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RESTRICTIONS ON TWO-HIGGS MODELS FROM HEAVY QUARK SYSTEMS*

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ABSTRACT

We obtain bounds on charged Higgs masses and couplings in models with two Higgs doublets by considering their effect on neutral B meson mixing. Even with the present fairly loose experimental constraints, the bounds are comparable to those obtained with additional assumptions from the neutral K system. Neutral Higgs effects on the spectrum and wave functions of toponium are examined in the same model. In the future they could lead to restrictions on, or discovery of, the corresponding neutral Higgs bosons if they have relatively low masses and enhanced couplings.

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1. Introduction

While even the single neutral physical Higgs boson of the standard model[1] is yet to be found, there is considerable speculation that the Higgs sector is to be enlarged,[2] if not to be replaced altogether by dynamically generated states which are only one manifestation of a whole spectrum of particles due to an additional kind of strong interaction.[3] At a less dramatic level, currently interesting models involving left-right symmetric gauge theories,[4] or supersymmetry,[5] for example, call for an enlargement of the Higgs sector to involve at least two Higgs doublets.

In a theory with two Higgs doublets we gain four more physical bosons, two charged and two neutral. At the same time there is an additional parameter in a second vacuum expectation value, or, more conveniently, a ratio of vacuum expectation values if we fix one appropriate combination to be that of the standard model. Tuning this ratio of vacuum expectation values allows one to enhance (or suppress) the strength of the physical Higgs couplings and thereby to increase (or decrease) the size of the effects these additional bosons have on various processes.

Abbott, Sikivie, and Wise[6] showed that useful bounds on the enhancement of the couplings of the charged Higgs bosons in such a model could be set by considering their effect on the $K_S^0 - K_L^0$ mass difference. Because the charged Higgs bosons couple proportionally to the mass of the fermion and their contributions are not subject to a GIM cancellation,[1] they potentially give a large short-distance contribution to this mass difference through their presence together with heavy quarks in the relevant one loop diagrams. In the case of the $K_S^0 - K_L^0$ mass difference it is the charm quark which is responsible for most of the short-distance contribution and therefore the charm quark mass which enters the bound derived in this manner.

More recently, the bounds derivable from the imaginary, i.e. CP violating, part of the neutral K mass matrix have been investigated.[7] Here the top quark plays a dominant role, and the resulting bounds are much stronger than those of Abbott, Sikivie and Wise,[6] if the assumption is again made that the short-distance contribution due to diagrams involving Higgs exchange is less than that due to W exchange. However, it is altogether possible to contemplate dropping this last requirement, in which case the Higgs exchange diagrams could become the primary source of CP violation in the neutral K mass matrix, and a fairly large range of Higgs masses and couplings is opened up.

In this paper we obtain the bounds on masses and couplings of charged Higgs bosons in a two doublet model that follow from their effect on neutral B meson mixing, i.e. the $B_S^0 - B_L^0$ mass difference. Again, virtual t quarks play the dominant role. However, in this case we obtain useful bounds independent of assumptions on the relative magnitude of the short distance contributions. Furthermore, as shown in Section II, even with the present fairly loose experimental constraints on $B^0 - \overline{B}^0$ mixing, we obtain quite stringent bounds. They are comparable to the best bounds[7] obtained previously in the neutral K system with the additional assumption discussed above on the relative magnitude of Higgs and W contributions.

In Section III we turn our attention to the neutral Higgs particles. We investigate in some detail a subject looked at previously: the effect of neutral Higgs boson exchange on the spectrum and wavefunctions of toponium.[8] We consider in particular the problem of unambiguously distinguishing the effects of the Higgs boson from the effects of different, but theoretically acceptable, potentials. The net restrictions following from having considered both charged and neutral Higgs bosons are summarized in Section IV.

2. Limits from $B^0 - \overline{B}^0$ mixing

As we have mentioned, many modifications and extensions of the standard model require extra Higgs multiplets. We shall be considering here the specific model with two Higgs doublets, although much of what we do can easily be extended to more drastic additions to the standard model.

In any model with extra Higgs doublets, care must be taken to preserve the property that there be no flavor changing neutral currents at tree level. This can be accomplished in two ways. First, we can have one neutral Higgs field coupled to charge $\frac{2}{3}$ quarks and another Higgs field coupled to charge $-\frac{1}{3}$ quarks.[9] In this case the coupling of the physical charged bosons is given by[6]

$$\mathcal{L}_{\text{int}} = \frac{g\phi^+}{2\sqrt{2}M_W}\overline{U}\left[\frac{\xi}{\eta}M_u K(1-\gamma_5) + \frac{\eta}{\xi}KM_d(1+\gamma_5)\right]D + H.c., \qquad (2.1)$$

where η and ξ are the vacuum expectation values of the unmixed Higgs fields coupled to charge $\frac{2}{3}$ and $-\frac{1}{3}$ quarks, respectively. The 3×3 matrix K is the Kobayashi-Maskawa (K-M) matrix,[10] and M_u and M_d are diagonal mass matrices for the three charge $\frac{2}{3}$ and $-\frac{1}{3}$ quarks U and D, respectively.

Second, we can avoid flavor changing neutral currents by having just one Higgs doublet couple to quarks,[11] as in the standard model. In this case the neutral Higgs couplings are diagonalized along with the mass matrix and the charged Higgs couplings are given by[6,11]

$$\mathcal{L}_{\text{int}} = \frac{g\phi^+}{2\sqrt{2}M_W}\overline{U}\left[\frac{\xi}{\eta}M_u K(1-\gamma_5) - \frac{\xi}{\eta}KM_d(1+\gamma_5)\right]D + H.c., \qquad (2.2)$$

Since for the second and third generations the mass of the charge $\frac{2}{3}$ quarks is much greater than that of the charge $-\frac{1}{3}$ quarks in the same generation, it is the term proportional to $\left(\frac{\xi}{\eta}\right) M_u$ in either Eq. (2.1) or (2.2) which gives the possibility of a significant enhancement of the Higgs couplings between light and heavy quarks. Therefore it is this term upon which we have the best possibility of imposing bounds from experimental constraints. Henceforth we shall concentrate on its effects on physical quantities, thereby bounding $\frac{\xi}{\eta}$.

The first bounds on $\frac{\xi}{\eta}$ in models with two Higgs doublets came[6] from looking at the $K_S - K_L$ mass difference and in particular the short-distance contributions to this mass difference arising from the box diagrams with heavy quarks and W's or Higgs bosons running around the internal loop (see Fig. 1). The usual contribution involving W's leads to an effective operator with a coefficient which because of the GIM cancellation[1] behaves as $G_F^2 m_q^2$, aside from factors coming from the K-M matrix. That involving Higgs bosons on the other hand, has no GIM cancellation and behaves as $G_F^2 \left(\frac{\xi}{\eta}\right)^4 \frac{m_q^4}{M_H^2}$ aside from the same K-M factors. Thus, if we impose the condition that the short-distance contribution from the diagrams involving Higgs bosons be less than that due to diagrams involving W's, we will characteristically arrive at bounds of the form $\left(\frac{\xi}{\eta}\right)^2 < \mathcal{O}\left(\frac{M_H}{m_q}\right)$. In the case of the $K_S - K_L$ mass difference, the K-M angle factors make the charm quark the origin of the most important short-distance contributions and the bound that results in this case[6] is $\left(\frac{\xi}{\eta}\right)^2 < \mathcal{O}\left(\frac{M_H}{m_c}\right)$.

If we turn instead to the imaginary, CP violating, part of the mass matrix for the neutral K system, then the top quark plays a leading role. The resulting bounds that follow[7] from making a similar assumption on the magnitude of Higgs exchange contributions versus those due to W exchange are of the form $\left(\frac{\xi}{\eta}\right)^2 < \mathcal{O}\left(\frac{M_H}{m_t}\right)$. Since $\frac{m_t}{m_c}$ appears experimentally[12] to be about 30, these bounds on $(\frac{\xi}{\eta})^2$ are "better" by approximately this factor. However, there is nothing sacred in making the assumption that the Higgs contributions are less than those due to W's. If we were to drop this assumption, and instead just demand consistency with the observed real and imaginary parts of the neutral K mass matrix, then the above bounds are no longer in force, and we are able to use the freedom in values of the K-M angles (particularly $\sin\delta$) to obtain a fairly wide range of Higgs masses and values of $\frac{\xi}{\eta}$.

We can avoid the necessity of making such a assumption by going to the neutral B meson system. Here the t quark contribution is completely dominant in the expression for the mass difference, since it is weighted by K-M angle factors whose magnitude is like those for the charm quark, but $m_t^2 \gg m_c^2$. Furthermore, the freedom in choosing matrix elements and in K-M angle related factors is considerably smaller (there is negligible dependence on $\sin\delta$) than in the K meson system. Thus we can expect a bound of the form $(\frac{\xi}{\eta})^2 < \mathcal{O}(\frac{M_H}{m_t})$ without additional assumptions on the relative magnitude of the Higgs and W exchange contributions.

Now we proceed to analyze the $B^0 - \overline{B}^0$ system in detail. The off-diagonal element of the mass matrix between states whose quark content is $b\bar{d}$ and $d\bar{b}$ has both a dispersive and an absorptive part. It was already known[13] that $|\Gamma_{12}/M_{12}| = \mathcal{O}(\frac{m_b^2}{m_t^2}) \ll 1$ for the box diagram contribution involving W's. We have checked that this also true for the Higgs contribution. Therefore $|\Gamma_{12}| \ll |M_{12}|$ and $\Delta M = M_{B_L} - M_{B_S} = 2|M_{12}|$. The short distance contributions to M_{12} are easy to transcribe from those for the K system:[6,13]

$$M_{12}^{WW} = \frac{G_F^2 f_B^2 m_B B_B}{12\pi^2}(U_{tb}^* U_{td})^2 m_t^2 \tag{2.3}$$

$$M_{12}^{WH} = \frac{G_F^2 f_B^2 m_B B_B}{3} (U_{tb}^* U_{td})^2 \left(\frac{\xi}{\eta}\right)^2 (8M_W^2 I_2 + 2I_3) m_t^4 \tag{2.4}$$

$$M_{12}^{HH} = \frac{G_F^2 f_B^2 m_B B_B}{3} (U_{tb}^* U_{td})^2 \left(\frac{\xi}{\eta}\right)^4 I_1 \frac{m_t^4}{M_W^2}. \tag{2.5}$$

Here matrix elements of the effective Hamiltonian have been taken, neglecting[14] terms involving external quark masses and momenta as small compared to the dominant term involving m_t^2 or m_t^4, which alone has been retained.

