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Finite-Size Core Effects in the Callan-Rubakov Model*

W. GOLDSTEIN AND M. SOLDATE
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

The interaction of the $J = 0$ partial wave of a massless isodoublet Dirac fermion with an $SU(2)$ gauge theory monopole of finite size is investigated by studying the original model of Callan and Rubakov with boundary conditions imposed at a core radius $r_0 \neq 0$. Both the chirality non-conserving and gauge-invariant "charged" condensates are studied with special attention to their leading dependence on r_0 and $\alpha \equiv e^2/4\pi$. It is found that, at fixed $r > r_0 > 0$, the $\alpha \to 0$ limit reproduces the free field values of both condensates, in contrast to the limit obtained with $r_0 = 0$. For finite α, however, the leading core effects are negligible for $r >> r_0/\alpha$. A gauge dependent, charge-carrying condensate is also considered and shown to vanish.

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1. Introduction

The catalysis of proton decay at strong interaction rates by a variety of Grand Unified Theory (GUT) monopoles, as predicted by Callan and Rubakov [1,2,3], is a quite surprising result with important implications for monopole phenomenology [4]. It is of interest to investigate the approximations made in the reduction of fermion–monopole interactions to the original [1,3], exactly soluble model. In particular, a major approximation of that model consisted in scaling the monopole's core radius, r_0,* to zero at fixed finite $\alpha = e^2/4\pi$. The solution in this approximation is similar to that of the Schwinger model. However, in contrast to the typical behavior of Schwinger–like systems, it is not possible to recover the free field theory in the limit $\alpha \to 0$.† Our results suggest that this is an artifact of the approximation of the monopole as a point–like object. It is instructive to examine in detail the physical situation, r_0 small but non-zero, to study how quantities of interest may tend to their free field values as $\alpha \to 0$, yet not be significantly suppressed at finite α.

In this letter we address the effects of a finite core radius on the behavior of the $J = 0$ partial wave of a massless Dirac isodoublet in the field of an $SU(2)$ monopole simply by imposing the standard boundary conditions at the core boundary, $r_0 \neq 0$, [5,6]. The validity of this approach will be discussed below.

Two condensates in the original model have attracted considerable attention. These are the electrically neutral, chirality [1] (or fermion number [3]) non–conserving and the 'charge violating' [5,6,7,8,9] condensates. Neutral condensates analogous to the former play a role in catalyzed proton decay by GUT monopoles. The 'charged'

* The monopole's 'radius' is the characteristic measure of the exponential approach of the Higgs field to its value in the vacuum sector. It is inversely proportional to the mass of the monopole configuration.

† For example, the model contains an interesting, chirality non-conserving fermion condensate, totally unsuppressed by factors such as exp $(-const/\alpha)$ or α.

condensate, when correctly interpreted [6], does not imply the breakdown of the electric charge superselection rule, but reflects the existence of very massive configurations of localized charge screened by the fermion vacuum. We reanalyzed these condensates for non–zero r_0. Their expressions evince a dependence on α, such that their free field values are recovered as $\alpha \to 0$. For finite α, though, the core effects become negligible at distances far from the monopole center. It is also possible to construct a genuinely charge carrying, gauge dependent condensate. A careful calculation shows that it vanishes.

2. The Model

The model describing the $J = 0$ partial wave of an isodoublet Dirac field in the presence of an $SU(2)$ monopole is well known. It is briefly reviewed below only to fix notations and to discuss the boundary conditions placed at the core radius. In $A_0 = 0$ gauge, the classical monopole configuration consists of adjoint Higgs and gauge fields of the form

$$\Phi_a^{(0)} = H(r)\,\hat{x}_a\,, \qquad A_{ai}^{(0)} = \epsilon_{aij}\,\hat{x}_j\,\frac{1-K(r)}{r}$$

where $a = 1,2,3$ in the $SU(2)$ index. $H(r)$ and $K(r)$ approach their asymptotic values exponentially:

$$H(r) \underset{r\to\infty}{\longrightarrow} h = \langle|\Phi|\rangle_0, \qquad K(r) \underset{r\to\infty}{\longrightarrow} 0$$

and satisfy $H(0) = 0$, $K(0) = 1$.

