SLAC-PUB-3228
September 1983
(T)

SUPERSYMMETRY BREAKING AT FINITE TEMPERATURE: THE GOLDSTONE FERMION

DANIEL BOYANOVSKY
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

In this paper it is shown that supersymmetry is broken at any finite temperature. The current algebra relations that indicate the presence of a Goldstone fermion are generalized to finite temperature in the real time formalism.

The existence of a massless pole in fermionic thermal Green's functions is thus predicted. Explicit calculations in several models confirm the existence of this Goldstone fermion at finite temperature. It is found that the residue at the $p_\mu = 0$ pole vanishes as the temperature goes to zero.

Submitted to Physical Review D

---

*Work supported in part by the Department of Energy, contract DE-AC03-76SF00515 and by the National Science Foundation, grant PHY80-18938.

INTRODUCTION

Supersymmetry plays a very important role in current attempts to construct realistic models in particle physics. (It might provide for solutions to outstanding problems like, for example, naturalness because it provides a natural mechanism for cancelling divergences.) However, despite its appealing features, supersymmetry (SUSY) must be broken in nature since we do not observe any bosonic partners of fermions. If SUSY is to be incorporated in the framework of unified theories it is necessary to understand the effect of finite temperature in these theories since they are expected to describe physics in the early universe.

Das and Kaku[1] were the first to realize that at high temperature SUSY behaves differently from other symmetries. While most symmetries – with few exceptions[2] – if broken at zero temperature are restored at sufficiently high temperature, unbroken SUSY at $T = 0$ breaks at high temperature.

In a more recent work, Girardello[3] *et al.,* have studied SUSY breaking at finite temperature. They conclude that such breaking is a natural consequence of different statistics for bosons and fermions.

However, despite this automatic breaking of SUSY at $T \neq 0$ they state that there are no Goldstone fermions associated with this breaking, instead they suggest that SUSY breaking is explicit due to boundary effects. Their argument relied heavily on the fact that in the Matsubara (imaginary time) formalism the minimum energy of a fermion is $2\pi T$.

Van Hove[4] has recently argued that there is a subtlety in the definition of the thermal averages of variations of operators under a SUSY transformation. This point has been further investigated by several authors[5,6,7] and they conclude that a careful treatment of the SUSY transformations leads to a "graded" thermal average in which **both** bosons **and** fermions obey **periodic** boundary conditions in imaginary time.

Thus they were led to conclude that if SUSY is unbroken at $T = 0$ it stays unbroken at any temperature.

Their proof relies on the fact that a SUSY transformation involves a constant anti-commuting (Grassman) parameter and proper account of this parameter in the density matrix leads to the "graded averages."

However, these authors compute Green's functions which do not have physical realization because they do not obey the physically correct boundary conditions in imaginary time. As it has been realized by Girardello *et al.*, this Grassman parameter **cannot** be constant in imaginary time if one decides to preserve the right boundary conditions for the physical Green's functions. In this paper we try to clarify some aspects of the problem looking at the behavior of Green's functions with the correct boundary conditions in imaginary time.

Although at finite temperature Lorentz invariance is lost, one can still quantize a theory in a Lorentz covariant way.[8] Indeed, it is known that there are two different formalisms that can be used to study field theories at finite temperature[8,9,10]: imaginary time (Matsubara) where the energy is discrete but momentum is continuous (non-covariant) or real-time where energy and momentum are continuous variables. Whereas the first is best suited to study the perturbative aspects of the theory, it is well known that in order to analyze real-time response functions the second is necessary.[11] There are subtleties in going from one formalism to the other, and we argue that when Green's functions are studied in real-time formalism, the essential physics is exposed clearly.

It is shown here that zero temperature Ward identities do translate with minor modifications to finite temperature in the real-time formalism and, as a consequence of this, a Goldstone fermion is associated with SUSY breaking at $T \neq 0$. Since Goldstone particles arise as excitations produced in a system as response to long wavelength perturbations, it is necessary to study the corresponding Green's functions in the real-time formulation to see the presence of these particles.

The paper is divided as follows: in Section 1 the zero temperature Ward identities and current algebra relations are reviewed. In Section 2 the finite temperature formalisms are reviewed exposing the differences between the imaginary and real-time approaches. The results of Section 1 are extended to finite temperature in the real-time approach.

Section 3 is devoted to the explicit computation of the fermion thermal Green's function for several models exposing the physical mechanism that gives rise to the massless pole and its limit as $T \to 0$.

Since some doubts have been raised as to the validity of the effective potential in terms of the auxiliary fields, an Appendix is devoted to this point.

The conclusions are summarized at the end of the paper.

1. REVIEW OF ZERO TEMPERATURE WARD IDENTITIES

Before analyzing the behavior of the SUSY theories at finite temperature, we will briefly review the standard $T = 0$ Ward identities and their relation to well known results in current algebra and Goldstone's theorem.

The theories we will deal with consist of supermultiplets (ϕ, ψ, F) of bosons, Majorana fermions and auxiliary fields.

The supersymmetry transformations are written as

$$\delta\phi = \delta\,\bar{\epsilon}\,\psi(x) \tag{1.1a}$$

$$\delta\psi(x) = [-i\,\partial\!\!\!/\,\phi(x) - F(x)]\delta\epsilon \tag{1.1b}$$

$$\delta F(x) = \delta\,\bar{\epsilon}\,i\,\partial\!\!\!/\,\psi(x) \tag{1.1c}$$

where $\delta\epsilon$ is a **constant** grassman (Majorana) parameter. These relations can be generalized to chiral theories in a straightforward manner. Under the transformations

(1.1a,c) the change in the action is:

$$\delta \int \mathcal{L}\, d^d x = \int \delta\epsilon\, \partial_\mu\, S_\mu(x)\, d^d x \tag{1.2}$$

where $S_\mu(x)$ is supercurrent.