We have reverted to the usual practice of expressing the matrix element as a factor B_B times its value in the vacuum insertion approximation, $\frac{4}{3} f_B^2 m_B$, where f_B is defined analogously to the pion or kaon decay constants, f_π and f_K, and m_B is the mass of the B meson. The quantities I_1, I_2, and I_3 depend on m_t and M_H and arise from the loop integration; they are given explicitly in the appendix of Ref. 6. The U_{ij} are elements of the Kobayashi-Maskawa matrix.[10] In the excellent approximation of setting the cosines of the angles θ_1, θ_2 and θ_3 equal to unity, the elements of relevance here are $U_{tb} \approx -e^{+i\delta}$ and $U_{td} = \sin\theta_1 \sin\theta_2$.

The connection to experiment is made through the observation that a non-zero value of M_{12} (or Γ_{12}) will result in mixing as the weak eigenstates B_L and B_S with masses M_L, M_S and widths Γ_L, Γ_S will be mixtures of the B^0 and the $\overline{B}^0$. If we use the sign of the lepton charge in the semileptonic decay as an indicator of whether the decaying meson contains a b or $\bar{b}$ quark, then a quantitative measure of the mixing[15] is given by the time integrated probability for decay into a "wrong" sign lepton compared to decay into a "right" sign lepton:[16]

$$r_0 = \frac{\Gamma(B^0 \to l^- + \cdots)}{\Gamma(B^0 \to l^+ + \cdots)}, \qquad \bar{r}_0 = \frac{\Gamma(\overline{B}^0 \to l^+ + \cdots)}{\Gamma(\overline{B}^0 \to l^- + \cdots)}. \tag{2.6}$$

Neglecting the effects of possible CP violation, which should be a good approximation in this case,[13] $r_0 = \bar{r}_0$ and we have the expression

$$r_0 = \frac{(\Delta M)^2 + (\Delta\Gamma/2)^2}{2\Gamma_{av}^2 + (\Delta M)^2 - (\Delta\Gamma/2)^2} \tag{2.7}$$

where $\Delta M = M_S - M_L$, $\Delta\Gamma = \Gamma_S - \Gamma_L$ and $\Gamma_{av} = (\Gamma_L + \Gamma_S)/2$. As noted previously, $|\Gamma_{12}| \ll |M_{12}|$ and so we can neglect $\Delta\Gamma$ compared to ΔM and obtain the result relevant to the case at hand,

$$r_0 = \frac{(\Delta M/\Gamma)^2}{2 + (\Delta M/\Gamma)^2} \tag{2.8}$$

In present experiments one does not tag individual initial B^0 or $\overline{B}^0$ mesons and follow their subsequent semileptonic decay. Instead one looks at production of a pair of hadrons containing initially a b and a $\bar{b}$ quark and measures the net number of same-sign and opposite-sign dileptons that result when both the heavy hadrons undergo semileptonic decay. In a situation where there is an uncorrelated pair of B^0 and $\overline{B}^0$ mesons, the ratio of same-sign to opposite-sign dileptons is[13,15,17]

$$r = \frac{N(l^+l^+) + N(l^-l^-)}{N(l^+l^-) + N(l^-l^+)} = \frac{2r_0}{1 + r_0^2} \tag{2.9}$$

Such would be the case generally at PEP and PETRA. However, when observing the same ratio near threshold where the B^0 and $\overline{B}^0$ are pair produced without other particles, the interference of the decay amplitudes (which are then coherent) results in[15,17]

$$r = r_0. \tag{2.10}$$

This is the situation at CESR where an upper limit on the mixing corresponding to[18]

$$r < 0.30 \tag{2.11}$$

for the $B_d^0 - \overline{B}_d^0$ system has been obtained. Applying Eqs. (2.10)and (2.8), this translates to the bound

$$|\Delta M/\Gamma| < .93. \tag{2.12}$$

With a B lifetime of 1.0 picosecond, we may alternately express this result as $|\Delta M| < 6.1 \times 10^{-13}$ GeV. Note that because the limit is obtained experimentally below the $B_s^0 = \bar{b}s$ threshold we need not worry about another origin[19,20] for the mixing other than that involving $B_d^0 = \bar{b}d$.

Since calculations of r in the standard model without extra Higgs contributions typically yield predictions[21] in the 0.01 to 0.1 range, it is clear already at this point that the short-distance Higgs contribution cannot be many times larger than that due to the usual W contribution, or we will be in violation of the experimental bound in Eq. (2.11). From Eqs. (2.1) and (2.3) we see that

$$\frac{M_{12}^{HH}}{M_{12}^{WW}} = 4\pi^2 \left(\frac{\xi}{\eta}\right)^4 m_t^2 I_1 \approx \frac{1}{4} \left(\frac{\xi}{\eta}\right)^4 \frac{m_t^2}{M_H^2} \tag{2.13}$$

where we have inserted[6] $I_1 = (16\pi^2 M_H^2)^{-1}$, which is good to order m_t^2/M_H^2. Thus we can see that we are headed for bounds of the general form $(\xi/\eta)^2 <$ several $\times$ (M_H/m_t).

Let us now make this more quantitative. For the moment we neglect M_{12}^{HW} and use the approximate expression for I_1 given above. Then noting that M_{12}^{WW}

and M_{12}^{HH} have the same phase, we have that

$$\Delta M = 2|M_{12}^{WW} + M_{12}^{HH}| = 2|M_{12}^{WW}| + 2|M_{12}^{HH}|, \tag{2.14}$$

and using Eqs. (2.1) and (2.3) this becomes:

$$\Delta M = \frac{G_F^2 f_B^2 m_B B_B s_1^2 s_2^2 m_t^2}{6\pi^2} \left(1 + \frac{1}{4}\left(\frac{\xi}{\eta}\right)^4 \frac{m_t^2}{M_H^2}\right). \tag{2.15}$$

With a "nominal" set of values (discussed below) of $m_t = 45$ GeV, $m_B = 5.3$ GeV, $f_B = f_K = 0.16$ GeV, $s_2 = 0.06$, $B_B = 1$, and a B lifetime[22] of 1.0 picosecond, this becomes the bound (shown in Fig. 2, dashed line)

$$\left(\frac{\xi}{\eta}\right)^2 < 4.1 \left(\frac{M_H}{m_t}\right), \tag{2.16}$$

when combined with Eq. (2.12) coming from the experimental bound on the mixing.

We now consider the bound obtained by including M_{12}^{WH} and keeping the full expressions for the quantities I_1, I_2, and I_3 in the equation

$$\frac{\Delta M}{\Gamma} = \frac{2|M_{12}^{WW} + M_{12}^{WH} + M_{12}^{HH}|}{\Gamma} < .93 \tag{2.17}$$

from combining $\Delta M = 2|M_{12}|$ with the experimental limit in Eq. (2.12). The bound that results from Eq. (2.17) is shown as the solid line in Fig. 2 using the same set of "nominal" values of the parameters as before. The approximate result of Eq. (2.15) is quite close to this exact bound, showing that it is M_{12}^{HH} rather than M_{12}^{HW} that is driving the bound. It should be noted at this point that although we have plotted the bound derived from the full expression in Eq.

(2.17) as a function of $\frac{M_H}{m_t}$ to facilitate comparison with previous bounds (e.g. Eq. (2.16) and Ref. 7), the analytic expression depends on M_H and m_t separately and not just on their ratio. We have set $m_t = 45$ GeV/c^2 in plotting Fig. 2, leaving M_H as the variable quantity.

A comment is in order here on the set of "nominal" values of the parameters which we have chosen, and their possible variation. The mass of the B meson is accurately fixed by experiment and we have taken $m_t = 45$ GeV/c^2, in the range suggested by present experimental evidence[12] for the t quark. We equate the B^0 meson lifetime with that determined for a mixture of hadrons containing the b quark, and take[22] 1.0 picoseconds for this "b quark lifetime." In fact, τ_b enters both the value for $\sin\theta_2$ (from the method of determining the K-M angles) and Γ_{B_0} in such a way as to cancel out in $\frac{\Delta M}{\Gamma}$, the quantity of relevance here to the mixing. So, if we use a given lifetime consistently there is no actual dependence on τ_b.

The value of $\sin\theta_2$ is extracted from τ_b, which yields[23] $|\sin\theta_3 + \sin\theta_2 e^{i\delta}| \approx 0.06(10^{-12}\,\text{sec}\,/\tau_b)^{\frac{1}{2}}$, and from the upper limit[24] on $(b \to u)/(b \to c)$, which limits $\sin\theta_3/|\sin\theta_3 + \sin\theta_2 e^{i\delta}| < 0.7$. This still allows considerable latitude in values of $\sin\theta_2$, from roughly $0.02(10^{-12}/\tau_b)^{\frac{1}{2}}$ to $0.10(10^{-12}/\tau_b)^{\frac{1}{2}}$.

The quantities f_B and B_B enter together in the form $\frac{4}{3}B_B f_B^2 m_B$ as the value of the matrix element of the effective operator relevant to the short-distance contribution to $B^0 - \overline{B}^0$ mixing. Several calculations of f_B indicate[25] that $f_B \approx f_K \approx f_\pi$, although substantially larger values[26] have also been used. One can separately argue[13] that $B_B \approx 1$. Alternatively one can look at the value of the whole matrix element. Recent estimates[27] can be rephrased as $B_B \approx \frac{1}{3}$ if we fix $f_B = f_K = 160$ MeV.