The lowest lying spherically symmetric excitations of this system can be parameterized by a collective coordinate, $\lambda(r,t)$:

$$A_i^\lambda = U_\lambda A_i^0 U_\lambda^{-1} + iU_\lambda \nabla_i U_\lambda^{-1}, \qquad \Phi^\lambda = U_\lambda \Phi U_\lambda^{-1} = \Phi^{(0)}$$

where

$$U_\lambda = e^{i\lambda(r,t)\hat{x}\cdot\vec{\tau}/2} \quad , \quad \lambda(0,t) = 0 \quad .$$

The $J = 0$ component of the Dirac isodoublet may be written as

$$\Psi_{A\alpha} = \frac{1}{\sqrt{2}}\left(\Psi^{(+)}_{A\alpha} + \Psi^{(-)}_{A\alpha}\right), \qquad \Psi^{(\pm)}_{A\alpha} = \begin{pmatrix} X^{(\pm)} \\ (\pm)X^{(\pm)} \end{pmatrix}_{A\alpha}$$

with [10]

$$X^{(\pm)}_{A\alpha} = \frac{1}{\sqrt{8\pi}\, r}[(g_\pm(r,t) + p_\pm(r,t)\,\hat{x}\cdot\vec{\sigma})\sigma_2]_{A\alpha}$$

where A and α are, respectively, spin and isospin indices taking on the values 1, 2, and $\gamma^5\Psi^{(\pm)} = \pm\Psi^{(\pm)}$.* It is convenient to express the action of the $J = 0$ fermions interacting with the collective degree of freedom, λ, in terms of a fermion field,

$$\chi^{(\pm)} = e^{\mp i\lambda\bar{\gamma}^5/2}\begin{pmatrix} g_\pm \\ \pm i p_\pm \end{pmatrix} \equiv e^{\mp i\lambda\bar{\gamma}^5/2}\chi^{(\pm)}_\lambda \tag{1}$$

which is invariant under the residual time independent gauge transformations of the theory;† in Minkowski space

$$S = \int dr\, dt \Big\{ i\,\bar{\chi}^{(+)}\Big[\bar{\gamma}^0\,\partial_t + \bar{\gamma}^1\Big(\partial_r + i\frac{\dot{\lambda}}{2}\Big) + \frac{K(r)}{r}\bar{\gamma}_5\Big]\chi^{(+)} \\ + i\,\bar{\chi}^{(-)}\Big[\bar{\gamma}^0\,\partial_t + \bar{\gamma}^1\Big(\partial_r - \frac{i\dot{\lambda}}{2}\Big) + \frac{K(r)}{r}\bar{\gamma}^5\Big]\chi^{(-)} + \frac{4\pi}{e^2}\Big[\frac{r^2}{2}(\dot{\lambda}')^2 + K^2(r)(\dot{\lambda})^2\Big]\Big\} \quad . \tag{2}$$

Here we have introduced the two-dimensional Dirac matrices $\bar{\gamma}^i$, $i = 1, 2$, $\bar{\gamma}^5 = \bar{\gamma}^0\,\bar{\gamma}^1$.

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* $\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix} \quad .$

† Under such a transformation, $\lambda(r,t) \to \lambda(r,t) + \delta\lambda(r)$ and $\chi^{(\pm)}_\lambda \to e^{\pm\delta\lambda(r)\bar{\gamma}^5/2}\chi^{(\pm)}_\lambda$.

In the limit of a point monopole ($r_0 \to 0$), contributions to the action proportional to $K(r)$ are dropped and a boundary condition imposed on the fermions. The condition is required to render the truncated Hamiltonian self-adjoint and is dictated by the behavior of the field deep within the core where λ is heavily suppressed [1,3]:

$$(1-\bar{\gamma}^0)\chi^{(\pm)}(0,t) = 0 \quad . \tag{3}$$

Thus far, the system with a finite-size monopole has proven intractable. In order to investigate the possible significance of core effects, we adopt the somewhat naive, 'first-order' procedure of imposing the standard boundary conditions on $\chi^{(\pm)}$ and λ at the finite radius r_0. This approach can be justified as follows. Field configurations with λ small throughout the core are liable to be dominant in the functional integral owing to the positive definite term in the Lagrangian proportional to $K^2(r)$. Thus, within the core the fermions essentially interact only with the fixed monopole background. Their eigenfunctions can be shown to have small lower components (in a basis where $\bar{\gamma}^0 = \tau_3$) throughout the core. It should be noted also that the exact form of the boundary conditions on the fermion fields is immaterial so long as they have the form $\bar{\Gamma}_\pm \chi_\pm|_{r=r_0} = 0$, and $[\bar{\gamma}_5, \bar{\Gamma}_\pm] \neq 0$. Nevertheless, the validity of the approximation remains somewhat problematic.

