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GRAVITATIONAL GAUGE FIELDS AND THE COSMOLOGICAL CONSTANT*

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ABSTRACT

We describe field theories for which the action is completely independent of the metric and connection of the space-time manifold. The metric in our approach is no more a fundamental field than a hadron field is a fundamental field in QCD. The fundamental fields in the action are 0(5) gauge fields and combinations of these fields are interpreted as the metric and connection so that conventional general relativity is obtained. Remarkably, all renormalizable matter actions for scalar, spinor and Yang-Mills gauge fields can be made metric independent. Significantly we find a new elementary symmetry of the action which implies the cosmological constant must vanish. Finally, we discuss the quantum theory resulting from these ideas.

1. Introduction

Suppose we consider a four dimensional space-time manifold with a metric $g_{\mu\nu}$ and connection $\Gamma^{\gamma}_{\alpha\beta}$. On this geometric manifold we define fields with total action S, a polynomial in these fields. Our fundamental assumption is that S is independent of the metric and the connection.

One the face of it this assumption seems physically absurd. After all, the total stress-energy tensor, $\theta_{\mu\nu}$ — a fundamental object in any field theory — is defined by the response of the action to a metric variation, $\delta S = \sqrt{g}\,\theta_{\mu\nu}\delta g^{\mu\nu}$. If the action S is independent of the metric how is it possible to include gravity? Remarkably, these objections can be circumvented if one adopts a new viewpoint.

The requirement that the action is polynomial in the fields[1] and independent of the metric and connection implies it is of the form

$$S = \int \epsilon^{\mu\nu\lambda\delta} L_{\mu\nu\lambda\delta} d^4x \tag{1}$$

Here $\epsilon^{\mu\nu\lambda\delta}$ is the permutation symbol, a tensor density with $\epsilon^{1234} = +1$, and $L_{\mu\nu\lambda\delta}$ is a covariant tensor independent of $g_{\mu\nu}$ and $\Gamma^{\gamma}_{\mu\nu}$ and a polynomial in any other fields. We show that the action of all conventional renormalizable field theories of matter can be reexpressed in this form and that the conventional theory of gravity is completely recovered. Furthermore, we find an exact symmetry of the gravitational action that implies the cosmological constant vanishes. The key to our construction procedure is the gravitational gauge field.

2. Gravitational 0(5) Gauge Fields

We adopt the following conventions: the metric signature is Euclidean (we assume we can rotate back to Minkowski space) and Greek indices $\mu = 1\ldots 4$

refer to space-time; upper case latin, $A = 1 \dots 5$ refer to 0(5) indices and lower case latin $a = 1 \dots 4$ to an 0(4) subgroup of 0(5); we also use the alternating notation on indices, $[AB] = AB - BA$.

We introduce the ten "gravitational gauge fields" $\omega_\mu^{AB} = -\omega_\mu^{BA}$ which transform as the adjoint representation of 0(5) and the five fields ϕ^A which transform as the vector representation. The field ϕ^A is assumed to be odd under CP and if we assume that the action S, given by (1), is CP even then S must have an odd number of ϕ^A fields.

The most general possible action that satisfies these criteria and exact 0(5) gauge invariance, is [2]

$$S = \int \mathcal{L}\, d^4x$$

with

$$\mathcal{L} = \epsilon^{\mu\nu\lambda\delta}\epsilon^{ABCDE}\Big(\alpha_1 R_{\mu\nu}^{AB} R_{\lambda\delta}^{CD}\phi^E + \alpha_2 R_{\mu\nu}^{AB}\epsilon_\lambda^C\epsilon_\delta^D\phi^E + \alpha_3\epsilon_\mu^A\epsilon_\nu^B\epsilon_\lambda^C\epsilon_\delta^D\phi^E\Big) \quad (2)$$

where the gauge field strength is

$$R_{\mu\nu}^{AB} = \partial_{[\mu}\omega_{\nu]}^{AB} + \omega_{[\mu}^{AC}\omega_{\nu]}^{CB}$$

