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IMPROVED BOUNDS ON SOME WEAK AMPLITUDES FROM THE b LIFETIME*

F. J. GILMAN AND J. S. HAGELIN
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

We use recent information on the b lifetime to improve bounds on short-distance contributions arising from the t quark to various weak amplitudes. In particular, our previous lower bound on ϵ'/ϵ is substantially increased, while the upper bound on $K \rightarrow \pi\nu\bar{\nu}$ is reduced.

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With three generations of quarks, the mixing between weak interaction eigenstates and quark mass eigenstates is parametrized by a 3×3 unitary (K-M) matrix[1] with three Cabibbo-like angles θ_i and a phase δ. For neutral Kaons, CP violating effects due to virtual transitions to c and t quarks can arise in the $K^0 - \bar{K}^0$ mass matrix and in non-leptonic decay amplitudes. These CP violating amplitudes always involve the combination $\sin\theta_2 \cos\theta_2 \sin\theta_3 \sin\delta \equiv s_2 c_2 s_3 s_\delta$.

In a previous paper[2] we have shown that using the short-distance contribution to the imaginary part of the $K^0 - \bar{K}^0$ mixing amplitude (proportional to ϵ) together with an upper bound on the short-distance contribution to $K_L \to \bar{\mu}\,\mu$, one is able to establish a lower bound on $s_2 c_2 s_3 s_\delta$. This results in a lower bound on the other CP violating amplitudes in the neutral Kaon system and in particular on the parameter ϵ' in terms of the matrix element of a single $(V-A) \times (V+A)$ type operator.

With the measurement of the b lifetime[3,4] this bound can be considerably improved by replacing the constraint coming from $K_L \to \bar{\mu}\,\mu$ with information on the K-M angles which follows from the b lifetime and from a bound[5] on $(b \to u)/(b \to c)$ that comes from measurements of semileptonic b decays. In this paper we derive this more stringent lower bound on $s_2 c_2 s_3 s_\delta$ and thus ϵ'/ϵ, with care to be on the conservative side in employing the experimental data. The same information is used to limit the short-distance contribution from virtual t quarks in other processes, and we explicitly derive bounds on $K^+ \to \pi^+ \nu\,\bar{\nu}$ and $B^0 - \bar{B}^0$ mixing as well.

We recall first of all that the short-distance contribution to the imaginary part of the $K^0 - \bar{K}^0$ mass matrix is given by[6,7]

$$\epsilon = \left[\frac{BG_F^2 f_K^2 m_K}{12\sqrt{2}\,\pi^2 \Delta M_K} Im\left(\eta_1 \lambda_c^2 m_c^2 + \eta_2 \lambda_t^2 m_t^2 + 2\eta_3 \lambda_c \lambda_t m_c^2 \ell n \frac{m_t^2}{m_c^2}\right) + \sqrt{2}\,\xi \frac{ReM_{12}^{sd}}{\Delta M}\right] e^{i\pi/4} \quad . \tag{1}$$

In Eq. (1), $\lambda_q \equiv U_{qs}^* U_{qd}$ is a product of K-M matrix elements, B parametrizes the matrix element of the $\Delta S = 2$ operator ($B = +1$ for vacuum insertion), and η_1, η_2, η_3 take account of the strong interaction corrections[8] to the effective $\Delta S = 2$ Hamiltonian relevant to $K^0 - \bar{K}^0$ mixing. These latter parameters have the values[8] 0.7, 0.6 and 0.4, respectively, for $M_W = 80$ GeV, $\Lambda_{QCD} = 0.1$ GeV, and $m_t = 30$ GeV.

The last term arises from shifting from a quark basis to a basis where A_0, the amplitude for $K^0 \rightarrow \pi\pi$ $(I = 0)$, is real. It involves the parameter ξ, proportional to CP violation in the $K^0 \rightarrow \pi\pi$ decay amplitude, which is related to the standard parameter ϵ' by

$$\left|\frac{\epsilon'}{\epsilon}\right| = \frac{1}{\sqrt{2}} \left|\frac{\xi}{\epsilon}\right| \left|\frac{A_2}{A_0}\right| = 15.6|\xi| \, , \tag{2}$$

where we have used the experimental values[9] of $|A_2/A_0| = 1/20$ and $|\epsilon| = 2.27 \times 10^{-3}$. The CP-violating contribution to $K^0 \rightarrow \pi\pi$ $(I = 0)$ decay is dominated by the contribution[10] from a single $(V - A) \times (V + A)$ operator, Q_6, in the effective Hamiltonian $\mathcal{H} = \sum_{i=1}^{6} C_i Q_i$. ImC_6 is proportional to the combination of K-M parameters $s_2 c_2 s_3 s_\delta$, in addition to the usual factor of $\frac{G_F}{\sqrt{2}} s_1$ characteristic of