We will generalize the transformations (1.1a,c) with $\delta\epsilon(x)$ a space-time dependent parameter. Define

$$\langle\varphi_1(x_1)\cdots\varphi_n(x_n)\rangle_J = \frac{\int D\varphi_a\varphi_1(x_1)\cdots \exp\left(\int d^d x \mathcal{L}[\varphi] + \int d^d x\, J_i\,\varphi_i\right)}{(\text{same for } \ J = 0)} \tag{1.3}$$

where $\varphi_a(x_i)$ stands for either bosonic or fermionic fields. Let us perform the infinitesimal transformation (1.1a-c) in the numerator of (1.3), this amounts to a change of variable in the functional integral and since it is invariant under this change we see that

$$\frac{\delta}{\delta\,\bar\epsilon\,(z)}\Big\langle\varphi_1(x_1)\cdots\varphi_n(x_n)\Big\rangle_J = 0 \tag{1.4}$$

writing $\delta\varphi_i(x) = (\partial\varphi_i/\partial x)\,\delta\epsilon(x)$, Eq. (1.4) reads:

$$\partial_{\mu_z}\langle S_\mu(z)\varphi_1(x_1)\cdots\varphi_n(x_n)\rangle_J + \delta(x_i - z)\Big\langle\varphi_1(x_1)\cdots\frac{\partial\varphi_i(x_i)}{\partial\epsilon(x_i)}\cdots\varphi_n(x_n)\Big\rangle_J$$
$$+ J_\phi(z)\langle\psi(z)\varphi_1\cdots\rangle + \langle(i\,\partial\!\!\!/\,\phi(z) - F(z))\,\varphi_1\cdots\rangle J_{\bar\psi}(z) + J_F(z)\langle i\,\partial\!\!\!/\,\psi(z)\varphi_1\cdots\rangle = 0\,. \tag{1.5}$$

This is the most general form of the Ward identities.[12] Consider the case $n = 1$, $\varphi_1 = \bar\psi$ and $J_i = 0$ in Eq. (1.4). This gives

$$\partial_{\mu_z}\langle S_\mu(z)\;\bar\psi\,(x)\rangle + \delta(z - x)\langle i\,\partial\!\!\!/\,\phi - F\rangle = 0 \tag{1.6}$$

or

$$\int d^d z\; \partial_{\mu_z}\langle S_\mu(z)\;\bar\psi\,(x)\rangle = \langle F\rangle\ . \tag{1.7}$$

In Eq. (1.7) we have assumed that the fields ϕ and F can have position independent vacuum expectation values. Equation (1.7) is the well known current algebra relation, if $\langle F\rangle \neq 0$ it implies that there is a Goldstone fermion in the spectrum. Another interesting relation can be derived. Consider Eq. (1.4) with $n = 0$ but $J \neq 0$.

$$\partial_{\mu_z}\langle S_\mu(z)\rangle_J + J_\phi(z)\langle\psi(z)\rangle_J + \langle i\,\partial\!\!\!/\,\phi(z) - F(z)\rangle_J J_\psi(z) + J_F(z)\langle i\,\partial\!\!\!/\,\psi(z)\rangle_J = 0 \quad . \tag{1.8}$$

Perform a Legendre transformation

$$\begin{aligned} \Gamma[\varphi_i] &= F[J] + \int J_i(x)\varphi_i(x) \\ J_i(x) &= \frac{\delta\Gamma[\varphi_i(x)]}{\delta\varphi_i(x)} \ , \end{aligned} \tag{1.9}$$

thus after integrating over z, this leads to the result:

$$\int dz \left\{ \frac{\delta\Gamma}{\delta\phi}\langle\psi\rangle_J + \langle i\,\partial\!\!\!/\,\phi - F\rangle_J \,\frac{\delta\Gamma}{\delta\,\bar\psi} + \frac{\delta\Gamma}{\partial F}\,\langle i\,\partial\!\!\!/\,\psi\rangle_J \right\} = 0 \quad . \tag{1.10}$$

Now take the functional derivative $\delta/\delta\psi(x)$, and set the sources to zero assuming $\langle\phi(z)\rangle_J \ \overrightarrow{J\to 0}\ v$ $\langle F(z)\rangle_J \ \overrightarrow{J\to 0}\ f$ we find:

$$\int dz \left\{ \frac{\delta\Gamma}{\delta\phi(z)}\delta(z-x) + f\,\frac{\delta^2\Gamma}{\delta\psi(x)\delta\,\bar\psi(z)} \right\} = 0 \tag{1.11}$$

or

$$0 = \frac{\delta V[F,\phi]}{\partial\phi}\bigg|_{\substack{\phi=v\\F=f}} = f\,S_\psi^{-1}(p=0) \ , \tag{1.12}$$

where $V[F,\phi] = -\Gamma[\phi,F]$ for constant fields ($\psi = 0$) and $S_\psi^{-1}(p)$ is the inverse of the full fermion propagator. Relation (1.12) is another expression of Goldstone's theorem, since it implies that whenever $f \neq 0$ the full fermion propagator has a pole at zero momentum.

2. FINITE TEMPERATURE FORMALISM

Although at finite temperature a field theory looses its Lorentz invariance because the plasma of excitations define a reference frame (its center of mass), the theory can still be quantized in a fully covariant fashion.[8]

One can write down a covariant density matrix operator $\hat{Z}_G$ (see Ref. 8) and the thermal averages of physical operators as $\langle O \rangle = (Tr\, O\ \hat{Z}_G / Tr\ \hat{Z}_G)$. The heat bath defines a reference time-like vector U_α with $U_\alpha U^\alpha = 1$. Thermal averages will depend upon the invariants $p_\mu U^\mu$, $p_\mu p^\mu$ and $\beta_\mu p^\mu$ where $\beta_\mu = (1/T) U_\mu$ and T is a Lorentz invariant quantity (temperature in the rest frame of the heat bath).[8] In the rest frame of the heat bath

$$\hat{Z}_G = Tr\, e^{-\beta H} \tag{2.1}$$

$$\langle O \rangle = \frac{Tr\, O\, e^{-\beta H}}{Tr\, e^{-\beta H}} \tag{2.2}$$

In Euclidean space (imaginary time) the partition function (2.1) can be written as a functional integral over fields[10,13]:

$$Z_G = \int D\phi \cdots \exp[-\int_0^\beta d\tau \int d^3x \mathcal{L}[\varphi(x), \tau)] \quad , \tag{2.3}$$

where the τ variable (imaginary time) is restricted to the interval $0 \leq \tau \leq \beta$ and the fields obey the periodicity conditions:

$$\begin{aligned} \phi(\beta, \vec{x}) &= \phi(0, x) \quad \text{(bosons)} \\ \psi(\beta, \vec{x}) &= -\psi(0, \vec{x}) \quad \text{(fermions)} \quad . \end{aligned} \tag{2.4}$$

An alternative way of quantizing the theory is the real time method in which, for example, the Minkowski space propagator is:

$$D(x) = \frac{Tr\{e^{-\beta H} T\, \hat{\varphi}(x)\, \hat{\varphi}(y)\}}{Tr\, e^{-\beta H}} \tag{2.5}$$

where $\hat{\varphi}(x,t) = e^{iHt}\, \varphi(x,0)\, e^{-iHt}$ is the Heisenberg field operator.