The resulting model remains essentially soluble. As usual, the fermion determinant can be explicity evaluated from the propagator in an arbitrary background field, λ. The reduced, two–dimensional Dirac equation is solved by

$$\chi^{(\pm)} = e^{\pm i(b+a\bar{\gamma}_5)}\chi_0^{(\pm)} \tag{4}$$

where $\chi_0^{(\pm)}$ is a free, massless, two-dimensional fermion, and

$$-\partial_t b = \partial_r a, \qquad \partial_r b + \partial_t a = -\frac{1}{2}\partial_t \lambda \quad . \tag{5}$$

The boundary conditions on $\chi^{(\pm)}$ and λ imply $a(r=r_0,t)=\partial_r b(r=r_0,t)=0$ and $(1-\bar{\gamma}^0)\chi_0^{(\pm)}(r=r_0,t)=0$. The difference of the gauge field action and the logarithm of the fermion determinant forms an effective action depending quadratically on b alone. In Euclidean space

$$S_E[b]=\int d^2r_E\Big[-\frac{8\pi}{e^2}r^2(\Box b)^2-\frac{1}{\pi}(\partial_\mu b)^2\Big] \quad . \tag{6}$$

This action, combined with the boundary condition on b at r_0 determines the b field propagator which is in turn the essential ingredient in the evaluation of fermion matrix elements.

3. Gauge Invariant Condensates

It has not been possible to evaluate the b propagator in closed form for all values of its arguments and of the coupling constant. However, if we restrict our interest to the fermion condensates mentioned above, it is only necessary to extract several limiting forms of this function. Thus, if we define

$$K(r,r';\tau,\tau')=\langle b(r,\tau)\,b(r',\tau')\rangle_0 \quad , \tag{7}$$

the chirality non-conserving condensate can be calculated from

$$\begin{aligned}
&\lim_{|\tau-\tau'|\to\infty}\langle 0|\,\bar{\Psi}^{(+)}\,\Psi^{(-)}(r,\tau)\,\bar{\Psi}^{(-)}\,\Psi^{(+)}(r',\tau')|0\rangle\\
&=\langle 0|\,\bar{\Psi}^{(+)}\,\Psi^{(-)}(r,\tau)|0\rangle\langle 0|\,\bar{\Psi}^{(-)}\,\Psi^{(+)}(r',\tau')|0\rangle\\
&=\Big(\frac{1}{2\pi r^2}\Big)\Big(\frac{1}{2\pi r'^2}\Big)\lim_{|\tau-\tau'|\to\infty} exp[2K(r,r;\tau,\tau)+2K(r',r';\tau',\tau')-4K(r,r';\tau,\tau')]\\
&\quad\times\langle 0|\,\bar{\chi}^{(+)}\,\chi_0^{(-)}(r,\tau)\,\bar{\chi}_0^{(-)}\,\chi_0^{(+)}(r',\tau')|0\rangle
\end{aligned} \tag{8}$$

where cluster decomposition has been used. The chirality conserving, "charged" condensate is obtained directly as

$$\langle 0|\,\bar{\chi}^{(\pm)}\,\bar{\gamma}^5\,\chi^{(\pm)}(r,\tau)|0\rangle = exp[2L(r,r;\tau,\tau)]\langle 0|\,\bar{\chi}_0^{(\pm)}\,\bar{\gamma}^5\,\chi_0^{(\pm)}(r,\tau)|0\rangle \tag{9}$$

where

$$L(r,r';\tau,\tau') = \langle a(r,\tau)a(r',\tau')\rangle_0 = -\int_{r_0}^{r} ds \int_{r_0}^{r'} ds'\;\partial_\tau\partial_{\tau'}K(s,s';\tau,\tau')\;. \tag{10}$$

It should be emphasized that the fields appearing in this bilinear ($\chi^{(\pm)}$ or both $\chi_0^{(\pm)}$ and a) are explicitly gauge invariant. Thus a nonzero result will not imply the breakdown of charge superselection.

