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ON THE FINITENESS OF θ_{QCD} RENORMALIZATION IN SUPERSYMMETRIC THEORIES*

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ABSTRACT

We point out that neither a softly broken Peccei-Quinn symmetry nor a softly broken supersymmetry per se suffices to insure the finiteness of θ_{QCD} renormalization. This is due to a logarithmic divergence that in general occurs in the higher loop corrections to the gluino mass.

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1. THE PROBLEM

The "Strong CP Problem" is a theorist's problem. Nevertheless it is important to consider this problem seriously; its solution could have powerful constraints on model building. Briefly stated, to account for the non-trivial topological structure of the QCD ground state one uses an effective Lagrangian [1]

$$\mathcal{L}_{eff} = \mathcal{L}_{QCD} + \frac{\theta g^2}{32\pi^2}\, F_{\mu\nu} \cdot \tilde{F}_{\mu\nu} \tag{1.1}$$

The second term in eq. (1.1), which violates P and T invariance, is expected to contribute to the electric dipole moment of the neutron. From the experimental upper bound on that quantity one infers [2]

$$\theta_{exp} \lesssim O(10^{-9}) \tag{1.2}$$

which is a very small number. A priori there is no reason why θ should be so small — this is the first part of the strong CP problem.

In principle one could just choose $\theta = 0$, but then one has to raise the question whether such a choice is stable under quantum corrections [3]. This will however not be the case in general because of the necessary presence of CP violation in the weak interactions. A fermion mass matrix which can be made real and diagonal at the classical level will not remain so under renormalization due to quantum corrections; a re-diagonalization will become necessary. With the unitary transformations U_L and U_R diagonalizing the mass matrix M

$$M_{Diag} = U_R^+ \, M \, U_L \tag{1.3}$$

one finds that the physically relevant parameters is $\bar{\theta}$ expressed as follows [3]

$$\bar{\theta} = \theta + arg\ det\ U_R^+ \, U_L \tag{1.4}$$

When the dynamics contain other sources of CP violation besides the $F \cdot \tilde{F}$ term, as it is the case in the standard Kobayashi-Maskawa ansatz, then the second term in eq. (1.4) will not vanish. CP violation residing in "hard", i.e. dimension $d = 4$ operators will lead to infinite mass renormalization at some order. Therefore the choice $\theta = 0$ does not imply that $\bar{\theta}$ vanishes; in general $\bar{\theta}$ can be infinite! Of course $\bar{\theta}$ can be renormalized to zero in such a case; however this would require a large redialing of parameters at each order of perturbation theory and is therefore considered to be "unnatural." This constitutes the second part of the strong CP problem: ideally quantum corrections should be allowed to shift $\bar{\theta}$ by only tiny amounts at most.

2. PARTIAL SOLUTIONS WITHIN SUPERSYMMETRIC MODELS

2.1 PECCEI-QUINN TYPE SOLUTIONS

Supersymmetry (SUSY) can, if not solve, at least alleviate the problem. First of all, SUSY offers a natural habitat for Peccei-Quinn type symmetries [4,5]. For example consider a minimal supersymmetric $SU(2) \times U(1)$ model which contains two $SU(2)$ doublets and one singlet Higgs field [6]. In the limit when certain mass parameters vanish it exhibits a R-symmetry [6,7] defined by its transformation on the chiral Higgs superfields:

$$\phi(x, \theta) \underset{R}{\rightarrow} e^{-2i\alpha/3} \phi(x, e^{i\alpha}\theta) \tag{2.1}$$

The corresponding R-current contains a color anomaly; thus R-symmetry acts as a Peccei-Quinn symmetry and allows θ to change by an arbitrary amount: thus $\bar{\theta}$ can always be rotated to zero. R-invariance, like SUSY, has to be broken. If it is broken spontaneously at a very high energy scale, a superlight and almost invisible axion [8] will exist in the spectrum of the theory as the price to be paid for solving the strong CP problem.

2.2 CAVEATS

It has been suggested [9] that if SUSY and thus R-invariance are broken by operators with dimension $d < 4$ then $\bar{\theta}$ stays finite to all orders of perturbation theory and may possibly be very small due to the "softness" of the breaking. This argument by itself is fallacious as can be seen from the connection between θ and chiral invariance due to a massless quark. Consider a simple quark model where the quark masses occur as explicit mass terms (rather than gaining mass through the Higgs mechanism). Now, θ is defined by the structure of the ground state. However, if one of the quarks, say the up quark, were massless, then there would be an extra chiral symmetry in the problem so that one could rotate θ via the anomaly to any arbitrary value, even zero, without changing the physical content of the theory. However the up quark can obtain its mass by a $d = 3$ (i.e. "soft") operator in this simplified model. θ_{QCD} is then a physical parameter and it appears as the coefficient of the $d = 4$, T-violating operator $F_{\mu\nu} \cdot \tilde{F}_{\mu\nu}$. Whenever CP or T-invariance is broken by some other $d = 4$ operator (as it is the case in the weak sector of the Kobayaski-Maskawa model [10]) $\bar{\theta}$ will in general be infinitely renormalized at some order [3]. The same thing happens when R-invariance or a Peccei-Quinn symmetry is broken "softly". At the end of sec. 2.3, we will give an explicit example of this point.