While the imaginary time formalism leads to non-covariant Feynmann propagators in which energies are discrete [$(2n+1)\pi/\beta$ for fermions, $2n(\pi/\beta)$ for bosons] and momenta are continuous , the real-time approach leads to fully covariant propagators with continuous energies and momenta.

The imaginary time method is best suited for the study of the perturbative expansion of the theory. However, in order to study the response of the system to external perturbations one has to examine the **real-time** linear response functions.[9] The real-time (Minkowski space) propagator $D(\vec{x}, t)$ is the analytical continuation of the imaginary-time (Euclidean) propagator $D(\tau, \vec{x})$[9,10] to $-\infty \leq t = i\tau \leq +\infty$. As has been pointed out in references 10 and 11, the fourier transform $D(k_0, \vec{k})$ is **not** the continuation of $D(\omega_n, \vec{k})$. $D(\omega_n, k)$ has to be continued to arbitrary Euclidean energy ω (this continuation is unique[9]), $D(\omega, k)$ is analytic in the right and left ω plane with possible discontinuities along the imaginary axis that yield the spectral density:

$$\rho(k_0, \vec{k}) = D(ik_0 - \epsilon, \vec{k}) - D(ik_0 + \epsilon, \vec{k}) \tag{2.6}$$

and finally:

$$D(k_0, k) = D[i(k_0 + i\epsilon), \vec{k}] + \frac{\rho(k_0, k)}{(e^{\beta k_0} - 1)} \quad . \tag{2.7}$$

The poles of $D(k_0, \vec{k})$ define the energy of excitations of momentum $\vec{k}$ in the reference frame of the heat bath.

The real-time free propagators for bosons and fermions (in the heat bath reference frame) read[8,10]:

$$D_\beta(k) = \frac{i}{k^2 - m^2 + i\epsilon} + 2\pi \, \frac{\delta(k^2 - m^2)}{e^{\beta E} - 1} \quad \text{(bosons)}$$

$$S_\beta(k) = \frac{i}{\not{k} - m + i\epsilon} - 2\pi \, (\not{k} + m) \, \frac{\delta(k^2 - m^2)}{e^{\beta E} + 1} \quad \text{(fermions)} \tag{2.8}$$

$$E = (\vec{k}^2 + m^2)^{1/2} \ .$$

Now we are in position to extend the result of Section 1 to finite T.

We start our discussion recalling Eq. (2.3). It has been recognized by Girardello[3] *et al.*, that the SUSY transformations (1.1a,b) with constant $\delta\epsilon$ are incompatible with the boundary conditions in Euclidean time for the physical fields, therefore one must impose

$$\delta\epsilon(0) = -\delta\epsilon(\beta) \ . \tag{2.9}$$

We generalize the transformations (1.1a-c) with $\delta\epsilon(\vec{x}\,,\,\tau)$ with the antiperiodicity condition (2.9) in the τ variable.

Define the thermal Green's functions in terms of the physical fields in Euclidean time:

$$\Big\langle \varphi_1(x_1,\tau_1)\cdots\varphi_n(x_n\tau_n)\Big\rangle_\beta^J = \int \, D\varphi_a\cdots \, \frac{\exp\left(-\int_0^\beta d\tau \int d^3x\mathcal{L} + \int_0^\beta d\tau \int d^3x J_i\varphi_i\right)}{(\text{same with } \ J=0)}$$

The steps leading to Eqs. (1.6) and (1.12) can be followed leading to:

$$\partial_{\mu_{z,\tau}}\Big\langle S_\mu(\vec{z}\,,\tau) \; \bar{\psi}\,(\vec{x}\,,\tau')\Big\rangle_\beta + \delta(\vec{z}-\vec{x})\;\delta(\tau-\tau')\Big\langle i\,\not{\partial}\,\phi(\vec{z}\,,\tau) - F(z,\tau)\Big\rangle_\beta = 0 \ . \tag{2.10}$$

In this expression we can continue analytically to **real-time** and integrate over $\vec{z}\,,t$ leading to

$$\int \, dzdt\partial_{\mu_{z,t}} \, \langle S_\mu(z,t) \; \bar{\psi}\,(x,t')\rangle_\beta = \langle F\rangle_\beta \ . \tag{2.11}$$

By the same token, introducing the Legendre transform Γ^{β}, and continuing to real time we find

$$\int dzdt \left\{ \frac{\delta\Gamma^{\beta}}{\delta\phi} \delta(\vec{z}-\vec{x})\,\delta(t-t') + \langle F \rangle_{\beta} \frac{\delta^2\Gamma^{\beta}}{\delta\psi(x,t')\,\delta\bar{\psi}(z,t)} \right\} = 0 \tag{2.12}$$

where we have assumed that ϕ and F may acquire position independent thermal averages.