The evaluation of $K(r,r';\tau,\tau')$ is similar to the calculation when $r_0 = 0$ [3]. We will concentrate, below, only on the new salient features. K satisfies the equation

$$\left[\frac{16\pi}{e^2}\Box(r^2\Box) - \frac{2}{\pi}\Box\right]K(r,r';\tau,\tau') = \delta(r-r')\delta(\tau-\tau') \quad . \tag{11}$$

Then

$$K(r,r';\tau,\tau') = -\frac{\pi}{2}\left[\Box^{-1}(r-r',\tau-\tau') + \Box^{-1}(r+r'-2r_0,\tau-\tau') - \tilde{K}\,(r,r';\tau,\tau')\right] \tag{12}$$

where

$$\Box^{-1}(r,\tau) = \frac{1}{4\pi}\,ln\left[(r^2+\tau^2)\mu^2\right] \tag{13}$$

and

$$\left(\Box - \frac{e^2}{8\pi^2 r^2}\right)\tilde{K}\,(r,r';\tau,\tau') = \delta(r-r')\delta(\tau-\tau') \quad , \tag{14}$$

$$\partial_r\tilde{K}\,(r=r_0,r';\tau,\tau') = 0 \quad . \tag{15}$$

Introduce the fourier transform of $\tilde{K}$ in $\tau - \tau'$ to solve these equations.

$$\tilde{K}(r, r'; \tau, \tau') = \frac{1}{\pi} \int_0^\infty d\omega \, cos[\omega(\tau - \tau')] \, \tilde{K}_F(r, r'; \omega) \tag{16}$$

$$\tilde{K}_F(r, r'; \omega) = -\sqrt{rr'} \begin{cases} [I_\nu(\omega r) + D(\omega r_0) K_\nu(\omega r)] K_\nu(\omega r') & \text{for} \quad r < r' \\ K_\nu(\omega r)[I_\nu(\omega r') + D(\omega r_0) K_\nu(\omega r')] & \quad r > r' \end{cases} \tag{17}$$

where

$$\nu = \frac{1}{2}\sqrt{1 + \frac{2\alpha}{\pi}} \approx \frac{1}{2} + \frac{\alpha}{2\pi} \tag{18}$$

and

$$D(\omega r_0) = -\frac{I_\nu(\omega r_0)}{K_\nu(\omega r_0)} - \frac{2[K_\nu(\omega r_0)]^{-1}}{[K_\nu(\omega r_0) + 2\omega r_0 K'_\nu(\omega r_0)]} \quad . \tag{19}$$

Inserting eq. (19) into eqs. (16) and (17),

$$\begin{aligned} \tilde{K}(r, r'; \tau, \tau') = & -\frac{1}{2\pi} Q_{\nu-1/2}\left(\frac{r^2 + r'^2 + (\tau - \tau')^2}{2rr'}\right) \\ & - \frac{\sqrt{rr'}}{\pi} \int_0^\infty d\omega \, cos[\omega(\tau - \tau')] D(\omega r_0) K_\nu(\omega r) K_\nu(\omega r') \end{aligned} \quad . \tag{20}$$

Here, $Q_{\nu-1/2}$ is the Lengendre function of degree $\nu - 1/2$ of the second kind [11].

The second term is the non-trivial correction to the result of the calculation in the original model. Qualitatively, the behavior of the b–propagator at large distances is controlled by small ω. In the limit of small ωr_0 and x, $D(\omega r_0) \to \frac{4}{\alpha}\omega r_0$. Consequently, one expects to see the correction terms to the b-propagator vanish at large r for finite α, though in a fashion non-analytic in α.

From eq. (20) it is clear that as $|\tau - \tau'| \to \infty$ the correction will vanish as $1/|\tau - \tau'|$. Then, only the limit $\tau' \to \tau$ is needed. In fact, the correction is not singular in this limit and one can simply set $\tau' = \tau$ in the integrand.