$\Delta S = 1$ weak amplitudes. Thus we write

$$\begin{aligned}\xi &= \frac{Im < \pi\pi(I=0)|\mathcal{H}|K^0 >}{A_0} \\ &\approx \frac{ImC_6 < \pi\pi(I=0)|Q_6|K^0 >}{A_0} \\ &\equiv (s_2c_2s_3s_\delta)(Im\,\tilde{C}_6)\frac{G_F}{\sqrt{2}}s_1 \; \frac{< \pi\pi(I=0)|Q_6|K^0 >}{A_0} \quad ,\end{aligned} \tag{3}$$

where $G_F s_1/\sqrt{2}$ and A_0, the $K^0 \to \pi\pi$ $(I=0)$ amplitude, have values directly determined by experiment, which we will use. As for ImC_6, the Wilson coefficients of the operators appearing in the effective $\Delta S = 1$ weak Hamiltonian have been derived in a number of analyses[10] of QCD corrections to the weak interactions, usually computed in the leading logarithm approximation to all orders in the strong interactions. These analyses[10] give $Im\,\tilde{C}_6 \approx -0.1$. Since ImC_6 in particular is generated at momentum scales between m_t and m_c, it is truly a short-distance effect susceptible to such a leading logarithm calculation in QCD and is quite stable with respect to changes in parameters (e.g., Λ_{QCD}).

For the matrix element $< \pi\pi(I=0)|Q_6|K^0 >$ where Q_6 is the $(V-A)\times(V+A)$ "penguin" operator

$$[\bar{s}_\alpha\,\gamma^\mu(1-\gamma_5)d_\beta]\;[\bar{u}_\beta\,\gamma_\mu(1+\gamma_5)u_\alpha + \bar{d}_\beta\,\gamma_\mu(1+\gamma_5)d_\alpha + \bar{s}_\beta\,\gamma_\mu(1+\gamma_5)s_\alpha] \;,$$

we choose the bag model value for reasons to follow. To use the bag model matrix element in the literature, we observe that Q_6 is related to the operator O_5 used by Donoghue *et al.*[11] by a factor of 9/16 when matrix elements between color singlet states are taken. Therefore

$$|\langle\pi\pi(I=0)|Q_6|K^0\rangle| \;= \frac{9\sqrt{3}}{16}\;|\langle\pi^0\pi^0(I=0)|O_5|K^0\rangle| \;= 1.4\,GeV^3\;. \tag{4}$$

In the same normalization $A_0 = 4.70 \times 10^{-4}$ MeV. Combining Eqs. (2), (3), and (4) we find

$$\left|\frac{\epsilon'}{\epsilon}\right| = 8.4(s_2 c_2 s_3 s_\delta)\left|\frac{Im\,\tilde{C}_6}{0.1}\right| \left|\frac{\langle\pi\pi(I=0)|Q_6|K^0\rangle}{1.4\ GeV^3}\right| . \qquad (5)$$

Thus a lower bound on ϵ'/ϵ follows from a lower bound on $s_2 c_2 s_3 s_\delta$. (We have previously shown[2] that $\epsilon'/\epsilon > 0$.) For this we turn back to the expression for ϵ in Eq. (1) and use our knowledge of the K-M angles coming from b decay.

We extract this information on the K-M angles from the b lifetime through the relation

$$\frac{1}{\tau_b} = \frac{\Gamma(b \to ce\nu)}{BR(b \to ce\nu)} = \frac{|U_{cb}|^2}{BR(b \to ce\nu)} \frac{G^2 M_b^5}{192\pi^3} F\left(\frac{m_c}{m_b}\right) , \qquad (6)$$

where $U_{cb} = c_1 c_2 s_3 + s_2 c_3 e^{i\delta}$ is the element of the K-M matrix connecting b to c and $F(m_c/m_b) = 1 - 8(m_c/m_b)^2 + 8(m_c/m_b)^6 - (m_c/m_b)^8 - 24(m_c/m_b)^4 \ell n(m_c/m_b)$ is a standard phase space factor to take account of the finite charm mass in the final state. We have the additional information from b decay that[5]

$$\frac{\Gamma(b \to ue\nu)}{\Gamma(b \to ce\nu)} = \left|\frac{U_{ub}}{U_{cb}}\right|^2 \frac{1}{F(m_c/m_t)} = \frac{s_1^2 s_3^2}{|c_1 c_2 s_3 + s_2 c_3 e^{i\delta}|^2 F(m_c/m_t)} < 0.05 . \qquad (7)$$

