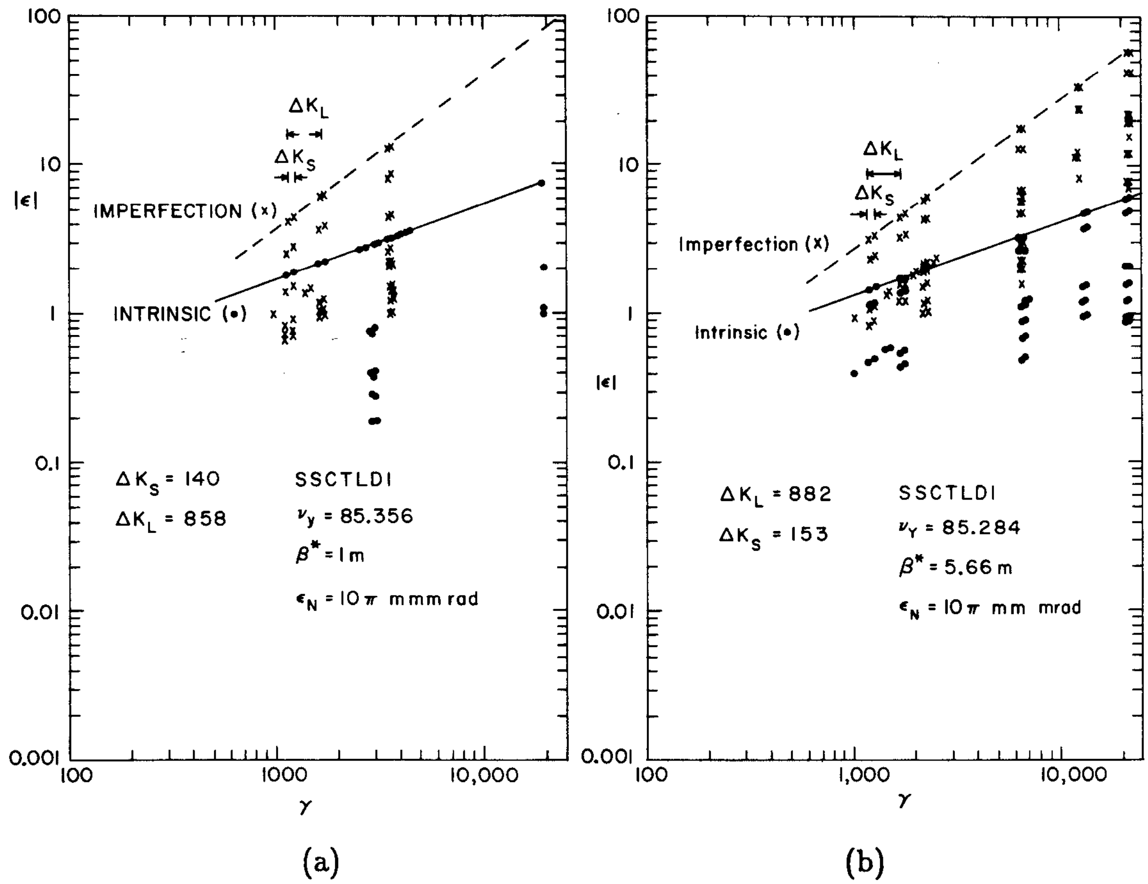

Fig. 1 The calculation for the SSC resonance strengths from Ref. 8.

yields estimates for the maximum resonance strength of about 6.2 for $\beta^\star = 1$ m and 4.6 for $\beta^\star = 5.66$ m.

The maximum imperfection resonance strengths calculated for an SSC lattice with an increased beta function at the interaction point ($\beta^\star = 5.66\,\mathrm{m}$) is $\epsilon_{\max} \simeq 60$, an order of magnitude larger than the intrinsic resonances. Since there was no correction of the resulting closed orbit, the imperfection resonances calculated are probably too large. With a proper orbit correction, they might be reduced by an order of magnitude to levels similar to the intrinsic resonances.

If this is not possible using standard orbit correction techniques, one could envisage an orbit correction scheme which uses the polarization of the beam as a feedback mechanism. This technique is familiar from experience with the ZGS and the AGS. However, with the very large resonances expected here the details would have to be somewhat different since resonance overlap would be a serious problem. In any case, the measurement time would have to be reasonably short. Since the acceleration time would be about 1000 sec in an SSC, we would like the polarization measurement in a much shorter time.