The terms non–analytic in α can be isolated by setting $\alpha = 0$ in the integrands wherever it is possible to do so without inducing any divergences. This can be done in the first term of $D(\omega r_0)$ in eq. (19). However, it cannot be done everywhere in the second term. Its denominator has the limiting forms

$$K_\nu(x) + 2xK'_\nu(x) \approx \begin{cases} -\sqrt{2\pi x}\, e^{-x} & \text{as} \quad x \to \infty \\ -\frac{\alpha}{\sqrt{2\pi x}} - \sqrt{2\pi x} + O(\alpha\sqrt{x}) & \text{as} \quad x \to 0 \quad . \end{cases} \tag{21}$$

Setting $\alpha = 0$ in this quantity induces an infrared divergence in the ω–integral. The non–analyticity in α of the corrections to the b–propagator arise accordingly. The behavior of the chirality–breaking condensate in the limit $r \to \infty$ with α and r_0 fixed, and $\alpha \to 0$ with r and r_0 fixed is determined by the limits shown in eq. (21). After some calculation involving eq. (8), *

$$\langle 0|\, \bar{\Psi}^{(+)}\, \Psi^{(-)}(r,\tau)|0\rangle \underset{r\to\infty}{\longrightarrow} \frac{1}{4\pi^2 r^3}\, e^{-\frac{\pi r_0}{\alpha r}} \tag{22}$$

$$\langle 0|\, \bar{\Psi}^{(+)}\, \Psi^{(-)}(r,\tau)|0\rangle \underset{\alpha\to 0}{\longrightarrow} \frac{\alpha}{4\pi^3 r^2 r_0}\sqrt{\frac{r - r_0}{r}}\, e^{\gamma} \tag{23}$$

up to a phase. Here, γ is Euler's constant. Note that as $\alpha \to 0$ at fixed r and r_0, the chirality–breaking condensate vanishes; the recovery of free field results in this limit is typical of Schwinger model physics. Through comparison with [1,3], we conclude that the limits $\alpha \to 0$ and $r_0 \to 0$ do not commute. The first result is of more practical interest and indicates that finite–size effects on the chirality–breaking condensates are negligible for $r \gg r_0/\alpha$.

The a–propagator is given formally by the combination of eqs. (10), (12), and (20). As before the leading terms in α can be found by setting $\alpha = 0$ in the integrands when doing so does not result in spurious divergences; again, only the denominatior of the second term in $D(\omega r_0)$ can not be assigned its value at $\alpha = 0$. The derivatives with respect to τ and τ', and integrations over s and s' indicated in eq. (10) can then

* The corrections to the argument of the exponent in eq. (22) can be found by expanding $D(\omega r_0)$ for $\omega r_0 \ll 1$, and evaluating the resulting integrals using [13], for arbitrary ν. The result is an expansion in $\frac{r_0}{\alpha r}$ and α, with the leading term as given.

be carried out leaving only an integral over ω to be done. The limit $\tau' \to \tau$ must be performed carefully as terms proportional to $ln(\tau - \tau')^2$ arise frequently, not just in the correction terms. An intermediate result is

$$L(r, r'; \tau, \tau') \underset{\substack{\tau' \to \tau \\ r' = r}}{\to} -\frac{1}{2} ln\Big(\frac{r - r_0}{2r_0}\Big) + \frac{1}{4} ln\Big[\frac{(\tau - \tau')^2}{r_0^2}\Big] + \frac{\pi}{2} \lim_{\tau' \to \tau} L_2(r, r; \tau, \tau') \tag{24}$$

where

$$L_2(r, r; \tau, \tau') = -\sqrt{\frac{2r_0}{\pi}} \int_0^\infty \frac{d\omega}{\sqrt{\omega}} \frac{cos[\omega(\tau - \tau')]}{K_\nu(\omega r_0) + 2\omega r_0 K'_\nu(\omega r_0)} \times \Big[e^{-\omega(2r - r_0)} - 2e^{-\omega r} + e^{-\omega r_0}\Big] \tag{25}$$