We will use the measured[5] semileptonic branching fraction in Eq. (6). This avoids the usual procedure of adding up all b decay widths theoretically, something which entails using somewhat uncertain strong interaction enhancement factors and phase space for the non-leptonic channels $b \to c\,\bar{u}\,d$, $b \to c\,\bar{c}\,s$, etc. To the extent that earlier calculations used factors which would result in semileptonic branching fractions in disagreement with experiment they will differ from our derived K-M matrix elements and mixing angles. In our calculations we assume that the spectator model is valid for semileptonic b decays, i.e. that the

b quark decays independently of the other quarks in the b-flavored hadron. We use $m_b = 4.7\,GeV$, $m_c = 1.5$ GeV and $BR(b \to ce\nu) = 13\%$ (present CESR average[5] $11.6 \pm 0.6\%$, PEP and PETRA average[5] $11.8 \pm 1.2\%$), all numbers tending to be on the conservative side with respect to our eventual lower bound on $s_2 c_2 s_3 s_\delta$. The alternative, of using the physical B meson mass for m_b and the mass which fits the electron spectrum in semileptonic decays for m_c, results in a larger lower bound. With the above masses and branching ratio Eqs. (6) and (7) become

$$|U_{bc}| = |s_3 + s_2 e^{i\delta}| = 0.059\ (10^{-12}\,\mathrm{sec}/\tau_b)^{1/2} \tag{8}$$

$$s_3 < 0.040\ (10^{-12}\,\mathrm{sec}/\tau_b)^{1/2}\ , \tag{9}$$

in the very good approximation of small s_2 and s_3. We shall use $\tau_b = 0.6$, 0.9, and 1.2×10^{-12} sec, again on the conservative side of the measurements (MAC[3], $1.8 \pm 0.6 \pm 0.4 \times 10^{-12}$ sec; MkII[4], $1.2\ ^{+0.45}_{-0.36} \pm 0.3 \times 10^{-12}$ sec) with respect to our eventual bound.

The lower bound on $s_2 c_2 s_3 s_\delta$ and hence ϵ'/ϵ follows from imposing Eqs. (8) and (9) as constraints together with Eq. (1), which rewritten with K-M matrix elements expressed in terms of (small) angles and appropriate values for the various masses and constants becomes

$$\begin{aligned}(2.19 \times 10^{-2})\ \mathrm{GeV}^2 = &\left(\frac{B}{0.33}\right) s_2 s_3 s_\delta \\ &\left[-\eta_1 m_c^2 + \eta_3 m_c^2 \ell n\left(\frac{m_t^2}{m_c^2}\right) + \eta_2 m_t^2 s_2 (s_2 + s_3 c_\delta)\right] .\end{aligned} \tag{10}$$

We have dropped the term proportional to ξ on the right-hand side, since we have previously shown[2] it is negative and its presence would only strengthen the bound on $s_2 c_2 s_3 s_\delta$. The parameter B has been explicitly divided by the value 0.33 obtained from a calculation[12] based on current algebra and SU(3) applied to the measured $\Delta I = 3/2$ contribution to $K \to \pi\pi$. Equation (10) is valid for $m_t^2 \ll m_W^2$. Although not expressly written, in our computations we have in fact used the full expression[13] for the right hand side of Eq. (10), valid for any value of m_t, and used the QCD corrections[8] calculated for the leading term in m_t^2.

The resulting lower bounds from Eq. (5) for ϵ'/ϵ are shown in Fig. 1 together with our previous lower bound[2] which was based on using[14] the short-distance contribution to $K_L \to \bar{\mu}\,\mu$ to bound the term proportional to m_t^2 on the right-hand side of Eq. (10). The lower bound is now much larger, typically of order 0.01 instead of 0.002.

The reason for the improved bound can be understood in the context of $K_L \to \bar{\mu}\,\mu$ as well. For $m_t \approx 35$ GeV, the measured b lifetime limits the short-distance contribution to the amplitude for $K_L \to \bar{\mu}\,\mu$ arising from t quark loops to be an order of magnitude smaller than if it saturated the dispersive part of the amplitude allowed by experiment.[14] (The short-distance contributions from c and t quarks to $K_L \to \bar{\mu}\,\mu$ are now comparable, and give a negligible contribution to the rate.) Conversely, the b lifetime limits the term proportional to m_t^2 on the right-hand side of Eq. (10) to be an order of magnitude smaller (for $m_t \approx 35$ GeV) than the upper bound based on saturating the dispersive part of $K_L \to \bar{\mu}\,\mu$.