Consequently we show in Fig. 3 what happens to the bound under reasonable pessimistic ($B_B = \frac{1}{3}$, $\sin\theta_2$=0.04, other parameters fixed) and optimistic ($B_B = \frac{3}{2}$, $\sin\theta_2$=0.08, other parameters fixed) excursions of the parameters. Even in the "pessimistic case," the bound is quite restrictive ($\left(\frac{\xi}{\eta}\right)^2 \lesssim 12M_H/m_t$). The "optimistic case" may alternatively be viewed as how the bound would improve if the experimental limit were lowered by about a factor of three with all the parameters fixed at their nominal values.

These limits are not far from what was obtained in Ref. 7 using the magnitude of CP violation in the neutral K system, but with the additional assumption in the K system that the Higgs contribution be less than that of the W to ϵ. This is seen in Fig. 4 where this previous bound is shown as the dotdashed line, and the new bound from the B system is shown as the solid line. In both cases we knew in advance that the t quark short-distance contribution is dominant over that of the c quark and consequently the bound will be of the qualitative form $\left(\frac{\xi}{\eta}\right)^2 < \mathcal{O}(M_H/m_t)$. The only question was the detailed number that replaces the order of magnitude: we have found that present limits of the $B^0 - \overline{B}^0$ mixing are already able to make the new bound comparable to the previous one.

Looked at the other way, from the viewpoint of the neutral K system, we see that the Higgs short-distance contribution to ϵ is not many times bigger than the standard short-distance contribution (involving W's). While the most extreme scenarios contemplated in Ref. 7 are thus ruled out, it is still quite acceptable with present limits on $B^0 - \overline{B}^0$ mixing to have a major part of ϵ come from the short-distance contribution involving charged Higgs bosons. In such a situation, as emphasized in Ref. 7, the ratio ϵ'/ϵ is correspondingly reduced from the value it would have in the standard model without additional Higgs.

Therefore small predicted values of ϵ'/ϵ are still possible through the introduction of a second Higgs doublet, even with the bound on the couplings derived here from the $B^0 - \overline{B}^0$ system.

3. Limits from Toponium Spectroscopy

We now move from a discussion of the effects of the charged Higgs to those of the neutral Higgs (with enhanced couplings), particularly on $t\bar{t}$ spectroscopy. Of all $q\bar{q}$ systems, $t\bar{t}$ is the best system to observe the neutral Higgs effects since the Higgs coupling to quarks is proportional to m_q, and relativistic effects are negligible. We begin with a review of heavy quarkonium systems. These systems are well described by treating the quarks as non-relativistic fermions interacting through a simple phenomenological potential, specified by a few parameters determined by fitting to the measured spectra. For the c and b quark systems, a wide range of successful forms have been proposed.[28] A few examples are:

1. Martin:[29]

$$V(r) = (5.82 \text{ GeV}) \left(\frac{r}{1(\text{ GeV})^{-1}} \right)^{.104} \tag{3.1}$$

2. Cornell:[30]

$$V(r) = \frac{-.48}{r} + \frac{r}{(2.34(GeV)^{-1})^2} \tag{3.2}$$

3. Richardson:[31]

$$V(r) = \frac{8\pi}{33 - 2n_f} \Lambda \left(\Lambda r - \frac{f(\Lambda r)}{\Lambda r} \right), \tag{3.3}$$

where

$$f(t) = \left[1 - 4 \int_1^\infty \frac{dq}{q} \frac{e^{-qt}}{[\ln(q^2 - 1)]^2 + \pi^2} \right], \tag{3.4}$$

and n_f is the number of quarks with mass less than the momentum of the bound heavy quarks (the relevant momentum scale for renormalization), and is taken to be 3.

The first potential is motivated purely by the $c\bar{c}$ and $b\bar{b}$ data, while the other two incorporate to some extent the short and long range behavior expected on theoretical grounds.

The consistency of present data with potentials having widely differing analytic forms is not as surprising as it might at first seem. If one adds an appropriate constant to each potential, one finds all potentials to be in very good agreement in the range .1 fm $< r <$ 1 fm—where the RMS radii of the observed charmonium and bottomonium states lie (see Fig. 2 of Ref. 28). Toponium, however, will discriminate between these potentials—its lowest lying state may have a radius of .05 fm or less, depending on the potential, and the predicted level spectra for top vary widely (see Table 1—note that the radii are specified in GeV^{-1}).

Into this somewhat murky situation of differing strong interaction potentials we now introduce the added effects of neutral Higgs boson exchange (Fig. 5). The analogue of Eq. (2.1) for charged Higgs is[32]

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} = & \frac{g}{2M_W}\phi_1\left\{\overline{U}\left[\frac{(\xi^2+\eta^2)^{1/2}}{\eta}M_u\right]U\cos\beta+\overline{D}\left[\frac{(\xi^2+\eta^2)^{1/2}}{\xi}M_d\right]D\sin\beta\right\} \\ & +\frac{g}{2M_W}\phi_2\left\{-\overline{U}\left[\frac{(\xi^2+\eta^2)^{1/2}}{\eta}M_u\right]U\sin\beta+\overline{D}\left[\frac{(\xi^2+\eta^2)^{1/2}}{\xi}M_d\right]D\cos\beta\right\} \\ & +\frac{g}{2M_W}\phi_3\left\{\overline{U}\left[\frac{\xi}{\eta}M_u\gamma_5\right]U+\overline{D}\left[\frac{\eta}{\xi}M_d\gamma_5\right]D\right\},\end{aligned} \tag{3.5}$$

where β is an unknown mixing angle between the two scalar physical fields, ϕ_1^0

and ϕ_2^0. We will concentrate in what follows on the effects of the exchange of the two scalar fields, whose couplings to t quarks are enhanced by factors of $\cos\beta(\xi^2+\eta^2)^{1/2}/\eta$ and $\sin\beta(\xi^2+\eta^2)^{1/2}/\eta$, respectively, over the coupling of the Higgs boson of the standard model. In as much as we are interested in bounds in the regime where ξ/η is large, $(\xi^2+\eta^2)^{1/2}/\eta \approx \xi/\eta$ and the respective couplings are enhanced by factors of approximately $(\xi/\eta)\cos\beta$ and $(\xi/\eta)\sin\beta$. If the two scalar bosons had the same mass, their combined effect would be equivalent to the exchange of a single scalar boson of that mass with a coupling enhanced by a factor ξ/η, the same ratio of vacuum expectation values we bounded previously. In the following we shall work with this latter, simplified situation, realizing that in general our results represent the weighted average of two Higgs boson exchange diagrams.

In momentum space, the diagram in Fig. 5 then corresponds to adding the following term to the spin independent part of the non-relativistic potential:

$$-\left(\frac{\xi}{\eta}\frac{gm_t}{2M_W}\right)^2\frac{1}{m^2+q^2}, \tag{3.6}$$

which gives[33]

$$-\left(\frac{\xi}{\eta}\frac{gm_t}{2M_W}\right)^2\frac{1}{4\pi r}e^{-rM_H} \tag{3.7}$$

in coordinate space. Again, this Yukawa-type attractive potential is to be added to whatever potential is chosen to represent the strong interactions for the $t\bar{t}$ system.

As has been noted before,[8] the energy levels and widths of toponium states will be noticeably shifted by the exchange of a Higgs with enhanced couplings. The qualitative features of its effects follow from it being attractive and having its strongest effect close to the origin (as it dies off exponentially with distance). It tends to pull in wave functions, decrease bound state radii, and increase wave functions at the origin, with its strongest effect being on the lowest lying states whose wave functions are already large in the neighborhood of the origin where the Higgs exchange potential lives.

Thus it is easy to understand the increased $E_{2S} - E_{1S}$ splitting in the presence of Higgs exchange, an effect already noted by Sher and Silverman:[8] the 1S state, with a bigger wave function at the origin to begin with, is pulled down deeper into the potential well than is the $2S$ state by the added Higgs term. However, an inspection of Table I reveals that comparable or larger differences in $E_{2S} - E_{1S}$ are obtained by changing from one strong interaction potential to another. By itself this effect does not decisively point to Higgs exchange as its unique origin.

What happens to the E(2S)-E(1P) separation is not quite as obvious. The situation is elucidated by a theorem of Martin:[34] if $\Delta V(r) = \frac{d}{dr} r^2 \frac{dV}{dr} > 0$ (true for all proposed quarkonia potentials), the nS state lies above the (n-1)P state, while if $\Delta V(r) < 0$ for all r such that $dV/dr > 0$ (true for the Higgs potential), the nS state lies below the corresponding P state. Here we have a qualitative signature of the presence of the Higgs. However, the theorem requires the given condition on $\Delta V(r)$ to hold for all r. (The condition $dV/dr > 0$ holds for both the Higgs and quarkonium potentials.) What happens in our case, where the Higgs only dominates near the origin? We might guess that the energy levels will be inverted if the Higgs term dominates below some relevant radius, perhaps that of the 2S or 1P. As M_H increases, the range of the Higgs potential decreases and we need a larger value of $\frac{\xi}{\eta}$ to keep $\Delta V < 0$. This does give a qualitative picture of what happens. To determine quantitatively the minimum value of $\frac{\xi}{\eta}$ for the level inversion, we numerically solve the Schrödinger equation. After obtaining E($2S$) and E($1P$) for various values of $\frac{\xi}{\eta}$, we interpolate to estimate the value of $\frac{\xi}{\eta}$ at which E($2S$)=E($1P$), which is shown in Fig. 6 for both the Richardson and Cornell potentials. The Cornell potential, which starts with a bigger wave function at the origin, requires a smaller Higgs coupling enhancement to affect the inversion. We find that for large M_H^0 the 2S level is depressed by Higgs-induced effects while the 1P remains much the same. As we decrease M_H the 2S becomes more and more depressed until for very small M_H the Compton wavelength of the neutral Higgs becomes comparable to the size of the $t\bar{t}$ system and the 1P starts to sink almost as fast as the 2S; hence the rise in the curves as we go to very small M_H.