3.2 Lattices Changes and Snake Design

K. Steffen discusses two alternative schemes for placing the snakes around the SSC in Ref. 9. In the case of distributed interaction regions (6-fold periodicity) he finds a solution which localizes the dispersive effects by leaving out every 45^{th} bending magnet. This yields 102 spaces for snakes.

For clustered interaction regions the solution is 40 missing magnets in each arc and 8 in each interaction region which yields a total of 96 spaces for snakes. Finally he discusses the interaction region design with a view towards making the IR spin transparent. Thus, it seems straightforward to obtain space for up to about 100 snakes in the SSC.

3.3 Other Problems and Possibilities

Nonlinear Resonances

During the workshop the width of a sextupole induced resonance was estimated and found to be quite small ($\epsilon \simeq 10^{-3}$). In addition, the resonance strengths in this case are independent of energy.

The resonance strengths induced by higher order multipole errors in the bending magnets should also be quite small and should decrease with increasing energy due to the adiabatic damping of the emittance.

Other Compensation Methods

During the workshop A. Chao studied a proposal by Derbenev and Kondratenko[10] to vary the bend angle from cell to cell in order to reduce resonance strengths. This technique was considered to be too complicated and would require excessively large variations in magnetic field ($\Delta B/B \simeq 0.01$).

Resonance jumps received very little attention because it is generally thought that $\Delta\nu \simeq 0.25$ is about the maximum practical tune change if we are to avoid betatron resonances. This tune change can only help those resonances for which $\epsilon \ll \Delta\nu$. Since these resonances are quite small anyway, they will be handled quite well by the snakes.

Terrain Following

There was some discussion at the workshop about terrain following. It was generally agreed that we *prefer a flat SSC.* However, J. Buon in Ref. 11 discusses a technique to render vertical bends harmless. Unfortunately, each bend requires about 1/2 of a snake to do this. But a few vertical bends could be handled effectively by this technique.

Beam-Beam Depolarization

Beam-Beam depolarization was not studied during the workshop. This is a very complicated and difficult question, and it is hampered by the general lack of understanding of the beam-beam effect on the particle *orbit*. However, there is evidence and theory to suggest that the beam will only depolarize at the beam-beam limit and thus polarized operation might not limit luminosity at all.

Spin Rotators

Spin rotators for the interaction regions were not studied at the workshop; however, these are well understood and should be straightforward. In addition, K. Steffen in Ref. 9 considered another possibility in an SSC with a vertical crossing angle; that is, using the vertical bend to orient the spin. This has some limitations in that it is energy dependent; however, it is an interesting and elegant alternative to separate spin rotators.

4. CONCLUSIONS

A primary uncertainty in the preceding analysis is the size to which the imperfection resonances can be reduced. To reduce them by an order of magnitude without orbit correction would require a tolerance of $\pm 10\,\mu$m on quadrupole placement. Of course, this value is an order of magnitude smaller than is typically achieved. A standard orbit correction scheme will certainly reduce the resonances somewhat, but probably not by an order of magnitude. The primary difficulty is that the imperfection resonance strength increases linearly with energy, while the effect of quadrupole misalignments on the orbit is independent of energy since the magnetic field scales proportional to the beam momentum. To achieve the desired sensitivity we probably must invoke a correction system using beam polarization as feedback. With this we may decrease the strong imperfection resonances to the range 5 to 10.

Using Eq. (11) described in Section 2, if we allow a maximum resonance strength of 5, we would need about 39 pairs of snakes *to avoid integer snake resonances.* This number could be reduced either by reducing resonance strengths or by crossing integer snake resonances.

In spite of the preceding considerations there is also some hope for reducing the number of snakes by reducing the periodicity of the snake configuration by using families of snakes larger than 2. This technique is suggested in Ref. 7. They also suggest that as few as 6 to 12 snakes might be needed in the SSC provided that higher order snake resonances are avoided. I believe that this estimate is too low, but with careful design of snake configurations and by crossing some integer snake resonances it may be possible to reduce the number of snakes further. However, in the continuing analysis of polarization possibilities for the SSC, it will certainly be necessary to include the effects of overlapping resonances, and it will be especially important to include machine errors which repeat only once each turn since these drive the integer snake resonances.

To conclude I would like to thank Alan Krisch and Owen Chamberlain for organizing a stimulating and interesting workshop and also the members of the working group for their many excellent contributions to the workshop proceedings.

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