The actual bounds shown in Fig. 1 turn out to be achieved when s_3 saturates the bound in Eq. (9) and $\cos\delta < 0$, with $\sin\delta$ relatively large (≈ 0.4 to 0.8). The lower cut-offs in m_t for some of the curves in Fig. 1 correspond to there being no solution to Eq. (10) for values of m_t below those points for the given values of B and b lifetimes. This has been emphasized previously by Ginsparg *et al.*,[15] with our cut-offs differing slightly because of the way we connect the lifetime to the K-M mixing angles and the use of different masses.

We have plotted in Fig. 1 the lower bounds on ϵ'/ϵ corresponding to $B = 0.66$ as well as $B = 0.33$, which we used previously. Note that with $B = 0.66$ the lower bounds on m_t do not add anything substantial to our knowledge beyond the direct limits from PETRA.[16] For all these curves we have used $|ImC_6| = 0.1$ and the bag model value[11] of 1.4 GeV^3 for $< \pi\pi(I=0)|Q_6|K^0 >$. We *do not assume* that the $\Delta I = 1/2$ rule is due to "penguin" contributions to $K \to \pi\pi$, which would require "boosting up" this matrix element of Q_6 by at least a factor of two given most calculations[10] of ReC_6. In this sense the bag model matrix element is small and therefore conservative. Indeed, Ginsparg and Wise[17] in calculations similar to these have proposed using ϵ'/ϵ measurements as a way of determining $< \pi\pi(I=0)|Q_6|K^0 >$.

Since the short-distance contribution[6] to $K^+ \to \pi^+\nu_i\bar{\nu}_i$ is dominated by second order weak diagrams involving c and t quarks, much of our analysis can be extended in a straightforward manner to this process as well. The branching ratio for $K^+ \to \pi^+\nu_i\bar{\nu}_i$ per lepton flavor can be normalized to that for $K^+ \to \pi^0 e^+\nu$ with the result[18]

$$\begin{aligned} BR(K^+ \to \pi^+\nu_i\bar{\nu}_i) &= \frac{0.61 \times 10^{-6}}{|U_{us}|^2}\left|\sum_{j=c,t} U^*_{js}U_{jd}D(x_j)\right|^2 \\ &= 0.61 \times 10^{-6}\left|D(x_c) + s_2(s_2 + s_3e^{i\delta})D(x_t)\right|^2 \end{aligned} \tag{11}$$

in the approximation of small mixing angles θ_i and where $x_j = m_j^2/m_W^2$ and[18]

$$D(x) = \frac{1}{8}\left[1 + \frac{3}{(1-x)^2} - \frac{(4-x)^2}{(1-x)^2}\right] x\,\ell n\,x + \frac{x}{4} - \frac{3}{4}\,\frac{x}{1-x}\ . \tag{12}$$

Since we already know[2] that $Re\, s_2(s_2 + s_3 e^{i\delta})$ is positive, the terms in Eq. (11) arising from the c and t quarks interfere constructively and the charm quark contribution alone provides a lower bound of $\approx 0.5 \times 10^{-11}$ on this process per lepton flavor. But we can do better in terms of both a lower and an upper bound by including the constructive interference with the t-quark contribution and using Eqs. (8) and (9) to bound[19] s_2:

$$0.019\,(10^{-12}\,\mathrm{sec}\,/\tau_b)^{1/2} < s_2 < 0.099\,(10^{-12}\,\mathrm{sec}\,/\tau_b)^{1/2} \tag{13}$$

and noting that $|s_2 + s_3 e^{i\delta}| = |s_3 + s_2 e^{i\delta}| = |U_{bc}| = 0.059(10^{-12}\,\mathrm{sec}\,/\tau_b)^{1/2}$. For a lower bound we take $\tau_b = 1.5 \times 10^{-12}$ sec making s_2 as well as $|s_2 + s_3 e^{i\delta}| = |U_{bc}|$ as small as possible. The resulting bound is the solid line in Fig. 2. Conversely, we use $\tau_b = 0.6 \times 10^{-12}$ sec for the upper bound on s_2 and $|s_2 + s_3 e^{i\delta}|$ to obtain the upper bound shown as the dash-dotted line in Fig. 2. The previous upper bound[14], obtained using $K_L \to \bar{\mu}\,\mu$ and still valid, is shown as a dotted line.

We can do even better by adding the additional constraint of making the mixing angles satisfy the equation for ϵ, Eq. (10). This "ϵ constraint" does not affect the upper bound on $BR(K^+ \to \pi^+ \nu_i \bar{\nu}_i)$ very much, since s_2 is not forced to be much less than its maximum value given in Eq. (13) when τ_b is "short" (recall we use $\tau_b = 0.6 \times 10^{-12}$ sec for our upper bound) and/or m_t is large. The result is within a few percent of the upper bound already plotted in Fig. 2. However, the minimum value of s_2 is much improved over that demanded just by Eq. (13). Even with $B = 0.66$ (which relaxes the "ϵ constraint" compared to using $B = 0.33$), the improved lower bound shown as the dashed line in Fig. 2 results.