Fairly spectacular effects can be produced in the wave function at the origin, particularly that of the lowest lying S-states. Here the part of the potential which is singular at the origin, i.e., which behaves as $\frac{1}{r}$, would be expected to play the main role. That this is indeed the case is shown in Fig. 7 where the dependence of $|\psi(0)|$ on $\frac{\xi}{\eta}$, for the 1S ground state of the $t\bar{t}$ system is plotted: there is only a very small difference between the results obtained from the full Cornell potential (solid line) and those obtained from its Coulomb-like part alone (dashed line)—note the suppressed zero. Similar results are found for the Richardson potential.

This suggests separating the portion of both the strong interaction and Higgs exchange potentials which are singular as $r \to 0$ and using this combination to determine (approximately) $\psi(0)$. This effective Coulomb potential $-\frac{\widetilde{\alpha}}{r}$ will have strength

$$\widetilde{\alpha} = \frac{4}{3}\alpha_s + \frac{1}{4\pi}\left(\frac{gm_t}{2M_W}\right)^2\left(\frac{\xi}{\eta}\right)^2 . \tag{3.8}$$

Since for the corresponding ground state, $|\psi(0)|^2 \propto (\tilde{\alpha} m_t)^3$, we might expect that

$$|\psi(0)|^{2/3} = |\psi(0)|^{2/3}_{\frac{\xi}{\eta}=0}\left[1 + c(\xi/\eta)^2\right], \tag{3.9}$$

where

$$c = \frac{3}{16\pi\alpha_s}\left(\frac{g m_t}{2M_W}\right)^2 . \tag{3.10}$$

In Fig. 7 we see that the linear behavior expected on the basis of Eq. (3.9) is a fairly good representation of the actual dependence. However, the deduced coefficient of $(\xi/\eta)^2$ is smaller than that predicted by Eq. (3.10), presumably because the characteristic factor of $e^{-M_H r}$ "screens" the full strength of the effective Coulomb piece of the Higgs exchange potential as we move out any finite distance from the point at $r = 0$. Be that as it may, thinking of the situation in terms of a single effective Coulomb potential leads to the qualitative or even semiquantitative understanding of the behavior of $\psi(0)$ shown in Fig. 7. For light neutral Higgs ($M_{H^0} \approx 5$ to 20 GeV/c^2) in particular, $\psi(0)$ changes appreciably, even for moderate values of ξ/η in the case of the Richardson potential (see Table 1).

Fig. 8 shows the effect on $|\psi(0)|$ of Higgs exchange with large $\frac{\xi}{\eta}$ through Z-toponium mixing[35] (which depends on $|\psi_n(0)|^2$) for entire spectrum of nS states (for the Richardson potential), while for comparison Figs. 9 and 10 show the spectra for the Richardson and Cornell potentials, with no Higgs. The differences are fairly striking, although the Cornell potential without Higgs (which has a larger coefficient of $\frac{1}{r}$) partly mimics the effect of adding Higgs exchange to the Richardson potential.

We also show, in Fig. 11, the bump due to the 1S state, smeared by beam energy spread, for various values of $|\psi(0)|_{1S}$, taking M_{V_0} fixed to be above the Z at 98 GeV (see Table 1 for a correspondence of these wavefunction values to $\frac{\xi}{\eta}$ and M_H). As discussed in Ref. 35, the bare width of the 1S is swamped by the width it acquires from mixing; this in turn is less than or near the machine resolution. Consequently the net effect of a larger $|\psi(0)|$ is simply to make the resonance more noticeable.

We conclude, however, that in general it may be far from easy to obtain a useful bound on $\frac{\xi}{\eta}$ from this effect. The study of $B^0 - \bar{B}^0$ mixing in the previous section already places a rather stringent bound on $\frac{\xi}{\eta}$: the changes in levels and wavefunctions in the remaining region of interest are mostly comparable to the differences in these quantities found from use of different potential models.

Still, a careful study, when toponium levels have been measured, might well yield information on the neutral Higgs. Certainly these effects must be borne in mind when the data has been taken, and one attempts to fit it to various potential models.

4. Conclusion

The bounds we have obtained from the $B^0 - \bar{B}^0$ system on the ratio of vacuum expectation values, ξ/η, in the two Higgs doublet model, is a fairly tight one. For charged Higgs masses below ≈ 0.5 TeV (where $\Gamma_H \ll M_H$), we have $\frac{\xi}{\eta} \lesssim 10$, even with some pessimism on the parameters entering the bound. If we narrow the region of interest for M_{H^+} to be the more accessible one below a couple of hundred GeV/c^2, then $\xi/\eta \lesssim 5$ with the nominal set of parameters we have been using. Furthermore, as experimental constraints on $B^0 - \bar{B}^0$ mixing continue to improve, so should the bound.

As we have noted several times, this is comparable to the bound obtained from the neutral K system, but with the added assumption there that the Higgs short-distance contribution to the CP violation parameter ϵ is less than the standard short-distance contribution involving W's. It is also comparable or better than bounds on ξ/η coming from other sources. For example, the bound $\xi/\eta \lesssim 2M_{H^+}/(9m_c m_t)^{1/2}$, derived in Ref. 8 from an assumed agreement of the t-quark semileptonic branching ratio with that of the standard model, is considerably less stringent than ours when $M_{H^+} > m_t$. Recently a bound on ξ/η which is independent of M_{H^+} has been derived[36] from the assumption of perturbative grand unification of SU(3) $\times$ SU(2) $\times$ U(1) with a desert between the weak and unification scales. For values of M_{H^+} below several hundred GeV the bound on ξ/η obtained from the $B^0 - \bar{B}^0$ system is smaller, while for larger M_{H^+} the bound of Ref. 36 is the more restrictive one. Quite tight bounds[37] on ξ/η, also follow from the requirement of stability of the Higgs potential when the lighter neutral scalar Higgs has a low mass.

The limits on ξ/η found from the $B^0 - \bar{B}^0$ system dampen the enthusiasm one feels at first sight for the potentially dramatic effects in the $t\bar{t}$ system due to exchange of a neutral Higgs boson with enhanced couplings, e.g., enlarged E(2S)-E(1S) splittings, enhanced $|\psi(0)|$, etc. Once we restrict ourselves to say, $\xi/\eta < 5$, the effects are not enormous unless M_{H^0} is quite small. Furthermore, exactly in cases where the effects are not large, they are qualitatively similar to the effects obtained by changing from one strong interaction potential to another with a stronger $\frac{1}{r}$ singularity. In this regard, we emphasized the inversion of the

2S and 1P levels as something which is qualitatively different in the presence of a Higgs exchange potential of sufficient strength. But even for this property, Fig. 6 shows that values of $\xi/\eta<5$ are not sufficient to cause this level inversion for the Richardson potential and do so only for small M_{H^0} in the case of the Cornell potential.

Nevertheless, a large value of $M_{H^\pm}$ (yielding a weaker bound on ξ/η) together with a small value of M_{H^0} for at least one of the neutral Higgs bosons in the two doublet model is a possible scenario to contemplate. In such a case, by carefully comparing the $t\bar{t}$ spectrum and wave functions in several of its aspects simultaneously, it still could be possible to sort out the effects of neutral Higgs exchange from those of differing strong interaction potentials.

Acknowledgments

We thank M. Gilchriese for asking the right question about the B meson system, and M. Nelson, T. Schaad, M. Sher, and D. Silverman for discussions about the experimental and theoretical situation.

Potential	M_H	$\frac{\xi}{\eta}$	E_{1s}	$\langle r \rangle_{1S}$	$E_{2s} - E_{1s}$	$E_{2s} - E_{1p}$	$\Psi(0)_{1s}$	$\frac{\Psi(0)_{2s}}{\Psi(0)_{1s}}$	$\Psi'(0)_{2p}$
Richardson	—	0	98.323	.24	.999	.102	8.5	.52	13.0
	5	2	98.277	.235	1.04	.099	9.0	.506	13.6
	5	7	97.609	.18	1.58	.061	15.1	.42	21.9
	10	3	98.259	.231	1.05	.093	9.3	.50	13.8
	10	6	98.027	.204	1.24	.063	12.3	.45	16.4
	10	8	97.708	.177	1.51	.032	16.2	.40	—
	10	11	96.739	.133	2.37	−.037	26.4	.306	29.3
	20	2	98.31	.238	1.009	.099	8.76	.51	13.2
	20	6	98.189	.22	1.103	.075	11.1	.48	14.6
	20	10	97.817	.18	1.41	.016	18.0	.41	17.9
	40	3	98.312	.238	1.007	.099	8.83	.51	13.2
	40	8.2	98.228	.224	1.071	.08	11.4	.49	14.3
	80	7	98.304	.237	1.013	.097	9.47	.51	13.5
	140	7	98.316	.238	1.004	.10	9.01	.51	13.3
Cornell	—	0	97.143	.144	2.226	.015	23.3	.372	30.8
	10	2	97.067	.141	2.29	.010	24.0	.368	31.6
	10	4	96.823	.133	2.51	−.005	26.4	.354	34.3
	40	4	97.055	.14	2.30	.005	24.9	.366	31.6
	100	6	97.09	.141	2.27	.008	24.9	.369	31.4
Martin	—	0	98.628	.419	.455	.127	2.72	.75	4.41
	10	5	98.58	.388	.482	.109	3.74	.70	5.11
	40	5	98.624	.416	.457	.125	3.00	.74	4.56
	100	5	98.627	.419	.456	.127	2.83	.75	4.47

Table 1. Calculated parameters of toponium, for a few different potentials, values of M_H, and $\frac{\xi}{\eta}$; $m_t = 50$ GeV (all units GeV to appropriate powers).