The first two terms of L_2 are identical to the term producing the non–analytic dependence of the b–propagator on α. The third term is slightly different in that it does not depend on r. In the limit $\tau' \to \tau$ it contributes a term $-\frac{1}{2\pi} ln[\frac{(\tau - \tau')^2}{r_0^2}]$. For small α as well its leading contribution is $-\frac{1}{2\pi}\{2\gamma + ln[\alpha(\tau - \tau')/2\pi r_0]^2\}$. It is then a matter of algebra to compute that

$$\langle 0|\, \bar{\chi}^{(\pm)} \bar{\gamma}_5\, \chi^{(\pm)}(r, \tau)|0\rangle \underset{\substack{r \to \infty \\ \alpha, r_0 \,\text{fixed}}}{\to} -\frac{2i}{\alpha} \frac{r_0}{r^2}\, exp\Big(-\frac{3\pi r_0}{\alpha r} - \gamma\Big) \tag{26}$$

$$\langle 0|\, \bar{\chi}^{(\pm)} \bar{\gamma}_5\, \chi^{(\pm)}(r, \tau)|0\rangle \underset{\substack{\alpha \to 0 \\ r, r_0 \,\text{fixed}}}{\to} -\frac{i}{2\pi(r - r_0)} \tag{27}$$

Again, the free field value [9] is obtained in the limit $\alpha \to 0$ at fixed r, r_0. The first result demonstrates the known [5,6] result that interactions heavily suppress the 'charge–violating' condensate. The reason is that the charge carried by the fermions is compensated by charge deposited on the core, as required by gauge invariance. The high Coulomb energy of charge on the core suppresses this condensate [6].

4. Charge Carrying Condensate

The field $\chi^{(\pm)}$, used thus far, is gauge invariant and electrically neutral. We would now like to consider the matrix element of an operator which carries the global $U(1)$ charge. To construct such an operator, define the charged fermion field [12,13]

$$\chi_{ch}^{(\pm)}(r,t) = e^{\pm i\lambda_\infty \bar{\gamma}^5/2}\, \chi^{(\pm)}(r,t)$$

where $\lambda_\infty \equiv \lambda(r=\infty,t)$. $\chi^{(\pm)}$ remains invariant under local gauge transformations, that is, transformations which vanish at spatial infinity, but responds as a charge $\pm 1/2$ object to global transformations, which entail a shift in the value of λ_∞ [12].

The Dirac equation for $\chi_{ch}^{(\pm)}$ is again solved by eqs. (4) and (5), but with λ replaced by $\lambda - \lambda_\infty$, The consequent boundary condition is $a(r=\infty,t)=0$. The a-field propagator is thus given by

$$L_{ch}(r,r';\tau,\tau') = -\int_r^\infty ds \int_{r'}^\infty ds' \partial_\tau\, \partial_{\tau'}\, K(s,s';\tau,\tau')$$

with K as in eqs (12) and (20). The charged operator we will consider is related to the operator of eq. (9):

$$\langle 0|\, \bar{\chi}_{ch}^{(\pm)}\; \bar{\gamma}^5\; \chi_{ch}^{(\pm)}(r,\tau)|0\rangle = exp[2L_{ch}(r,r;\tau,\tau)]\langle 0|\, \bar{\chi}_0^{(\pm)}\, \bar{\gamma}^5\, \bar{\chi}_0^{(\pm)}\,(r,\tau)|0\rangle \qquad (28)$$

We have

$$L_{ch}(r,r';\tau,\tau') = \frac{1}{8}\int_r^\infty ds \int_{r'}^\infty ds' \partial_\tau\ \partial_{\tau'}\Big\{\ell n[(s-s')^2+(\tau-\tau')^2]$$

$$+ [\ell n(s+s'-2r_0)^2+(\tau-\tau')^2]\Big\}$$

$$+\frac{1}{4}\int_r^\infty ds \int_{r'}^\infty ds'\ \partial_\tau\ \partial_{\tau'}\ Q_{\nu-1/2}\left[\frac{s^2+s'^2+(\tau-\tau')^2}{2ss'}\right]$$

$$+\frac{1}{2}\int_r^\infty ds \int_{r'}^\infty ds'\ \sqrt{ss'}\ \partial_\tau\ \partial_{\tau'}$$

$$\int_0^\infty d\omega\ \cos\ \omega(\tau-\tau')D(\omega r_0)K_\nu(\omega s)K_\nu(\omega s')$$

$$= \frac{1}{8}\ \ell n\frac{(r-r')^2+(\tau-\tau')^2}{(r+r'-2r_0)^2+(\tau-\tau')^2}$$