Summing over three generations of leptons, the expected range of the branching ratio for $K^+ \to \pi^+ \nu\,\bar{\nu}$ is between 3×10^{-11} and 9×10^{-11} when $m_t = 35$

GeV. This is well below a previous[14] "lower bound" of several times 10^{-10} which relied upon a short-distance explanation for the real part of the $K^0 - \bar{K}^0$ mixing amplitude, ΔM_K, in order to constrain the K-M angles. The use of the real part, which was standard procedure in the past, results in values of s_2 and s_3 which are typically much larger than those allowed by the recent measurements of τ_b. With the benefit of hindsight we can see what went wrong. The small values of s_2 and s_3 that result from the b lifetime mean that the t quark contribution to ΔM_K is negligible, and one is left with the contribution coming from the c quark as calculated by Gaillard and Lee.[6] However, if as expected the value of B is significantly less than the vacuum insertion value of unity (such as $B = 0.33$), then this short-distance contribution is completely inadequate to explain the measured ΔM_K. One is forced to conclude that the real part of the $K^0 - \bar{K}^0$ mixing amplitude is not short-distance dominated for such values of B. (In fact, this possibility was mentioned in Ref. 14.) Note that processes such as $K^+ \to \pi^+ +$ photinos or $K^+ \to \pi^+ +$ Higgsinos are also[14,20] proportional to $U_{ts}^* U_{td} m_t^2$ and therefore reduced by the b lifetime in the same proportion as $K^+ \to \pi^+ \nu \bar{\nu}$.

The neutral B meson system involves a different combination of mixing angles since t quark loops will now involve the product of K-M matrix elements $U_{tb} U_{td}^* = (c_1 s_2 s_3 - c_2 c_3 e^{i\delta})(s_1 s_2)^*$. A particular property of interest is $B^0 - \bar{B}^0$ mixing, which results in same sign dileptons when both B mesons produced in e^+e^- annihilation decay semileptonically.

It has been shown[21] that $\Gamma_{12}/M_{12} = O(m_b^2/m_t^2) \ll 1$ and that Γ_{12} and M_{12} have the same phase (up to terms of order m_c^2/m_b^2) for the $B^0 - \bar{B}^0$ system. Either of these conditions makes CP violation small, so to good approximation the number of same sign dileptons divided by all dileptons is given by $2r/(1+r)^2$ where

$$r = \frac{(\Delta M)^2 + (\Delta\Gamma/2)^2}{2\Gamma^2 + (\Delta M)^2 - (\Delta\Gamma/2)^2} . \tag{14}$$

ΔM and $\Delta\Gamma$ are the $B_S^0 - B_L^0$ mass and width differences, respectively, and Γ is the average decay rate. As noted above $\Delta\Gamma/\Delta M$ is expected to be $\ll 1$ and Eq. (14) simplifies to

$$r = \frac{(\Delta M/\Gamma)^2}{2 + (\Delta M/\Gamma)^2} . \tag{15}$$

To leading order in m_t^2, the short-distance contribution to ΔM when $\Delta M \gg \Delta\Gamma$ is given by[21,22]

$$\Delta M = 2\,|M_{12}| = \eta_{QCD}\, \frac{G_F^2 f_B^2 B_B m_B m_t^2}{6\pi^2}\, |(U_{tb}^2 U_{td}^{*2})| , \tag{16}$$

where η_{QCD} is the QCD correction factor[21] (≈ 0.85) while f_B and B_B are the analogues of f_K and B for K mesons. For small s_2 and s_3 the quantity of relevance, $(\Delta M/\Gamma)^2$, becomes

$$\left(\frac{\Delta M}{\Gamma}\right)^2 = (0.058)\left(\frac{f_B}{f_K}\right)^4 B_B^2 \left(\frac{m_t}{35\ GeV}\right)^4 \left[\frac{s_2}{0.099\,(10^{-12}\,\text{sec}\,/\tau_b)^{1/2}}\right]^4 . \tag{17}$$

In Eq. (17) we have normalized f_B relative to f_K since theoretical investigation[23] of the decay constants f_D and f_B indicates their values should not be significantly different from f_π or f_K. We expect $B_B = O(1)$.

s_2 has been normalized in Eq. (17) by its upper bound from Eq. (13): An upper bound on $(\Delta M/\Gamma)^2$ is therefore obtained by replacing the square bracket in Eq. (17) by unity. This upper bound is thus independent of τ_b, and depends only on the ratio $(b \to u)/(b \to c)$. Unless m_t is much larger than 35 GeV we see that $B^0 - \bar{B}^0$ mixing and (same sign dileptons)/(all dileptons) $\approx (\Delta M/\Gamma)^2$ should be an effect of at most several percent.