and

$$\epsilon_\mu^A \equiv D_\mu\phi^A = \partial_\mu\phi^A + \omega_\mu^{AB}\phi^B$$

where ϵ^{ABCDE} is the 0(5) anti-symmetric permutation symbol and the α_i are polynomials in $(\phi^A)^2$. If we drop the requirement of the discrete CP invariance then we may add to the action a term[3] $\alpha_4\epsilon^{\mu\nu\lambda\delta} R_{\mu\nu}^{AB}\phi^B R_{\lambda\delta}^{AB}\phi^B$ which is even in the ϕ^A field; but we do not do so here. The fact that the action is necessarily restricted to only three terms is a consequence of the requirement that it be independent of the geometry of the manifold. The restrictive power of this natural condition is apparent.

Before proceeding we will motivate our approach by remarking that the ten components of the 0(5) gravitational gauge fields ω_μ^{AB} will be subsequently identified as the six components of the connection ω_μ^{ab} and the four components, ω_μ^{5a}, with the vierbein, e_μ^a. Then the terms in the action (2) can be identified as a topological invariant, the Hilbert action and the cosmological term. In this way the relation to general relativity is established.

The magnitude of the vector ϕ^A is arbitrary. In the following we will impose the gauge invariant constraint

$$\phi^A \phi^A = M^2 \tag{3}$$

which can be implemented as the equation of motion of an additional 0(5) scalar, a Lagrange multiplier. Alternatively if we left the magnitude arbitrary (3) could follow as an equation of motion and then the magnitude fluctuates about this minimum. This is a different theory which we do not consider here.[4]

Imposing the normalization (3) has as an immediate consequence that ϕ^A becomes a gauge artifact. Using the 0(5) gauge rotation freedom we have $\phi^5 = M$, $\phi^a = 0$. We refer to this gauge choice as the "physical gauge".[5] This does not completely fix the full 0(5) gauge invariance because we are still free to perform arbitrary 0(4) gauge rotations.

A second consequence of the normalization condition (3) is that the α_i in the action (2) are just constants. We also note that $\phi^A \epsilon_\mu^A = \phi^A D_\mu \phi^A = 0$.

In the physical gauge this theory retains an exact 0(4) invariance and it is convenient to introduce a few definitions that will aid us in exposing this 0(4) gauge structure. We define

$$\hat{R}^{AB}_{\mu\nu} = R^{AB}_{\mu\nu} + \frac{1}{M^2}\epsilon^{[A}_{\mu}\epsilon^{B]}_{\nu} \tag{4}$$

and a new gauge derivative ∇_μ which acts as 0(5) gauge vectors ξ^A according to

$$\nabla_\mu \xi^A = D_\mu \xi^A + \frac{1}{M^2}\phi^{[A} D_\mu \phi^{B]} \xi^B \tag{5}$$

with obvious generalizations to gauge tensors. One can check that

$$\nabla_\mu \phi^A = 0\,. \tag{6}$$

The Bianchi identity

$$\epsilon^{\mu\nu\lambda\delta} D_\nu R^{AB}_{\lambda\delta} = 0 \tag{7}$$

implies

$$\epsilon^{\mu\nu\lambda\delta}\epsilon^{ABCDE}\phi^A \nabla_\nu \hat{R}^{BC}_{\lambda\delta} = 0 \tag{8}$$

It is easily checked that in the physical gauge $\hat{R}^{ab}_{\mu\nu}$ is just the 0(4) field strength,

$$\hat{R}^{ab}_{\mu\nu} = \partial_{[\mu}\omega^{ab}_{\nu]} + \omega^{ac}_{[\mu}\omega^{cb}_{\nu]} \tag{9}$$

establishing the utility of the definitions.