An independent argument [11] notes that $\bar{\theta}$ renormalization is driven by fermion mass renormalization; it then invokes the famous non-renormalization theorems of SUSY [12] to conclude that $\bar{\theta}_{ren} < \infty$. This could solve the second part of the strong

CP problem (although not the first one). The argument is certainly correct for unbroken SUSY (where $\delta\theta = 0$) and for a spontaneously broken SUSY. In the case of SUSY that is broken explicitely by operators of dimension $d < 4$ a more detailed analysis is required. As shown by Girardello and Grisaru [13], certain SUSY breaking operators of dimension $d = 3$ can introduce new quadratic divergences into the unrenormalized theory; this happens when one inserts an explicit mass term for the fermionic component of a chiral superfield (such as a quark mass term). Other $d = 3$ operators such as mass terms for the gauginos, or $d = 2$ operators such as mass terms for the scalar components of chiral superfields will still lead to new logarithmic divergences (these operators are called "soft supersymmetry breaking terms" in ref. [13]). Assuming that CP violation resides in $d = 4$ operators, the imaginary part of fermionic mass terms and thus $\delta\theta_{ren}$ could in general exhibit such logarithmic divergences. In the next section, we illustrate some ways by which divergent fermion mass terms could arise and discuss whether an infinite $\delta\theta_{ren}$ would occur.

The results of this section suggest that a logarithmically divergent θ (coefficient of a dimension-4 operator) could result from the addition of a soft symmetry-breaking $d < 4$ operator. At first, this seems like a violation of Symanzik's Theorem [14] (which would claim that no new divergences could be introduced with dimension greater than that of the symmetry breaking term). The theorem is evaded in this case for a subtle reason. The symmetry limit in question does not require $\bar{\theta} = 0$, but requires that the physics be independent of $\bar{\theta}$. In fact, $\bar{\theta}$ could be infinite, but in the symmetry limit, an "infinite" rotation to $\bar{\theta} = 0$ does not effect the physics. Thus, an infinite $\bar{\theta}$ in the presence of the soft symmetry-breaking term is not a new divergence and Symanzik's Theorem does not apply.

2.3 INFINITE RENORMALIZATION OF FERMION MASSES DUE TO SOFT SUSY BREAKING

In a theory with exact global SUSY, the fermion masses which appear at tree level are unrenormalized to all orders in perturbation theory. This is a consequence of the non-renormalization theorem of SUSY [12] (which states that F-terms do not get renormalized). Logarithmically divergent renormalization of fermion mass terms can be introduced by soft SUSY breaking terms. We present three such examples and comment on whether these can result in an infinite renormalization of $\bar{\theta}$.

First, let us introduce an explicit mass term for the scalar fields which breaks SUSY via $d = 2$ operators:

$$\mathcal{L}_{soft} = \frac{1}{2}\mu^2(A_1^2 + A_2^2) \tag{2.2}$$

where $A = (A_1 + iA_2)/\sqrt{2}$ is the spin zero component field of a chiral multiplet ϕ. Here we use the usual notation for chiral superfields [15]

$$\phi(x,\theta) = A(x) + \sqrt{2}\,\theta\,\psi(x) + \theta^2 F(x) \tag{2.3}$$

where F is a complex auxiliary field. Equation (2.2) can be rewritten as follows [13]:

$$\mathcal{L}_{soft} = \int d^4\,\theta\, U\,\bar{\phi}\,\phi \tag{2.4}$$

where $U = \mu^2\ \theta^2\ \bar{\theta}^2$ is a dimension zero spurion superfield; θ and $\bar{\theta}$ are Grassmann variables not to be confused with θ_{QCD}. Using this spurion formalism one sees that this SUSY breaking is represented by a D-term and thus the mass term μ is infinitely renormalized. Effectively, what happens is that logarithmically divergent terms will be generated by loop corrections.

Relevant for our discussion here is the fact that eq. (2.4) also generates:

$$\int d^4\,\theta\, U(D^2\theta + \bar{D}^2\,\bar{\theta}) = \mu^2 F \quad . \tag{2.5}$$

The implication is that the addition of eq. (2.2) to a SUSY theory leads to a generation of a term linear in the auxiliary field F. Power counting indicates that the coefficient of this term will be logarithmically divergent.

The term linear in the auxiliary field F can be transformed away by redefining A:

$$A(x) \rightarrow A(x) + c \tag{2.6}$$

where c is a suitably chosen constant. This shift in the field A leads however to a shift in mass which is common to all physical components of the chiral superfield ϕ, including the fermions. Since the coefficient of the linear F-term will in general be logarithmically divergent, so will the constant c and thus also mass renormalization for the component fermion. However F and therefore ϕ have to be charge neutral (i.e., singlets under the gauge group) since otherwise a linear F-term could not appear in the Lagrangian. In particular, ϕ does not carry any color quantum numbers; therefore no infinite $\bar{\theta}$ renormalization can occur this way.