Equation (2.11) can be written:

$$\left.\frac{\partial V^{\beta}(F(\phi))}{\partial\phi}\right|_{\substack{<\phi>_{\beta}=v \\ <F>_{\beta}=f}} = f\, S^{-1}_{\beta\psi}(k_0 = 0,\, \vec{k} = 0) = 0 \ . \tag{2.13}$$

The physical meaning of Eq. (2.11) is that if the auxiliary fields acquire a non-vanishing thermal average a zero momentum (long wavelength) fermionic (collective) excitation can be created with zero energy, this is the analogue of the Goldstone theorem at $T = 0$.[14]

The above analysis indicates that whenever $f \neq 0$ there is a "massless" excitation; however there remains the question of under which circumstance $f \neq 0$. To understand this we recall the zero temperature relation[15]

$$m_B^2 - m_F^2 \sim \langle F \rangle \tag{2.14}$$

where m_B = mass of a boson and m_F = mass of the fermion. This relation can be seen to hold at finite temperature leading to the result that $\langle F \rangle_{\beta} \neq 0$ at $T \neq 0$. Indeed at finite temperature the left-hand side of (2.12) is replaced by the temperature dependent "effective" masses and the right-hand side by $\langle F \rangle_{\beta}$. However, since the thermal bath of excitations treats fermions different from bosons through the statistics we expect their "effective" masses to be different, indicating that $\langle F \rangle_{\beta} \neq 0$ at $T \neq 0$, and indeed explicit calculation shows this expectation to be correct.

We are, therefore, led to conclude that at any finite $T \neq 0$ SUSY is broken and as a consequence there is a massless fermionic excitation. Mass here is defined as the value of the energy necessary to create a long wavelength excitation in the reference frame of the heat bath.

In the following section we will carry out the calculations outlined above for some specific models, to show how the Goldstone fermion arises in these examples.

3. EXPLICIT CALCULATIONS IN SOME MODELS

In this section we will compute explicitly the fermion propagator at finite temperature in the real-time formalism as well as the effective potential for the scalar fields in different theories and the relation (2.13) will be checked. We study examples for which SUSY is unbroken at $T = 0$ and show the mechanism of breaking at $T \neq 0$ and the appearance of the massless fermion.

Model A: Wess-Zumino in $D = 4$.[16]

The model is defined by the supermultiplet $\Phi = (\mathcal{Z}, \Psi, \mathcal{H})$ where $\mathcal{Z} = 1/\sqrt{2}\ (\mathcal{A} + i\mathcal{B})$, Ψ a majorana spinor and $\mathcal{H} = 1/\sqrt{2}\ (\mathcal{F} + i\mathcal{G})$ an auxiliary field. $\mathcal{A}$ and $\mathcal{F}$ are scalar and $\mathcal{B}$ and $\mathcal{G}$ are pseudoscalar fields. The Lagrangian is:

$$\mathcal{L} = \partial_\mu \mathcal{Z}\, \partial_\mu \mathcal{Z}^* + \frac{1}{2}\bar{\psi}\, i\, \partial\!\!\!/\, \psi + \mathcal{H}^*\mathcal{H} + \mathcal{H}P'(\mathcal{Z}) + \mathcal{H}^* P'(\mathcal{Z}^*) - \frac{1}{2}\bar{\psi}\,[\gamma_+ P''(\mathcal{Z}) + \gamma_- P''(\mathcal{Z}^*)]\psi \tag{3.1}$$

where $\gamma_\pm = [1/2(1 \pm \gamma_5)]$ and $P(\mathcal{Z})$ is a polynomial of at most third order in $\mathcal{Z}$. We choose:

$$P(\mathcal{Z}) = -\ell \mathcal{Z} + \frac{g}{6}\mathcal{Z}^3 \tag{3.2}$$

with ℓ and g positive constants. In order to calculate the effective potential, we assume that the scalar fields $\mathcal{A}$ and $\mathcal{F}$ can acquire expectation values A and F, respectively.

We shift the fields $\mathcal{A} = \mathcal{A}' + A$ $\mathcal{F} = \mathcal{F}' + F$, the induced masses for the particles can be read off:

$$m_{\mathcal{A}'}^2 = \frac{g}{\sqrt{2}}\left(\frac{g}{\sqrt{2}}A^2 - F\right) \tag{3.3a}$$

$$m_{\mathcal{B}}^2 = \frac{g}{\sqrt{2}}\left(\frac{g}{\sqrt{2}}A^2 + F\right) \tag{3.3b}$$

$$m_\psi = \frac{g}{\sqrt{2}}A \tag{3.3c}$$

The one-loop effective potential in terms of A and F is:

$$\begin{aligned} V_{eff}[A,F] =& V_{tree}[A,F] + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\ell n[k^2 + m_{\mathcal{A}'}^2] + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\ell n[k^2 + m_{\mathcal{B}}^2] \\ & - \int \frac{d^2k}{(2\pi)^4}\ell n[k^2 + m_\psi^2] \end{aligned} \tag{3.4}$$

$$V_{tree}[A,F] = -\frac{1}{2}F^2 - \sqrt{2}F\left[\frac{g}{4}A^2 - \ell\right]$$

Following the methods of Dolan and Jackiw[10] the finite temperature effective potential can be written as $V_{eff} = V_{T=0} + V_\beta$ with:

$$\begin{aligned} V_\beta &= \frac{1}{\beta}\int \frac{d^3k}{(2\pi)^3}\left\{\ell n\left[1 - e^{-\beta E_A}\right] + \ell n\left[1 - e^{-\beta E_B}\right] - 2\ell n\left[1 + e^{-\beta E_\psi}\right]\right\} \\ E_i &= (k^2 + m_i^2)^{1/2} \quad . \end{aligned} \tag{3.5}$$

To apply the results of Section 2 we are interested in $\partial V_{eff}/\partial A$. From Eqs. (3.4) and (3.5) we find:

$$\begin{aligned} \frac{\partial V_{eff}}{\partial A} =& -Fm_\psi + Fm_\psi\frac{g^2}{2}\int \frac{d^4k}{(2\pi)^4}\left\{\frac{1}{(k^2 + m_A^2)(k^2 + m_\psi^2)} - \frac{1}{(k^2 + m_B^2)(k^2 + m_\psi^2)}\right\} \\ & + \frac{1}{2}g^2A\int \frac{d^3k}{(2\pi)^3}\left\{\frac{1}{E_A(e^{\beta E_A} - 1)} + \frac{1}{E_B(e^{\beta E_B} - 1)} + 2\frac{1}{E_\psi(e^{\beta E_\psi} + 1)}\right\} \quad . \end{aligned} \tag{3.6}$$

It is interesting to note that the first two terms (the $T = 0$ contributions) vanish for $F = 0$, thus there is a supersymmetric solution at $T = 0$; however, the finite temperature contribution does not vanish at $F = 0$. A similar result is found for $\partial V_{eff}/\partial F$, so that at $T = 0$ there is a solution for the set of equations $\partial V_{eff}/\partial A = 0$ $\partial V_{eff}/\partial F = 0$ with $F = 0$; at finite temperature these equations cannot be satisfied with $F = 0$.