In terms of $\hat{R}^{AB}_{\mu\nu}$ the action density $\mathcal{L}$ becomes

$$\mathcal{L} = \epsilon^{\mu\nu\lambda\delta}\epsilon^{ABCDE}\Big(\alpha_T \hat{R}^{AB}_{\mu\nu}\hat{R}^{CD}_{\lambda\delta}\phi^E + \alpha_H \hat{R}^{AB}_{\mu\nu}\epsilon^{C}_{\lambda}\epsilon^{D}_{\delta}\phi^E + \alpha_C \epsilon^{A}_{\mu}\epsilon^{B}_{\nu}\epsilon^{C}_{\lambda}\epsilon^{D}_{\delta}\phi^E\Big) \tag{10}$$

with $\alpha_T = \alpha_1$, $\alpha_H = \alpha_2 - 4\alpha_1/M^2$, $\alpha_C = \alpha_3 - 2\alpha_2/M^2 + 4\alpha_1/M^4$. The first term of (10) is the 0(5) topological density and can be used to classify the field configurations by an index n corresponding to a member of the set of integers z according to $\pi_3(0(5)) = z$. One finds for its contribution to the action

$$S_T = 128\pi^2 \alpha_T M n \ . \tag{11}$$

The other two terms in (10) are not topological densities and require a further interpretation.

3. Geometrical Interpretation

The metric tensor has two related but distinct roles in physics. First, it determines the overall geometry of space-time—the arena of various quantum fields. Second, it is a field in its own right. However the metric in conventional field theory is unlike any other field in so much as it is necessarily present in the action. In our approach this distasteful dualism between the metric and other fields is eliminated. The metric is not even present in the action as a fundamental field—it is derived from the gravitational gauge fields. In some ways our approach resembles that taken in the strong interaction field theory. Quantum chromodynamics, the gauge theory of the strong interaction, in principle describes the dynamics of the hadrons. Yet hadronic fields do not appear in the fundamental action. Hadronic fields are complicated representations in terms of the fundamental quark and gauge fields that do appear in the action. From this modern viewpoint it was a mistake to ever put hadronic fields in an action.

The viewpoint we adopt here is similar except that we apply it to gravity instead of the strong interaction. The fundamental fields are the gravitational gauge fields while the action is independent of the metric and connection. But the metric and connection may be related to these gauge fields and we do so by making two postulates. These postulates are definitions.

First is the tetrad postulate

$$\mathcal{D}_\lambda \epsilon_\nu^A \equiv \nabla_\lambda \epsilon_\nu^A - \Gamma^\alpha_{\lambda\nu} \epsilon_\alpha^A = 0 \tag{12}$$

which defines the connection $\Gamma^\alpha_{\lambda\nu}$; second is the metric postulate

$$M^4 g_{\mu\nu} \equiv \epsilon_\mu^A \epsilon_\nu^A \tag{13}$$

which defines the metric tensor. The consistency of the tetrad postulate is easily checked,

$$\phi^A \mathcal{D}_\lambda \epsilon_\nu^A = 0 \ .$$

It follows from these postulates that the covariant derivative of the metric vanishes

$$\mathcal{D}_\lambda g_{\mu\nu} = 0$$

so we have a metric space; but the torsion tensor

$$S^\alpha_{\lambda\nu} = \frac{1}{2} \Gamma^\alpha_{[\lambda\nu]} \tag{14}$$

does not in general vanish.

The torsion tensor has 24 independent components and is a reducible representation under the local 0(4) group according to $24 = 4^V + 4^A + 16$. Introducing the vector S_ν, the axial vector A_ν and the 16 dimensional representation tensor $G_{\alpha\lambda\nu}$ which satisfies

$$G_{\alpha\lambda\nu} + G_{\alpha\nu\lambda} = 0$$

$$G_{\alpha\lambda\nu} + G_{\lambda\nu\alpha} + G_{\nu\alpha\lambda} = 0$$

$$G^\alpha_{\alpha\nu} = 0$$

one can write the torsion tensor

$$S^{\alpha}_{\lambda\nu} = \delta^{\alpha}_{[\lambda} S_{\nu]} + \epsilon^{\alpha}_{\lambda\nu\delta} A^{\delta} + G^{\alpha}_{\lambda\nu} \tag{15}$$