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FIGURE CAPTIONS

1. Box diagrams contributing to $B^0 - \overline{B}^0$ mixing in a two-Higgs doublet model. H is the physical, charged Higgs.

2. Limit on $\left(\frac{\xi}{\eta}\right)^2$ versus the charged Higgs mass from $B^0 - \overline{B}^0$ mixing, for the "nominal" values of parameters given in the text. The dashed line is the approximate bound (see Eq. 2.16), while the solid curve is the full bound.

3. Possible variations due to the use of different parameters in the limit given in Fig. 2. The upper curves correspond to the "pessimistic" case described in the text; the lower to the "optimistic." The corresponding approximate bounds are denoted by dashed lines.

4. Comparison of our limit from Fig. 2 (solid curve) with those of Ref. 7(dot-dash).

5. Neutral Higgs exchange diagram contributing to the binding potential in the $t\bar{t}$ system.

6. Minimum value of $\frac{\xi}{\eta}$ for which $E_{1P} > E_{2S}$, versus Higgs mass, for the Richardson and Cornell potentials.

7. $|\psi(0)|^{2/3}$ versus $(\frac{\xi}{\eta})^2$ for the Cornell potential (solid curve), and its Coulomb part alone (dashed curve), (the light dotted line is straight, for comparison). M_{H^0}=40 GeV.

8. $R(e^+e^- \to \mu^+\mu^-)$ resulting from toponium-Z mixing for the Richardson potential, with m_t=47.5 GeV/c^2, $\frac{\xi}{\eta}$=12, M_H = 10 GeV, convoluted with a gaussian appropriate for σ_{beam}=40 MeV.

9. $R(e^+e^- \to \mu^+\mu^-)$ resulting from toponium-Z mixing for the Richardson potential, with m_t=47.5 GeV/c^2, but no Higgs exchange, convoluted with

a gaussian appropriate for σ_{beam}=40 MeV.

10. $R(e^+e^- \rightarrow \mu^+\mu^-)$ resulting from toponium-Z mixing for the Cornell potential, m_t=47.5 GeV/c^2, but no Higgs exchange, convoluted with a gaussian appropriate for σ_{beam}=40 MeV.

11. $R(e^+e^- \rightarrow \mu^+\mu^-)$ resulting from the 1S resonance, smeared by $\sigma_{beam} = 40$ MeV, for various values of $|\psi(0)|_{1S}$, and a fixed $M_{V_0} = 98 GeV$.