This interesting result can be traced back to the fact that bosons and fermions obey different statistics in agreement with the conclusions reached in Section 2 and by Girardello[3] *et al.* From Eq. (3.6) we find that the solution to $\partial V_{eff}/\partial A = 0$ gives rise to:

$$F \sim C_\beta \tag{3.7}$$

where C_β is the third contribution (temperature correction) in (3.6).

At low temperatures we can set $m_A = m_B = m_\psi = m$ in C_β and we find:

$$\int \frac{d^3k}{(2\pi)^3} \frac{\exp(-\beta\sqrt{k^2+m^2})}{\sqrt{k^2+m^2}} \approx m^2 \left(\frac{T}{m}\right)^{3/2} e^{-(m/T)} \tag{3.8}$$

$$F \approx 2g^2 A m \left(\frac{T}{m}\right)^{3/2} e^{-(m/T)} \tag{3.9}$$

for $T \ll m$.

The next step is to compute the real-time inverse fermion propagator at $p_0 = 0$, $\vec{p} = 0$.

$$\begin{aligned} S^{-1}(p) &= S_0^{-1}(p) - \Sigma(p) \\ S_0^{-1}(p) &= -i(\not{p} - m_\psi) \ . \end{aligned} \tag{3.10}$$

Using the real-time propagators quoted in Eq. (2.8) we find for one loop:

$$\Sigma = \Sigma_{T=0} + \Sigma_\beta$$

$$\Sigma_{T=0}(p_\mu = 0) = ig^2 \, \frac{m_\psi}{2} \int \frac{d^4k}{(2\pi)^4}$$

$$\times \left[\frac{1}{(k^2+m_A^2)(k^2+m_\psi^2)} - \frac{1}{(k^2+m_B^2)(k^2+m_\psi^2)} \right] \tag{3.11}$$

$$\Sigma_\beta(p_0 = 0, \vec{p} = 0) = \frac{ig^2 A}{2F} \int \frac{d^3k}{(2\pi)^3}$$

$$\times \left[\frac{1}{E_A(e^{\beta E_A} - 1)} + \frac{1}{E_B(e^{\beta E_B} - 1)} + \frac{2}{E_\psi(e^{\beta E_\psi} + 1)} \right]$$

where we have used the relations (3.3a,c). Comparing Eqs. (3.11) and (3.6) we find that indeed:

$$\frac{\partial V_{eff}}{\partial A} = F\,(iS^{-1}(p_0 = 0, \vec{p} = 0)) = 0 \quad . \tag{3.12}$$

Therefore, since $F \neq 0$ (see (3.9)) $S^{-1}(p_0 = 0, \ \vec{p} = 0) = 0$; thus there is a pole at $p_0 = 0, \ \vec{p} = 0$ in the fermion propagator. It is interesting to notice that due to conditions (3.7) and (3.9) Σ_β in (3.11) turns out to be temperature independent (at low temperatures).

Although we have not mentioned the renormalization procedure for this theory the reader can be easily convinced that renormalization will not affect our results, since renormalization can be performed in a temperature independent fashion.

Model B:

Now we will study the non-linear sigma model in two dimensions in the large N limit.[17] The model is described by a supermultiplet (n^a, ψ^a, F^a) of real fields and $a = 1, 2 \ldots N$. The Lagrangian for the theory is:

$$\mathcal{L} = \frac{1}{2g^2} \left[n^a(-\partial^2)n^a + \bar{\psi}^a \; i \; \partial\!\!\!/ \; \psi^a + F^a F^a \right] \tag{3.13}$$

with the constraints

$$n^a n^a = 1 \tag{3.14a}$$

$$\psi^a n^a = 0 \tag{3.14b}$$

$$2n^a F^a = \bar{\psi}^a \psi^a \tag{3.14c}$$.

We introduce a supermultiplet of real Lagrange multipliers (A_0, F_0, ψ_0) and write the Lagrangian of the theory as (after integrating over F^a):

$$\mathcal{L}' = \frac{1}{2}\frac{F_0}{g^2} + \frac{1}{2}\, n^a \left[-\partial^2 - A_0^2 - F_0\right] n^a + \frac{1}{2}\, \bar{\psi}^a \left[i\, \not{\partial} - A_0\right] + \bar{\psi}_0\, n^a \psi_a \tag{3.15}$$

where we have integrated over F^a and rescaled the supermultiplets (n^a, F^a, ψ^a) $\rightarrow$ $1/g(n^a, F^a, \psi^a)$ and $(A_0, F_0, \psi_0) \rightarrow 1/g^2(A_0, F_0, \psi_0)$. The leading order in the large N expansion is obtained integrating out the n^a and ψ^a fields. We end up with an effective theory in terms of the fields A_0, F_0 and ψ_0 with effective Lagrangian:

$$\mathcal{L}_{eff} = \frac{1}{2}\frac{F_0}{g^2} - \frac{iN}{2}\, Tr\ell n(i\, \not{\partial} - A_0) + \frac{iN}{2}\, Tr\ell n\left(-\partial^2 - A_0^2 - F_0 - \bar{\psi}_0\, \frac{i}{i\, \not{\partial} - A_0}\, \psi_0\right) \tag{3.16}$$

and effective potential:

$$V_{eff}[A_0, F_0] = -\frac{1}{2}\frac{F_0}{g^2} + \frac{N}{2}\int \frac{d^2k}{(2\pi)^2}\, \ell n\left[k^2 + A_0^2 + F_0\right] - \frac{N}{2}\int \frac{d^2k}{(2\pi)^2}\, \ell n[k^2 + A_0^2] \ . \tag{3.17}$$