$$+\frac{1}{4}\int_r^\infty ds \int_{r'}^\infty ds'\ \left(\partial_s^2 - \frac{e^2}{8\pi^2 s^2}\right)Q_{\nu-1/2}\ \left[\frac{s^2+s'^2+(\tau-\tau')^2}{2ss'}\right]$$

$$+\frac{1}{2}\int_r^\infty ds \int_{r'}^\infty ds'\ \sqrt{ss'}$$

$$\int_0^\infty d\omega\ \omega^2 \cos\ \omega(\tau-\tau')D(\omega r_0)K_\nu(\omega s)K_\nu(\omega s') \tag{29}$$

where eqs. (13) and (14) have been used. As in sect. 3, this expression turns out to be finite at $\tau = \tau'$; it does, however possess infrared singularities in the regions of large s or s'. The presence of this divergence has been previously noted [7] and explored in detail [6] using the bosonization formalism of Callan [2]. Therefore, our exposition here will be brief, and will be limited to the leading behaviour as $\alpha \to 0$.

The term involving ∂_s^2 and that involving $D(\omega r_0)$ are separately finite for $\alpha > 0$. At $\alpha = 0$, however, they develop cancelling logarithmic divergences according to

$$\lim_{\alpha\to 0}\ -r^{\alpha/2\pi}\ \frac{\sqrt{\pi}}{4}\ \frac{\Gamma(2+\alpha/2\pi)}{\Gamma(3/2+\alpha/2\pi)}\int_R^\infty \frac{ds'}{s'^{1+\alpha/2\pi}} + \frac{1}{2}\ell n\frac{\alpha}{\pi r_0}$$

where $R \gg r$. (At $\alpha = 0$, the $\ell n\alpha$ contribution is equivalent to $\frac{1}{2}\int^\infty ds'/s'$, as can be easily seen by working out the last term of eq. (29) at $\nu = 1/2$.)

Thus, at $\alpha = 0, \nu = 1/2, L_{ch}$is found to be finite and in agreement with free field theory. In particular, eq. (27) is obtained whether $\chi^{(\pm)}$ or $\chi^{(\pm)}_{ch}$ is used.

On the other hand, for $\alpha > 0$, the remaining integral in (29), proportional to $\alpha/2\pi$, turns out to contain an α-independent large distance divergence. Specifically, for $\lambda \gg 1$ [11],

$$-\frac{1}{4}\frac{\alpha}{2\pi}\int_r^\infty \frac{ds}{s^2}\int_{\lambda s}^\infty ds'\, Q_{\nu-1/2}\left(\frac{s^2+s'^2}{2ss'}\right)$$

$$\underset{\lambda\gg 1}{\longrightarrow} -\frac{\alpha}{8\pi}\int_r^\infty \frac{ds}{s^2}\int_{\lambda s}^\infty ds'\,\sqrt{\pi}\,\frac{\Gamma(\nu+1/2)}{\Gamma(\nu+1)}\left(\frac{s'}{s}\right)^{-\nu-1/2}$$

$$= -\frac{\alpha}{8\pi}\sqrt{\pi}\,\frac{\Gamma(\nu+1/2)}{\Gamma(\nu+1)}\,\frac{\lambda^{1/2-\nu}}{\nu-1/2}\int_r^\infty \frac{ds}{s} = -\frac{1}{2}\int_r^\infty \frac{ds}{s} + O(\alpha)$$

Evidently, this divergence has the correct sign to drive the expectation valve, eq. (28), to zero, as required by unbroken charge symmetry. (The apparent persistence of this divergence at $\alpha = 0$ is not to be taken seriously. Clearly the free field limit must be taken before the divergent integrals are performed, in which case this term does not contribute. This behavior is in agreement with that found in ref. [6].)

5. Conclusion

We have identified the leading corrections in α to the chirality and "charge" non-conserving condensates of $J = 0$ fermions about an SU(2) monopole of non-zero size. When considering the effects of high frequency fermionic modes on the condensates, one should include higher partial waves, and the theory becomes renormalizable, not super-renormarlizable. In the context of an SU(5) monopole this raises the question of whether results of the sort we have found are consistent with the renormalization group. Probably, the basic conclusions that we have reached will remain correct in any case: the coupling constant dependence of finite-size core corrections to gauge-invariant quantities is such that the free-field results are recovered when the coupling constant is taken to zero; otherwise, the corrections are negligible sufficiently far from the core.

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