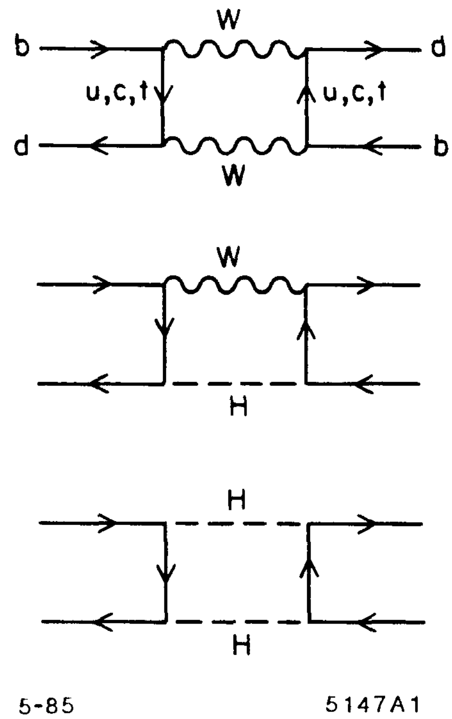


Fig. 1

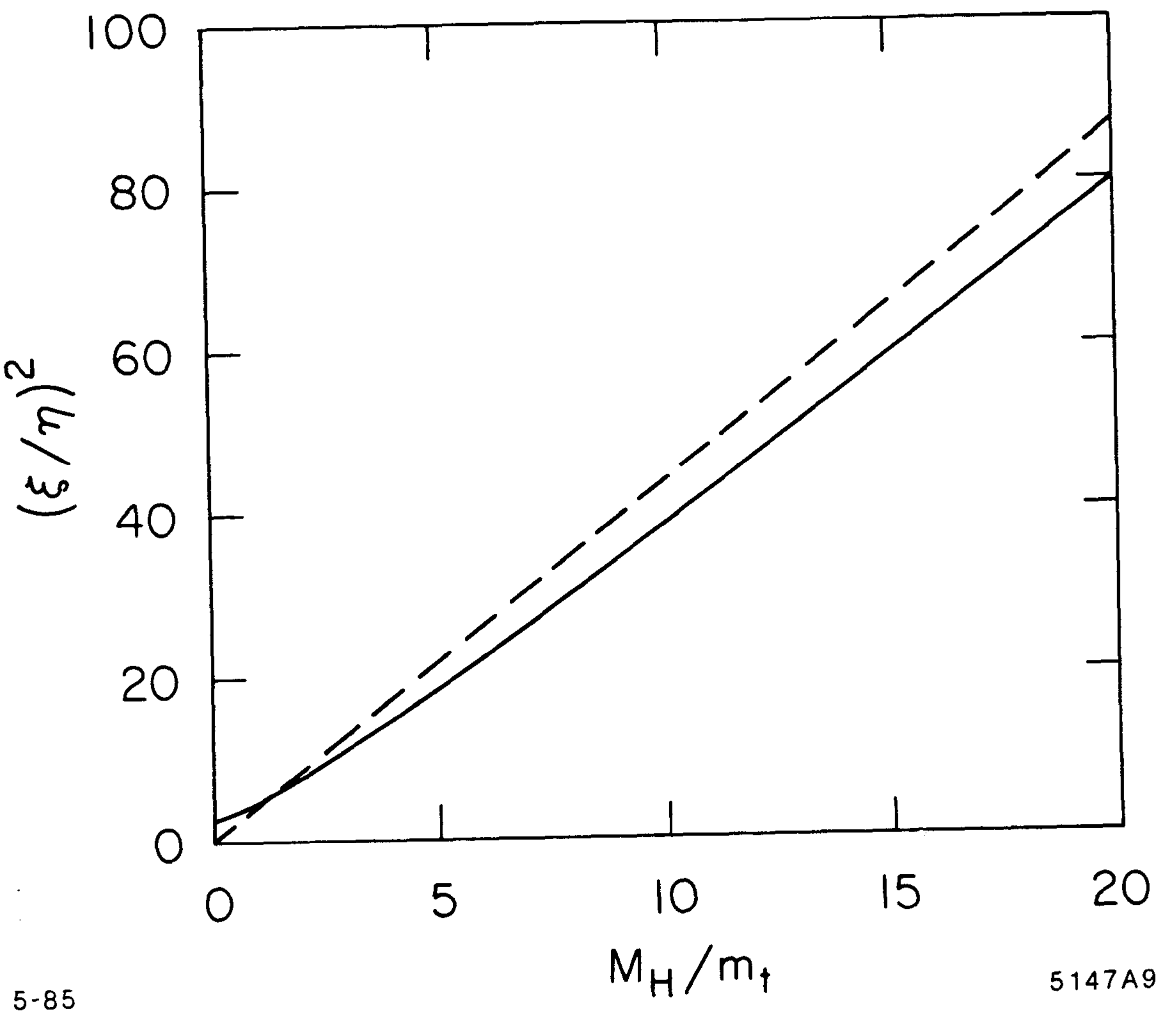


Fig. 2

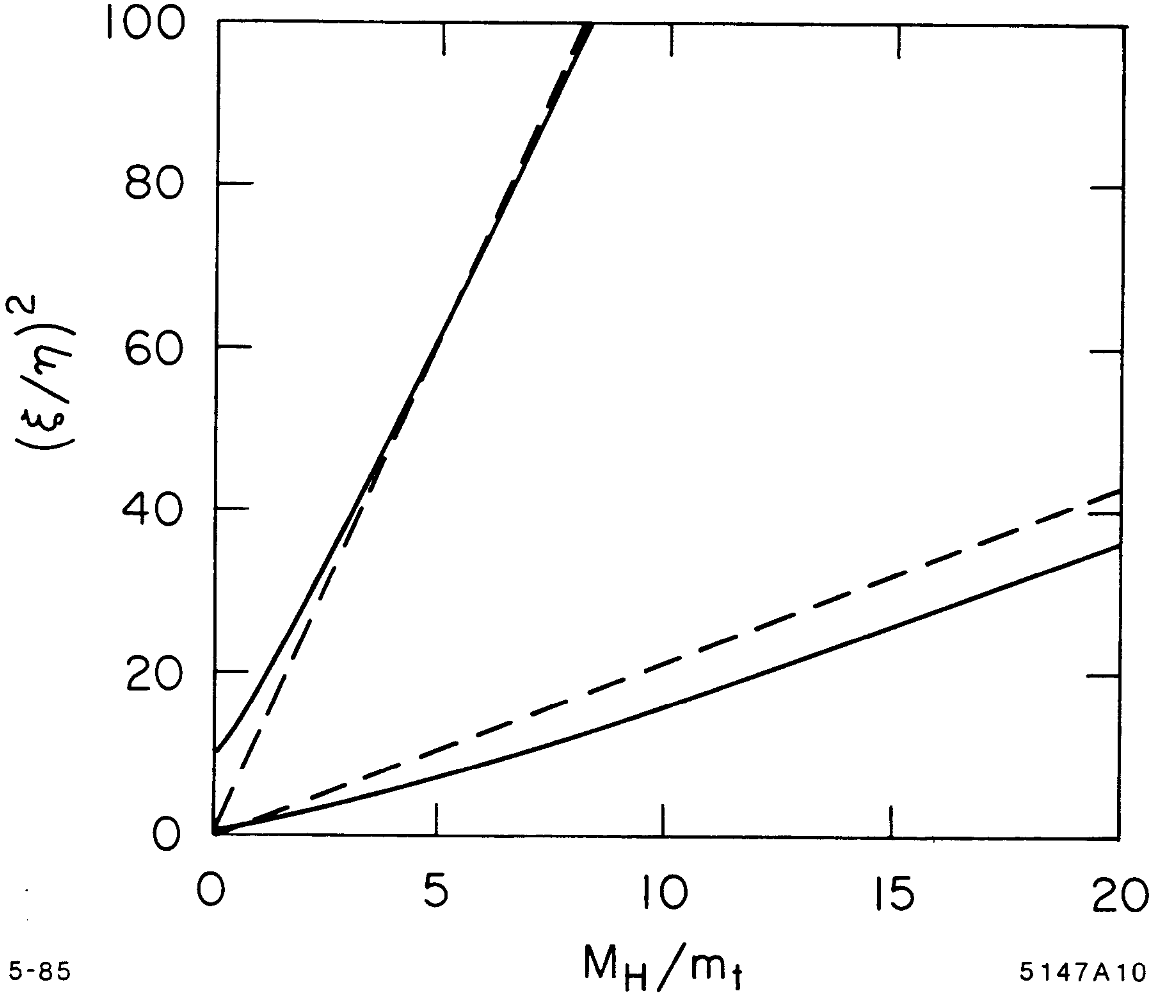
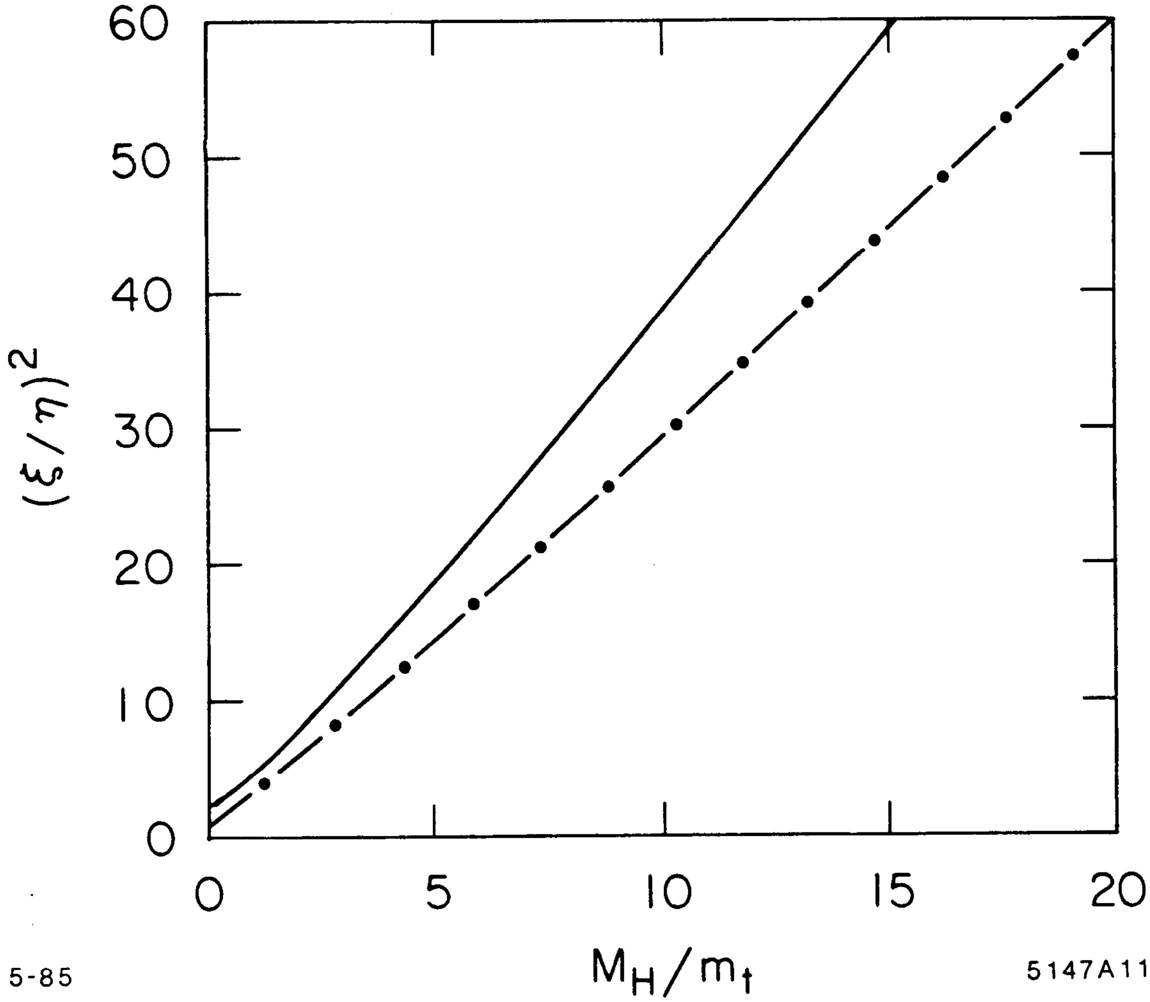


Fig. 3



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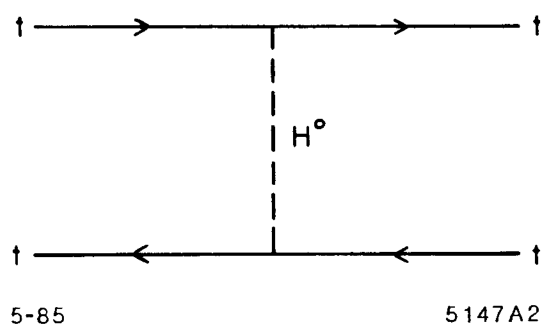

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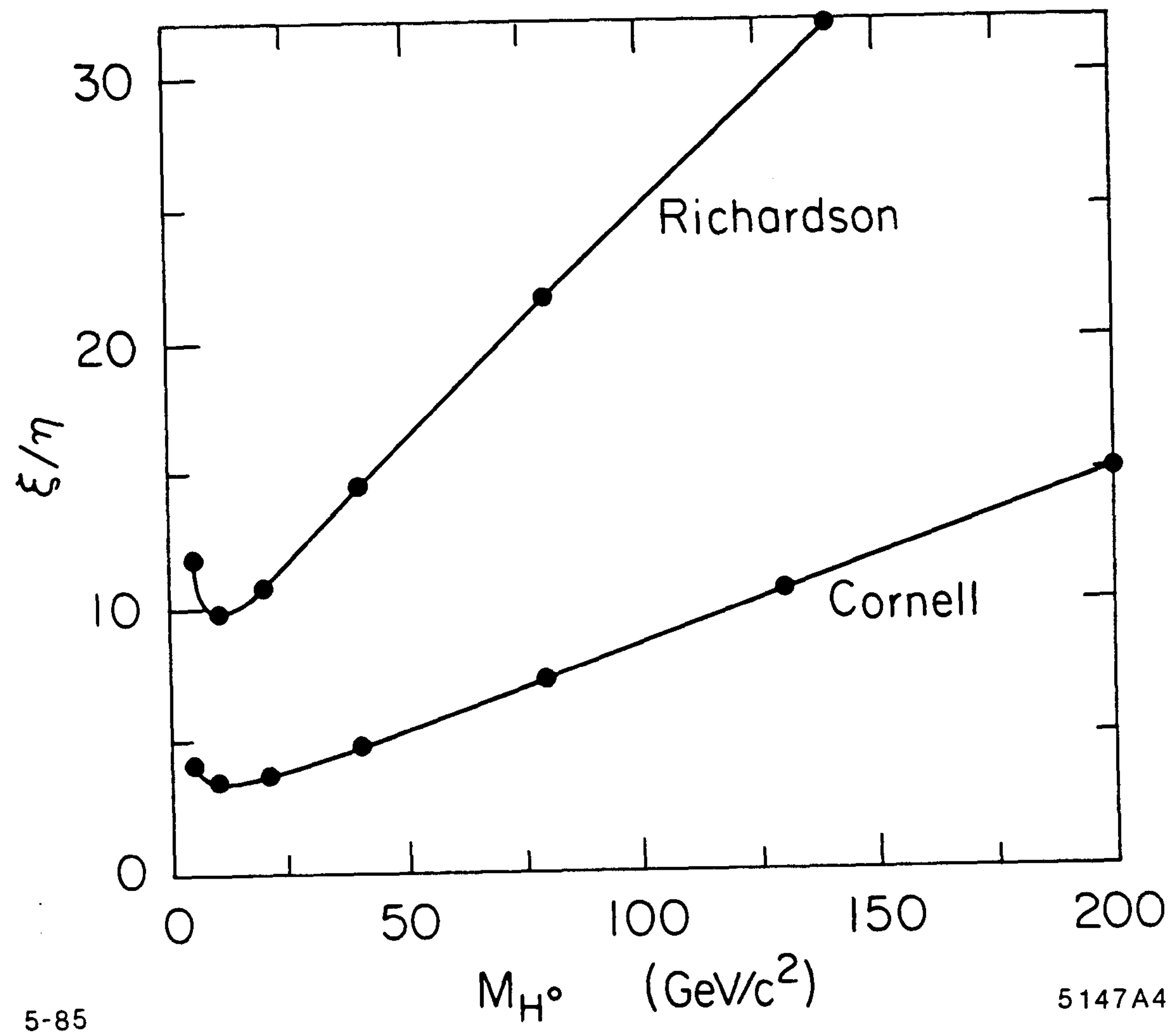