The finite temperature effective potential is written as $V_{eff} = V_{T=0} + V_\beta$ with

$$\begin{aligned} V_\beta &= \frac{N}{\beta}\int \frac{dk}{(2\pi)}\left\{\ell n\left[1 - e^{-\beta E_B}\right] - \ell n\left[1 + e^{\beta E_\psi}\right]\right\} \\ E_{B,\psi} &= \left(k^2 + m_{B,\psi}^2\right)^{1/2} \qquad m_B^2 = A_0^2 + F_0 \\ & \qquad\qquad\qquad\qquad\quad m_\psi^2 = A_0^2 \end{aligned} \tag{3.19}$$

The extremum condition for A_0 reads:

$$\begin{aligned} 0 = \frac{\partial V}{\partial A_0} = m_\psi N \Bigg\{ &\int \frac{d^2k}{(2\pi)^2}\left[\frac{1}{k^2 + m_B^2} - \frac{1}{k^2 + m_\psi^2}\right] \\ &+ \int \frac{dk}{(2\pi)}\left[\frac{1}{E_B(e^{\beta E_B} - 1)} + \frac{1}{E_\psi(e^{\beta E_\psi} + 1)}\right]\Bigg\} \ . \end{aligned} \tag{3.20}$$

At zero temperature it has been recognized by Alvarez[17] that the particle associated to the field A_0 is a fermion-fermion bound state created by the operator $\bar{\psi}^a \psi^a$ and ψ_0 is associated to a fermion-boson bound state created by the operator $n^a i \not{\partial} \psi^a$ and is the super-partner of A_0, their propagators have a pole (and branch cut) at $k^2 = 4m^2$ with $m = \langle A_0 \rangle$, the ground state being supersymmetric ($m_B = m_\psi$) the extremum equations allow a solution with $\langle F_0 \rangle = 0$.

For $T \neq 0$ the extremum conditions $\partial V/\partial A_0 = 0$, $\partial V/\partial F_0 = 0$, cannot be satisfied with $\langle F_0 \rangle_\beta = 0$ and we find $\langle F_0 \rangle_\beta \approx m\, e^{-m/T}$.

To calculate the self-energy of the fermion ψ_0 we use the real-time propagators given in Eq. (2.8). The interaction vertex is $\mathcal{L}_{int} = \bar{\psi}_0\, n^a\, \psi^a$. The leading contribution in large N is:

$$\begin{aligned}\Sigma_\beta(p_0, \vec{p}) = -N \int \frac{d^2k}{(2\pi)^2} \Bigg\{ & -\frac{(\not{p} + \not{k} + m_\psi)}{[(p+k)^2 - m_\psi^2](k^2 - m_B^2)} \\ & + 2\pi i \frac{(\not{p} + \not{k} + m_\psi)\, \delta(k^2 - m_B^2)}{(e^{\beta E_B} - 1)\, [(p+k)^2 - m_B^2]} \\ & - 2\pi i \frac{(\not{p} + \not{k} + m_\psi)\, \delta[(p+k)^2 - m_\psi^2]}{(k^2 - m_B^2)(e^{\beta E_\psi} + 1)} \Bigg\} \ .\end{aligned} \tag{3.21}$$

Using (3.19) and (3.20) we find ($\langle F_0 \rangle_\beta \neq 0$):

$$0 = \frac{\partial V}{\partial A} = \langle F_0 \rangle_\beta\, S_{\psi_0}^{-1}\, (p_0 = 0, \vec{p} = 0) \ . \tag{3.22}$$

In this case the "Goldstone fermion" arises as a boson-fermion bound state. In leading order, the model is renormalized by wave function renormalization for F_0 and, as can be seen, this does not modify our results.[18]

At this point there are two puzzling questions that can be raised, the first is: how is it that at $T \to 0$ SUSY is explicit and the fermion is massive, and at $T \neq 0$ SUSY is broken and the fermion Green's function acquires a massless pole even at very low T? The second question is: how is the current algebra relation (2.11) realized?[19]

To answer these questions we will study a simple model in two dimensions in which the physics of these phenomena will be clearly exposed.

Model C:

The Lagrangian for this model reads

$$\mathcal{L} = \frac{1}{2}\Big[(\partial_\mu \phi)^2 + \bar{\psi}\, i\, \partial\!\!\!/ \,\phi + F^2 - 2mF(\phi^2 - b) - 2m\phi\, \bar{\psi}\, \psi\Big] \tag{3.23}$$

As usual to calculate the effective potential we shift the fields $\phi \to \phi + \varphi$, $F \to F + f$, $\langle F \rangle = 0$. Using by now standard techniques we find at $T \neq 0$:

$$\begin{aligned} 0 = \frac{\partial V}{\partial \phi}\bigg|_{\varphi, f} &= m_{\psi f} - 4m^2 f\, m_\psi\, C_0 + 2m\, m_\psi\, C_\beta \\ \frac{\partial V}{\partial \phi}\bigg|_{\varphi, f} &= f\, S_\psi^{-1}(p_0 = 0, \vec{p} = 0)\ . \end{aligned} \tag{3.24}$$

where

$$m_\psi = 2m\varphi$$

$$m_B^2 = 2mf + 4m^2\varphi^2$$

$$C_0 = \int \frac{d^2k}{(2\pi)^2} \frac{1}{(k^2 + m_\psi^2)(k^2 + m_B^2)}$$

$$C_\beta = \int \frac{dk}{(2\pi)} \left\{ \frac{1}{E_B(e^{\beta E_B} - 1)} + \frac{1}{E_\psi(e^{\beta E_\psi} + 1)} \right\}$$

with the definitions of E_B, E_ψ given in Eq. (3.19). From (3.24) we find at low T:

$$f \simeq -2m\, e^{-(2m/T)} \times (\text{power of } m/T)\ . \tag{3.25}$$

Since the inverse propagator vanishes at $p_0 = 0$ $\vec{p} = 0$ it can be written at small p_0, $\vec{p}$ as:

$$S^{-1} \approx -i\, p\!\!\!/ \, A(p^2 = 0) - i\, \gamma_0\, p_0\, B_\beta - i\, \vec{\gamma} \cdot \vec{p}\, D_B \tag{3.26}$$

where B_β, D_β are temperature dependent (momentum independent) constants. Since at finite temperature the theory is not Lorentz invariant $B_\beta \neq D_\beta$ and we will calculate $S^{-1}(p_0, \vec{p} = 0)$. The one loop contribution to the self-energy Σ can be calculated using again the real-time propagators. We find the finite temperature contribution to Σ linear in p_0:

$$4m^2 i\, \Sigma'_\beta(p_0, \vec{p} = 0) = \frac{4m^2 i(2\gamma_0 p_0)}{(m_B^2 - m_\psi^2)^2} \int \frac{dk}{(2\pi)} \left[\frac{E_B}{(e^{\beta E_B} - 1)} + \frac{E_\psi}{(e^{\beta E_\psi} + 1)}\right] \tag{3.27}$$