On the other hand, inserting the lower bound for s_2 of 0.019 $(10^{-12}\,\mathrm{sec}/\tau_b)^{1/2}$ from Eq. (13) results in totally negligible mixing for any reasonable parameters. However, if we impose the "ϵ constraint" of Eq. (10), s_2 is restricted to be much bigger than its lower bound. For example, with $m_t = 35$ GeV, $\tau_b = 1.5 \times 10^{-12}$ sec, and $B = 0.66$ (the last condition designed to relax the "ϵ constraint"), $s_2 \gtrsim 0.06$ while the corresponding bounds from the lifetime alone are $0.081 > s_2 > 0.016$. Thus with $m_t = 35$ GeV and τ_b fixed, the actual range of s_2 is quite small if the ϵ constraint is also imposed. Consequently the amount of $B^0 - \bar{B}^0$ mixing is restricted to lie in a rather limited range compared to what might have been expected[24] from just imposing the b-lifetime constraint.

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REFERENCES

1. M. Kobayashi and T. Maskawa, Prog. Theo. Phys. 49, 652 (1973).
2. F. J. Gilman and J. S. Hagelin, Phys. Lett. 126B, 111 (1983).
3. E. Fernandez *et al.*, Phys. Rev. Lett. 51, 1022 (1983).
4. N. S. Lockyer *et al.*, SLAC preprint SLAC-PUB-3165 and Phys. Rev. Lett. (in press), 1983.
5. S. Stone, invited talk at the 1983 International Symposium on Lepton and Photon Interactions at High Energies, August 4-9, 1983, Ithaca, NY (unpublished) reviews the world data on weak decays of heavy quarks.
6. M. K. Gaillard and B. W. Lee, Phys. Rev. D10, 897 (1974).
7. J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B109, 213 (1976).
8. F. J. Gilman and M. B. Wise, Phys. Lett. 93B, 129 (1980); and Phys. Rev. 27D, 1128 (1983).
9. Particle Data Group, Phys. Lett. 111B (1982).
10. F. J. Gilman and M. B. Wise, Phys. Rev. D20, 2392 (1979); B. Guberina and R. D. Peccei, Nucl. Phys. B163, 289 (1980); R.D.C. Miller and B.H.J. McKellar, Aust. J. Phys. 35, 235 (1982); F. J. Gilman and M. B. Wise, Ref. 8.
11. J. F. Donoghue *et al.*, Phys. Rev. D21, 186 (1980) and D23, 1213 (1981).
12. J. F. Donoghue *et al.*, Phys. Lett. 119B, 412 (1982).
13. J. S. Hagelin, Phys. Rev. D23, 119 (1981); T. Inami and C. S. Lim, Prog. Theo. Phys. 65, 297 (1981).
14. J. Ellis and J. S. Hagelin, Nucl. Phys. B217, 189 (1983).

15. P. H. Ginsparg *et al.*, Phys. Rev. Lett. 50, 1415 (1983).

16. E. Lohrmann, talk at the Topical Conference of the 1983 SLAC Summer Institute on Particle Physics, July 18-29, 1983 (unpublished) reviews the PETRA data.

17. P. H. Ginsparg and M. B. Wise, Harvard University preprint HUTP-83/A027, 1983 (unpublished).

18. T. Inami and C. S. Lim, Ref. 13 and J. Ellis and J. S. Hagelin, Ref. 14. In the calculations in Ref. 14 as well as here we keep higher order terms in m_t^2/m_W^2 as well as the QCD corrections to the leading terms, which are significant, even though this is not explicitly shown in our equations.

19. This is similar to the bounds on s_2 which are also found in K. Kleinknecht and B. Renk, Dortmund preprint UNIDO-83/276, 1983 (unpublished). See also the fit by E. Paschos, B. Stech, and U. Turke, CERN preprint TH.3601-CERN 1983 (unpublished) who also use the real part of the $K^0-\bar{K}^0$ amplitude.