30
20
10
0
ξ/η
Richardson
Cornell
0
50
100
150
200
M_{H° (GeV/c^2)
5-85
5147A4

Fig. 6

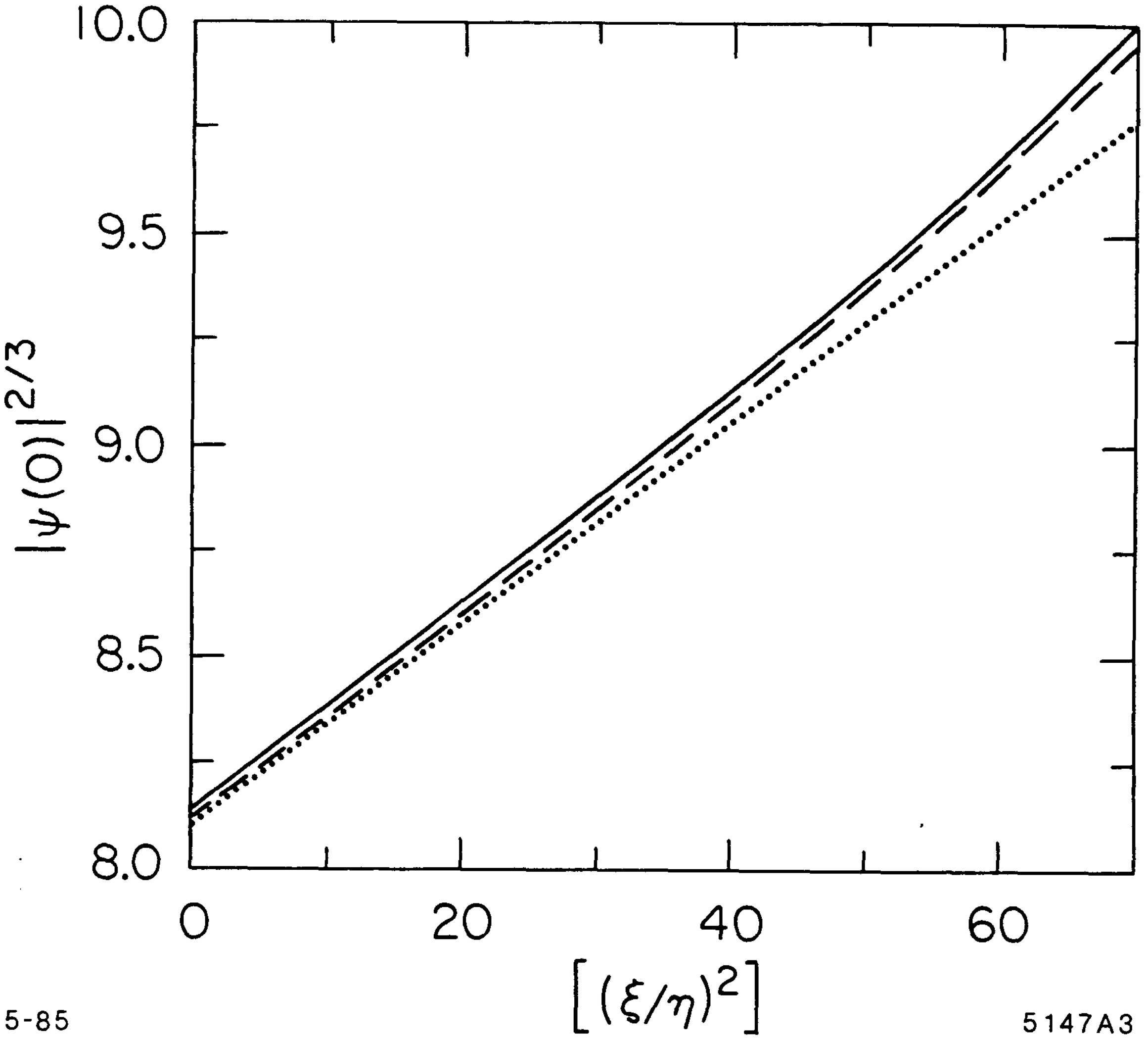
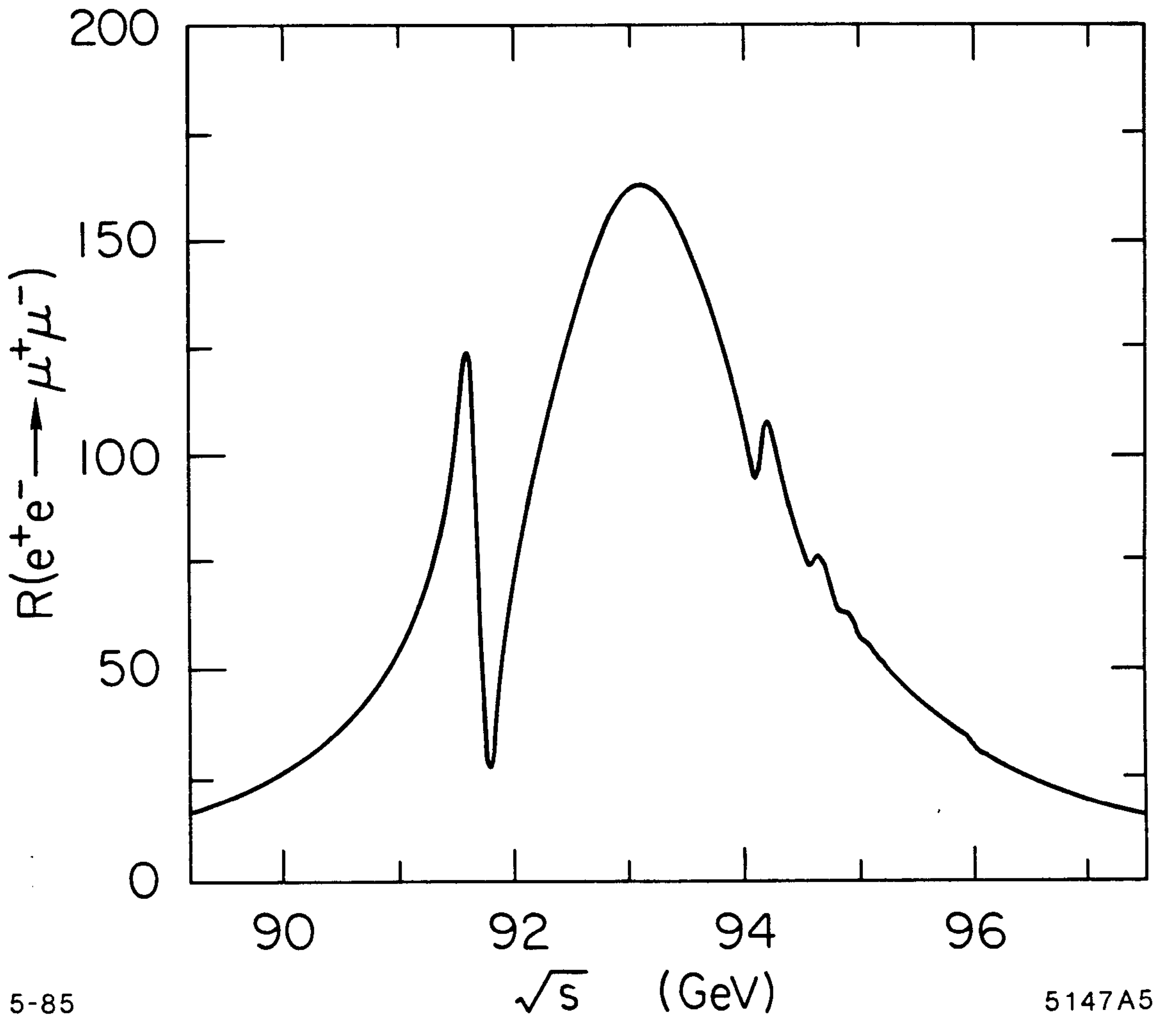


Fig. 7


200
150
100
50
0
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90
92
94
96
$\sqrt{s}$ (GeV)
5-85
5147A5