Once again, we appear to have a violation of Symanzik's Theorem. In this case, we have added a soft symmetry breaking dimension-2 term and have generated a new divergence in a dimension-3 fermion mass term. Actually, what has happened is that the introduction of the term in eq. (2.2) has led to the generation of an infinity in a dimension-1 term linear in A [see eq. (2.6)] which is allowed by Symanzik's Theorem. That is, the A-field has acquired an infinite vacuum expectation value. Because of the existence of $A\bar{\psi}\psi$ terms in the theory, when the shift in eq. (2.6) is made, a divergent fermion mass term results. However, after introducing the symmetry-breaking it is sufficient to renormalize only the dimension-1 and -2 terms to render the theory finite.

Second, an explicit mass term can be given to the gluino, which is the (Majorana) spinor component of a vector superfield:

$$\mathcal{L}_{\mu} = \mu \ \bar{\lambda} \ \lambda \tag{2.7}$$

The mass μ gets logarithmically renormalized, in general both in its real and its imaginary parts. It does not affect the quark mass term directly and therefore, at first sight, might appear to be irrelevant for the neutron electric dipole moment. However gluinos appear in the color anomaly of the R-current; massless gluinos therefore allow $\bar{\theta}$ to be rotated to zero. On the other hand, when gluinos are massive, the $\bar{\theta}$ dependence cannot be rotated away anymore and $\bar{\theta}$ is a physical parameter. A logarithmically divergent imaginary part of μ can be transformed into a logarithmically divergent $F \cdot \tilde{F}$ term via the color anomaly of the R current and such a $F \cdot \tilde{F}$ term (before renormalization) will contribute a divergent amount to the electric dipole moment of the neutron.

Third, it has also been observed that a logarithmically divergent gluino mass is generated in certain supergravity theories [16]. Such theories appear at low energies as globally supersymmetric theories accompanied by various soft-SUSY-breaking terms [17]. Among the soft-SUSY-breaking terms generated is a three-scalar interaction $H\,\tilde{q}\,\tilde{q}$ (involving one Higgs scalar and two scalar quarks). This interaction term also breaks the R-invariance of eq. (2.1). Since the R-invariance protects the gluinos from gaining a mass, the effect of the $H\,\tilde{q}\,\tilde{q}$ vertex is the generation of a finite gluino mass at one loop. In fact, at two loops, the diagram of fig. 1 generates a logarithmically divergent gluino mass. The conclusion is then the same as above, namely the $\bar{\theta}$ parameter contains a logarithmically divergent piece which must be renormalized away. Note that this is an explicit example where a softly-broken R-invariance fails to prevent $\delta\theta$ from being infinite.

3. CONCLUSIONS

Softly broken SUSY, being more flexible that spontaneously broken SUSY, can conveniently be employed to construct models for particle physics [18]. The price to be paid for such convenience lies in the *ad hoc* nature of the symmetry breaking. Here we have concentrated on a more specific issue, namely the strong CP problem. Despite some claims in the literature, we have found that softly broken SUSY does not yield a natural solution of the strong CP problem: in general $\bar{\theta}$ will be subjected to infinite renormalizations. In a more ambitious approach, soft-SUSY breaking terms in the low energy theory can be a result of a spontaneously broken supergravity theory coupled to matter. In these theories, one can argue that these infinities should be cut off at the Planck scale (M_P); however the contributions from the finite one loop term [19] and from the two loop term cut off at M_P tend to be too large by orders of magnitude than the experimental upper limit $\theta_{exp} \lesssim 10^{-9}$. Even in a minimal version where certain mixing angles are put to zero as much as possible one still finds $\bar{\theta} \sim 10^{-9}$ [20]. In the calculations of ref. [19], it was shown that the computed value of $\bar{\theta}$ could be made sufficiently small if the masses of the SUSY partners of the known states were much heavier than 1 TeV. However, in this case, it would be very difficult to use supersymmetry to explain why $m_W \ll M_P$ (i.e., the hierarchy problem) — the very reason why supersymmetry has been so intensely studied in recent years [18]. We conclude that in models of softly-broken SUSY, it is difficult to understand why $\bar{\theta}$ is smaller than its experimental upper limit.

Therefore we see three reasonable options to address the strong CP problem in the context of SUSY theories:

(a) Ignore it since it can be renormalized away.

(b) Arrange a spontaneously broken global SUSY model in such a way that $\bar{\theta}$ is calculable and sufficiently small [11].

(c) Insist on having an R-invariance broken spontaneously at some large mass scale which would lead to the existence of superlight axions.

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FIGURE CAPTION

1. Logarithmically divergent contribution to the gluino mass which occurs in certain low energy supergravity theories (see ref 16) .

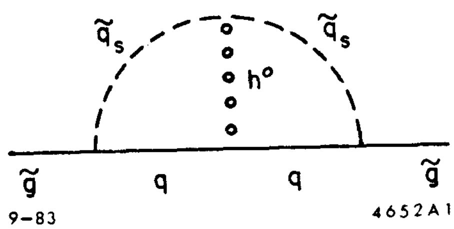

Fig. 1