20. M. K. Gaillard *et al.*, Phys. Lett. 123B, 241 (1983).

21. J. S. Hagelin, Nucl. Phys. B193, 123 (1981) and references to previous work therein.

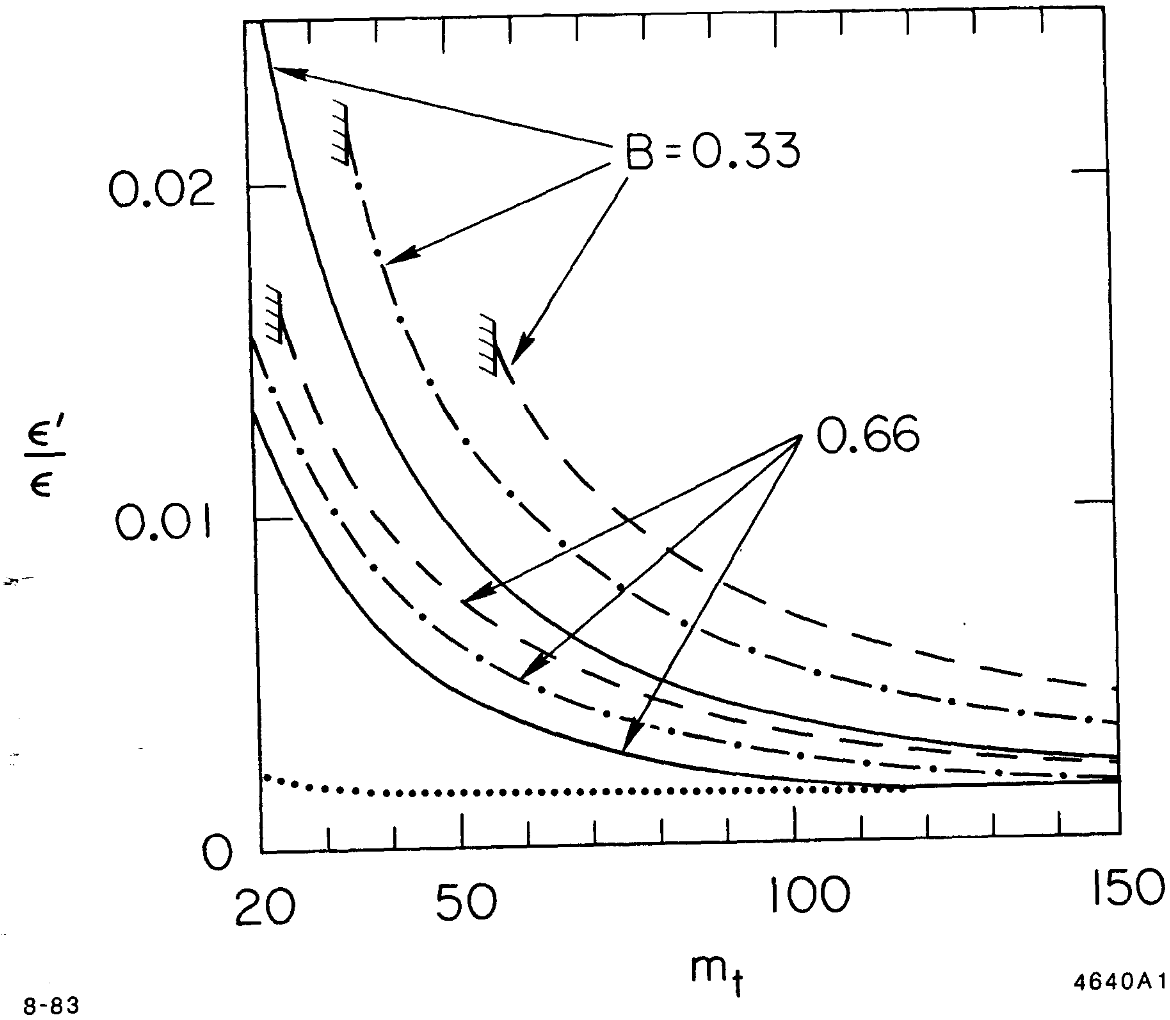
22. J. Ellis *et al.*, Nucl. Phys. B131, 285 (1977).

23. H. Krasemann, Phys. Lett. 96B, 397 (1980). However, see E. Golowich, Phys. Lett. 91B, 271 (1980) and V. Novikov *et al.*, Phys. Rev. Lett. 38, 626 (1977) for lower and higher values respectively.