Fig. 8

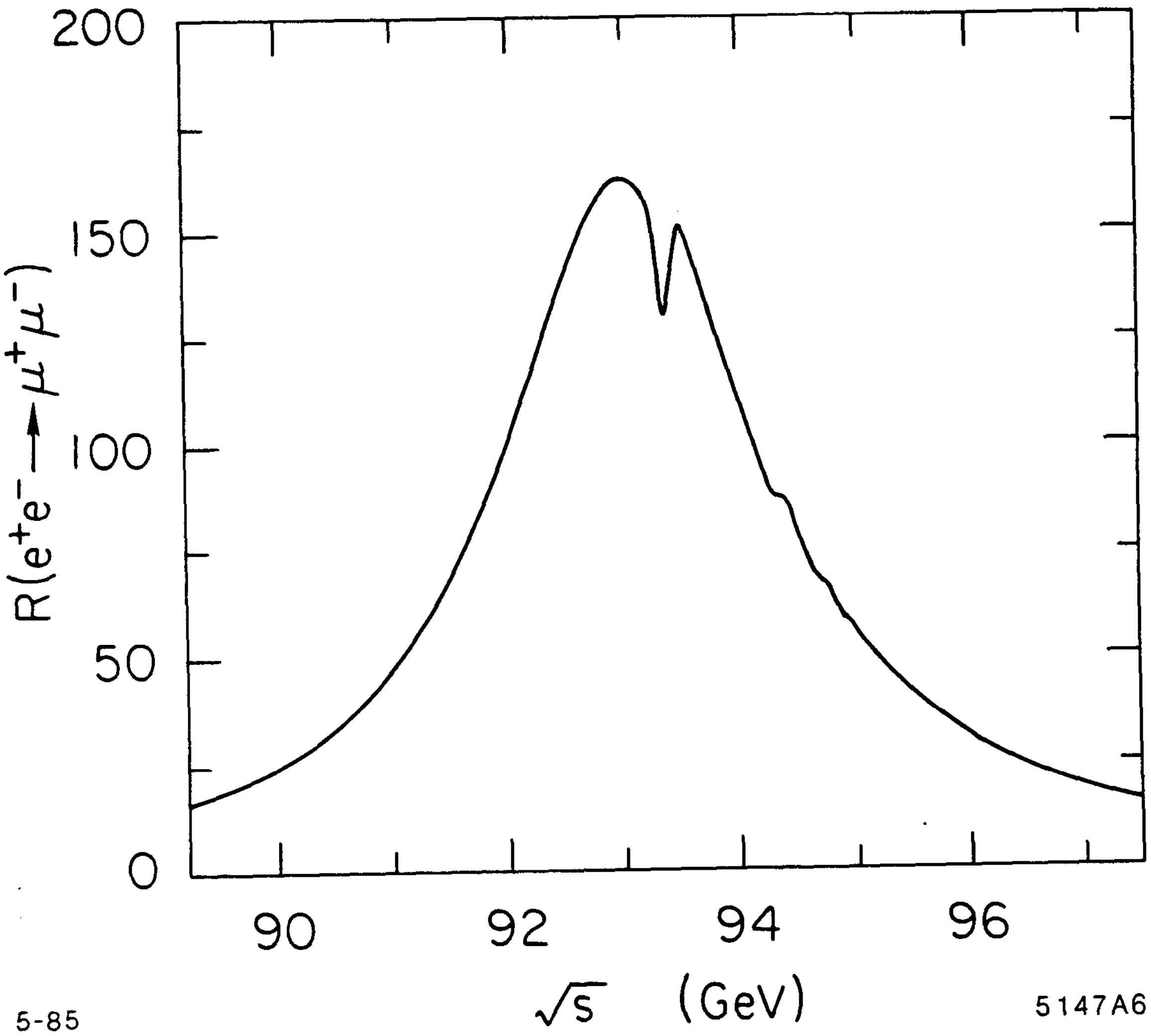
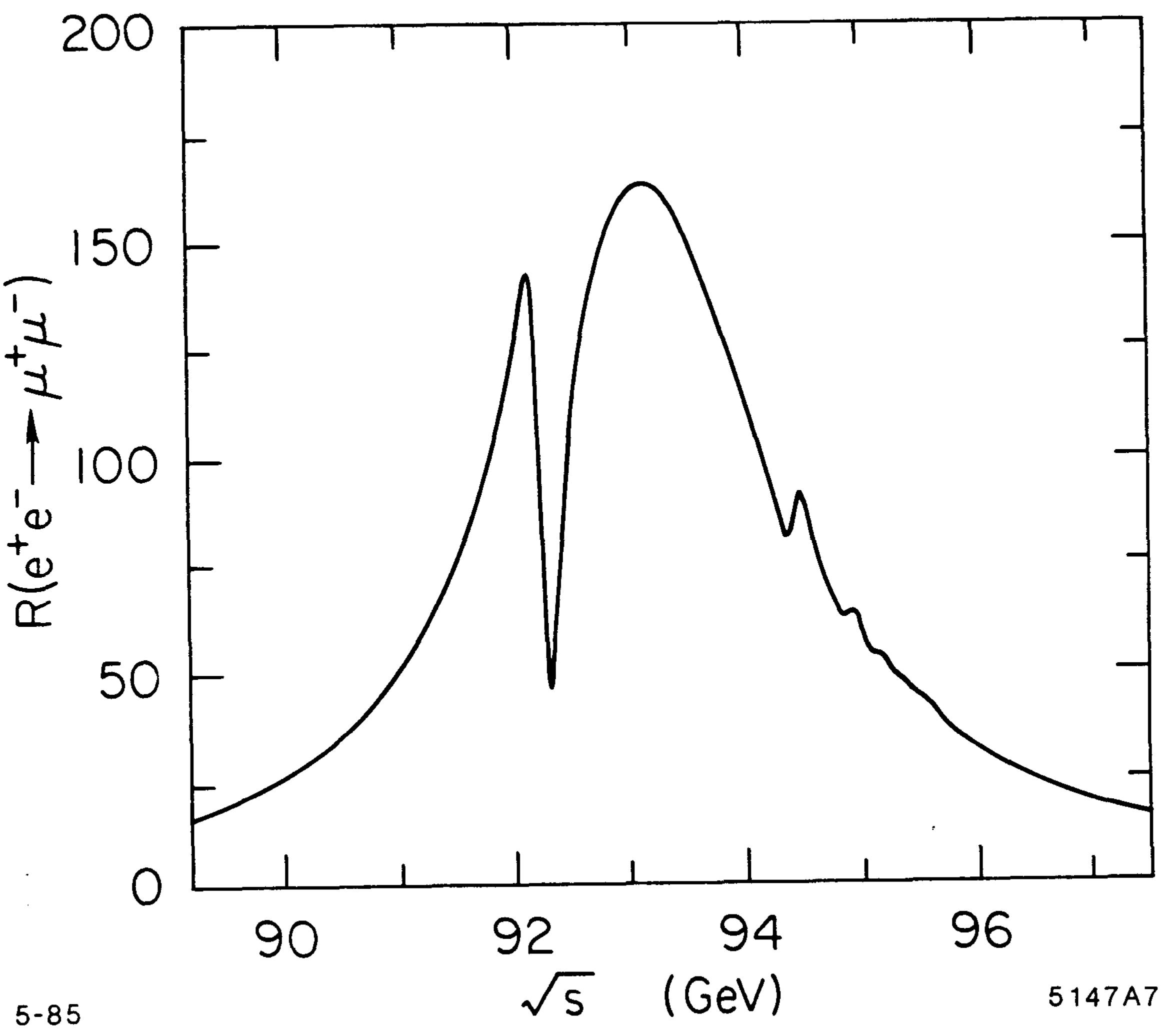


Fig. 9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150
100
50
0
$R(e^+e^- \longrightarrow \mu^+\mu^-)$
90
92
94
96
$\sqrt{s}$ (GeV)
5-85
5147A7

Fig. 10

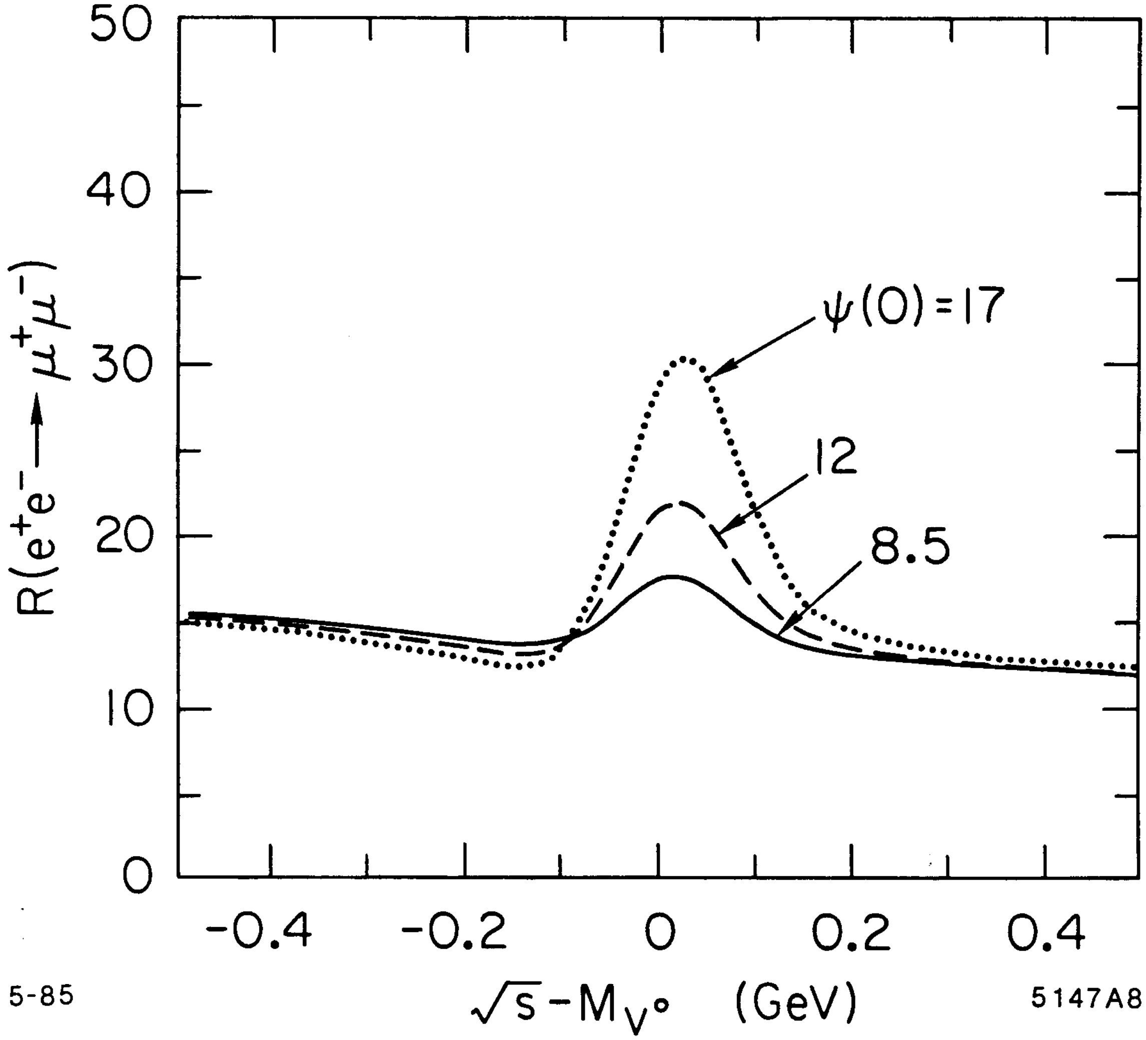
50
40
30
20
10
0
$R(e^+e^- \longrightarrow \mu^+\mu^-)$
$\psi(0)=17$
12
8.5
-0.4
-0.2
0
0.2
0.4
$\sqrt{s}-M_{V^\circ}$ (GeV)
5-85
5147A8

Fig. 11