From Eqs. (3.24) and (3.25) Σ_β can be written as

$$\Sigma'_\beta \propto i\gamma_0 p_0\, e^{m/T} \quad . \tag{3.28}$$

Therefore we see that as $T \to 0$ this term overwhelms the zero temperature contribution to $S^{-1}(p)$, hence

$$S(p_0, \vec{p} = 0)_{\substack{p_0 \to 0 \\ T \to 0}} \frac{e^{-(m/T)}}{\gamma_0 p_0} \approx \frac{f}{\gamma_0 p_0} \times (\text{powers of } T/m) \quad . \tag{3.29}$$

This expression indicates that the residue of the Goldstone pole vanishes as $T \to 0$ (from (3.27) we see that the residue is positive) clearly exposing the fact that the Goldstone contribution should vanish as $T \to 0$.

To study the way in which the current algebra relation (2.11) is realized we notice that in this model the supercurrent is:

$$S_\mu = (\not{\partial}\phi + iF)\gamma^\mu \psi \quad . \tag{3.30}$$

After shifting the fields Eq. (3.30) can be written as:

$$S_\mu = i f \gamma^\mu \psi + (\not{\partial}\phi + iF)\gamma^\mu \psi \quad . \tag{3.31}$$

Since $\langle \bar{\psi}\, \psi \rangle$ is proportional to f as $p \to 0$, it is easy to see that the first term in (3.31) contributes to higher order in $e^{-(m/T)}$ to (2.11). The second term gives rise to:

$$\partial_\mu S_{\mu_g} = (\Box\phi + i\,\not{\partial} F)\psi + (\not{\partial}\phi + iF)\,\not{\partial}\psi \quad . \tag{3.32}$$

Using the linearized equation of motion for F:

$$F = m_\psi \phi + \dots \tag{3.33}$$

and adding and subtracting the mass terms for ϕ and ψ we find:

$$\partial_\mu S_\mu = \left(m_\psi^2 - m_B^2\right)\phi\psi + \left\{(\Box\phi + m_B^2\phi)\,\psi + (\not{\partial}\,\phi + im_\psi\phi)\,(\not{\partial}\,\psi + im_\psi\psi)\right\} \ . \tag{3.34}$$

Up to one loop (2.11) can be written as:

$$\begin{aligned} I(p) = & - i2m \int \frac{d^2k}{(2\pi)^2}\left\{(-2mf) - (k^2 - m_B^2) + (\not{k} + m_\psi)\,(\not{p} + \not{k} - m_\psi)\right\} \\ & \times D(k)\,S(p+k)\,S(p)\,\delta(p) \end{aligned} \tag{3.35}$$

where $D(k)$, $S(k)$ are the real-time boson and fermion propagators respectively. In Appendix B it is shown that the p_μ independent contribution cancels out and as $p_\mu \to 0$ and $T \to 0$

$$I(p) = \lim_{p\to 0} i4m^2 f \Sigma'_\beta(0)\,S(p) \tag{3.36}$$

where $\Sigma'_\beta(0)$ is the linear term in p_0 of the temperature correction to the self energy.

From Eqs. (3.26) and (3.27) we find the relation

$$S_\psi^{-1} = 4m^2 i \Sigma'_\beta(0) \ . \tag{3.37}$$

Therefore the relation (2.11) is fulfilled.

4. CONCLUSIONS

In this paper we have shown that in any theory with unbroken SUSY at $T = 0$, the symmetry is broken at any temperature, $T \neq 0$ due to different statistics for fermions and bosons in agreement with previous work.

Furthermore, looking at the real-time thermal Green's functions, we established that the breaking of SUSY is associated to a massless pole in fermionic Green's functions – Goldstone fermion – as a consequence of the Goldstone phenomenon. Therefore, there is a definite physical observable as a consequence of this breaking.

We have shown that at very low temperatures the residue at this massless pole is of the form $e^{-(m/T)}$ where m is the common mass of the supermultiplet at $T = 0$, hence the contribution of this pole vanishes at $T = 0$.

We have also shown how current algebra relations are fulfilled at finite temperature.

ACKNOWLEDGEMENTS

It is a pleasure to thank Helen Quinn and Michael Peskin to whom I am deeply indebted for their illuminating remarks and encouraging comments. I would also like to thank the Theory Division and C.N.L.S. at Los Alamos National Laboratory for their hospitality while part of this work was done.

REFERENCES

1. A. Das and M. Kaku, Phys. Rev. **D 18** (1978) 4540.
2. S. Weinberg, Phys. Rev. **D 9** (1974) 3357;

   R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **42** (1979) 1651, Phys. Rev. **D 20** (1979) 3390.
3. L. Girardello, M. T. Grisaru and P. Salomonson, Nucl. Phys. **B178** (1981) 331.
4. L. Van Hove, Nucl. Phys. **B207** (1982) 15.
5. M. B. Paranjape, A. Taormina and L.C.R. Wijewardhana, Phys. Rev. Lett. **50** (1983) 1350.
6. D. A. Dicus and Xerxes R. Tata, University of Texas preprint (1983).
7. T. E. Clark and S. T. Love, Nucl. Phys. **B217** (1983) 349.
8. H. A. Weldon, Phys. Rev. **D 26** (1982) 1394, and references therein.
9. A. L. Fetter and J. D. Walecka, Quantum Theory of Many Particle Systems (McGraw-Hill, New York, 1971) page 291.