24. S. Pakvasa, KEK preprint KEK-TH 66, 1983 (unpublished).

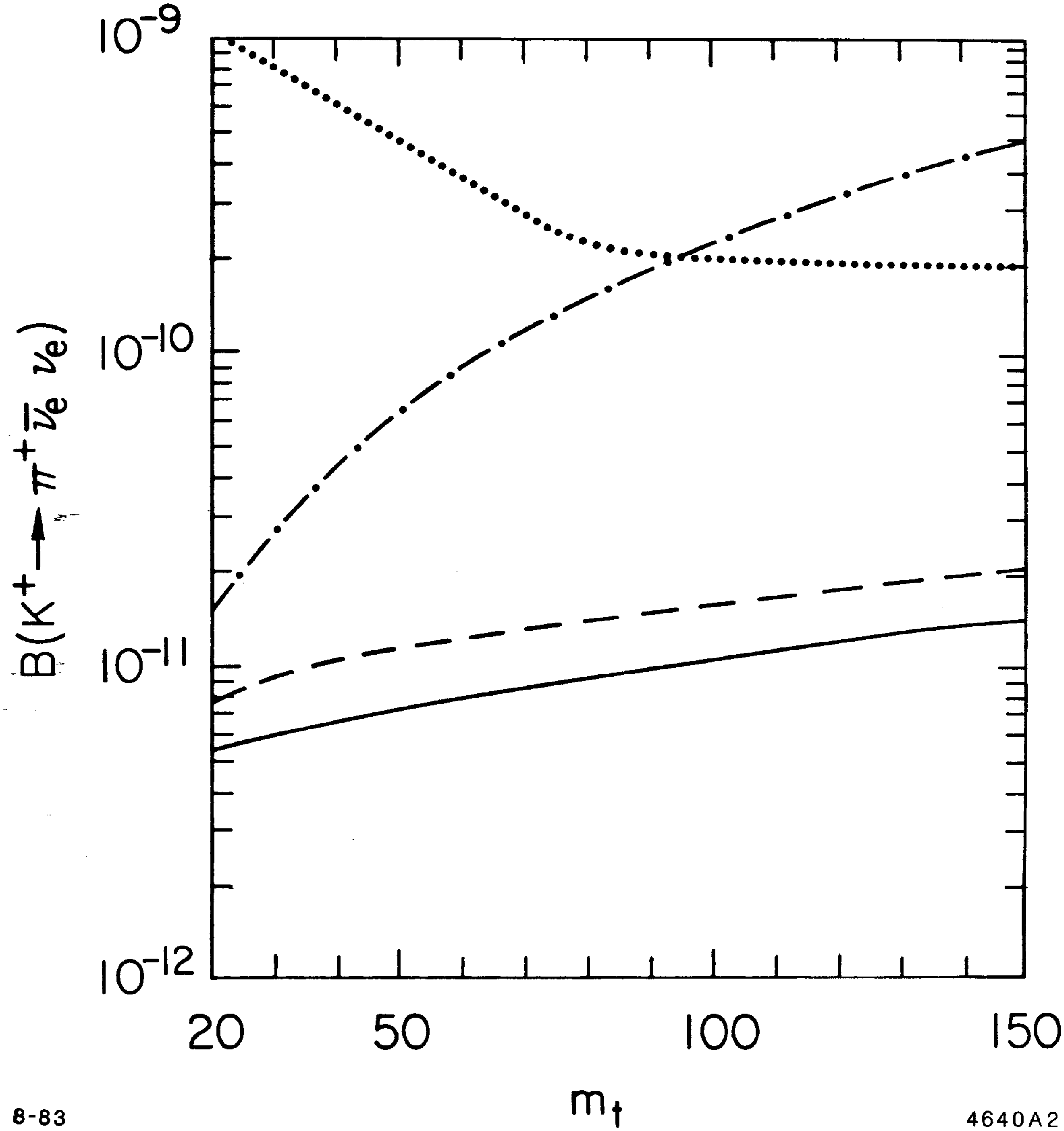
FIGURE CAPTIONS

1. Lower bounds on ϵ'/ϵ for $\tau_b = 0.6 \times 10^{-12}$ sec (solid line), 0.9×10^{-12} sec (dash-dotted line), 1.2×10^{-12} sec (dashed line) and values of the matrix element parameter $B = 0.33$ and 0.66. Shown as a dotted line is the previous lower bound (still valid) for $B = 0.33$ that utilized the short-distance contribution to $K_L \rightarrow \bar{\mu}\mu$ instead of information on the b lifetime.
2. Lower and upper bounds on $BR(K^+ \rightarrow \pi^+\nu_e\bar{\nu}_e)$: solid line – lower bound for $\tau_b \leq 1.5 \times 10^{-12}$ sec; dashed line – lower bound for $\tau_b \leq 1.5 \times 10^{-12}$ sec and K-M angles satisfying Eq. (10), the "ϵ constraint"; dash-dotted line – upper bound for $\tau_b \geq 0.6 \times 10^{-12}$ sec; dotted line – previous upper bound (still valid) using the short-distance contribution to $K_L \rightarrow \bar{\mu}\mu$.



B = 0.33
0.66
0.02
0.01
0
$\frac{\epsilon'}{\epsilon}$
20
50
100
150
m_t
8-83
4640A1

Fig. 1



10^{-9}
10^{-10}
10^{-11}
10^{-12}
$B(K^+ \rightarrow \pi^+ \bar{\nu}_e \nu_e)$
20
50
100
150
m_t
8-83
4640A2

Fig. 2