   See also L. Kadanoff and G. Baym, Quantum Statistical Mechanics (Benjamin, New York, 1962).
10. L. Dolan and R. Jackiw, Phys. Rev. **D 9** (1974) 3320.
11. D. J. Gross, R. Pisarski and L. G. Yaffe, Revs. of Mod. Phys., Vol. 53, No. 1, (1981) 43.
12. This derivation is due to S. Coleman, see his lectures "The Uses of Instantons" at the 1977 International School of Subnuclear Physics, Ettore Majorana.
13. C. Bernard, Phys. Rev. **D 9** (1974) 3312.
14. I am grateful to Helen Quinn for having shared with me some of her unpublished results on this point.
15. E. Witten, Nucl. Phys. **B202** (1982) 253, Nucl. Phys. **B185** (1981) 513.

16. This is one of the models studied in Ref. 3.

17. O. Alvarez, Phys. Rev. **D 17** (1978) 1123.

18. As usual the theory is renormalized with zero temperature counterterms (see, for example Ref. 11).

19. I am grateful to Michael Peskin for having raised this question and showing me the correct path to the answer.

20. L. Alvarez-Gaumè, D. Z. Freedman and M. T. Grisaru, Harvard preprint HUTMP 81/B111 (1981).

21. T. Murphy and L. O'Raifeartaigh, Nucl. Phys. **B218** (1983) 484.

APPENDIX A

The Controversy of the Dummy Field

In a recent paper by Alvarez-Gaumè[20] *et al.*, there was a conflict regarding the effective potential as a function of two varibles f and ϕ. In their computations, the effective potential as a function of f and ϕ was different from the one obtained after eliminating the field f at the classical level. It is shown in this Appendix that in the loop expansion there is no such puzzle which we believe is due to an inconsistency in their solution to the extremum equations for f. The Lagrangian of the theory is given by Eq. (3.23) without the Fermi fields.

After integrating out the field F using the equations of motion and shifting $\phi \rightarrow \phi + \varphi$ we find up to one loop:

$$V_{eff}[\varphi] = V_{tree}[\varphi] + \frac{\hbar}{2} \int \frac{d^2k}{(2\pi)^2} \ell n[k^2 + 2m^2(3\varphi^2 - b)] \qquad (A.1)$$

where we explicitly wrote $\hbar$ in the one loop contribution.

If we keep F as a variable the effective potential is calculated as usual after shifting $F \rightarrow F + f$, $\phi \rightarrow \phi + \varphi$ the result is:

$$V_{eff}[f, \varphi] = V_{tree}[f, \varphi] + \frac{\hbar}{2} \int \frac{d^2k}{(2\pi)^2} \ell n\,[k^2 + 2mf + 4m^2\varphi^2] \qquad (A.2)$$

the extremum conditions read

$$\frac{\partial V}{\partial \varphi} = 0 = 2mf\varphi + \frac{\hbar}{2} \int \frac{d^2k}{(2\pi)^2} \frac{8m^2\varphi}{k^2 + 2mf + 4m^2\varphi^2} \qquad (A.3a)$$

$$\frac{\partial V}{\partial f} = 0 = -f + m(\varphi^2 - b) + m\hbar \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 + 2mf + 4m^2\varphi^2} \quad . \qquad (A.3b)$$

Equation (A.3b) has to be solved iteratively in powers of $\hbar$, and we find up to order $\hbar$

$$f = m(\varphi^2 - b) + m\hbar \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 + 4m^2\varphi^2 + 2m^2(\varphi^2 - b)} \quad . \qquad (A.4)$$

Using this result for f in (A.2) and keeping terms up to order $\hbar$ we find Eq. (A.1).

This result generalizes to higher orders. This is in agreement with what has been suggested recently by Murphy and O'Raifeartaigh.[21]

APPENDIX B

Here we perform the computations leading to Eq. (3.36) in the text.

Using the real-time propagators (2.8) the term:

$$B_1 = i2m \int \frac{d^2k}{(2\pi)^2}(k^2 - m_B^2)\, D(k)\, S(p+k) \tag{B.1}$$

can be written as

$$B_1 = -2m\, m_\psi \left\{ \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 + m_\psi^2} - \int \frac{dk}{(2\pi)} \frac{n_\psi}{E_\psi} \right\} . \tag{B.2}$$

The term

$$B_2 = -i2m \int \frac{d^2k}{(2\pi)^2} \,(\not{k} + m_\psi)\,(\not{p} + \not{k} - m_\psi)\, D(k)\, S(p+k)$$

is written as

$$B_2 = 2m\, m_\psi \left\{ \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 + m_B^2} + \int \frac{dk}{(2\pi)} \frac{n_B}{E_B} \right\} . \tag{B.3}$$

Therefore

$$\begin{aligned} B_1 + B_2 = &- 4m^2 f\, m_\psi \int \frac{d^2k}{(2\pi)^2} \frac{1}{(k^2 + m_B^2)(k^2 + m_\psi^2)} \\ &+ 2m\, m_\psi \int \frac{dk}{2\pi}\left[\frac{n_B}{E_B} + \frac{n_\psi}{E_\psi}\right] . \end{aligned} \tag{B.4}$$

The $p_\mu = 0$ part of the term

$$\Sigma(p) = i4m^2 f \int \frac{d^2k}{(2\pi)^2} D(k)\, S(k+p) \tag{B.5}$$

yields

$$\begin{aligned} \Sigma(p=0) = &4m^2 f\, m_\psi \int \frac{d^2k}{(2\pi)^2} \frac{1}{(k^2 + m_\psi^2)(k^2 + m_B^2)} \\ &- 2m\, m_\psi \int \frac{dk}{(2\pi)} \left[\frac{n_B}{E_B} + \frac{n_\psi}{E_\psi}\right] \end{aligned} \tag{B.6}$$

therefore $B_1 + B_2 + \Sigma(p=0) = 0$. Hence as $p \to 0$, $T \to 0$, the leading contribution comes from the term in $\Sigma_\beta(p)$ linear in p_0 (at $\vec{p} = 0$) giving rise to Eq. (3.36) in the text.