where we use the metric to raise and lower indices. If one puts matter into some geometrical gravity theories which are elementary extensions of Einstein's theory one finds the torsion does not vanish.[6] Scalar fields, through their angular momentum, contribute to S_{ν}; fermions, through their axial currents contribute to A_{δ}; gauge fields contribute to $G^{\alpha}_{\lambda\nu}$. Torsion will be a part of this gauge theory of gravity as well, for requiring it to vanish would imply arbitrarily restricting gauge degrees of freedom.

We note that the metric, connection and torsion are related in the usual way for metric spaces. The connection may be written

$$\Gamma^{\kappa}_{\mu\lambda} = \left\{ {\kappa \atop \mu\lambda} \right\} + S_{\mu\lambda}{}^{\kappa} - S_{\lambda}{}^{\kappa}{}_{\mu} + S^{\kappa}{}_{\mu\lambda}$$

where the Christoffel symbol is

$$\left\{ {\kappa \atop \mu\lambda} \right\} = \frac{1}{2}\, g^{\kappa\sigma}\Big(\partial_{\mu} g_{\lambda\sigma} + \partial_{\lambda} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\lambda}\Big)$$

Our choice of definition for the metric and connection, (12) and (13), is not gratuitous. These definitions, if used in the field equations for the gravitational gauge fields, imply the metric and connection satisfy the Einstein equations with a cosmological term. This is most easily seen if we go to the physical gauge. Then $\hat{R}^{ab}_{\mu\nu}(\omega)$ is the 0(4) field strength given by (9) and

$$\epsilon^{5}_{\mu} = 0 \qquad \epsilon^{a}_{\mu} = M^2 e^{a}_{\mu} = M\omega^{a5}_{\mu} \tag{16}$$

where e^a_μ is the vierbein. Using $\epsilon^{5abcd} = \epsilon^{abcd}$ the action density (10) becomes

$$\mathcal{L} = M\epsilon^{\mu\nu\lambda\delta}\epsilon^{abcd}\Big(\alpha_T \hat{R}^{ab}_{\mu\nu}\hat{R}^{cd}_{\lambda\delta} + M^4\alpha_H \hat{R}^{ab}_{\mu\nu} e^c_\lambda e^d_\delta + M^8\alpha_C e^a_\mu e^b_\nu e^c_\lambda e^d_\delta\Big) \tag{17}$$

The definition of the Riemann tensor is

$$R^\mu_{\nu\rho\sigma}(\Gamma) = \partial_\rho \Gamma^\mu_{\nu\sigma} - \partial_\sigma \Gamma^\mu_{\nu\rho} + \Gamma^\lambda_{\nu\sigma}\Gamma^\mu_{\nu\rho} - \Gamma^\lambda_{\nu\rho}\Gamma^\mu_{\lambda\sigma} . \tag{18}$$

Using the vierbein e^a_μ and its inverse, $e^{a\mu}$ to raise and lower indices, using the tetrad and metric postulates and (16) one can show by a lengthy but direct calculation

$$\hat{R}^{ab}_{\mu\nu}(\omega) = R^\alpha_{\tau\mu\nu}(\Gamma) e^a_\alpha e^{b\tau} . \tag{19}$$

The action density (17) becomes

$$\begin{aligned}\mathcal{L} = {} & M\alpha_T (g)^{-1/2}\epsilon^{\mu\nu\lambda\delta}\epsilon^{\alpha\beta\gamma\tau} R_{\alpha\beta\mu\nu} R_{\gamma\tau\lambda\delta} \\ & - 4M^5\alpha_H (g)^{1/2} R + 24 M^9 \alpha_C (g)^{1/2}\end{aligned} \tag{20}$$

where we have used $\epsilon^{\mu\nu\lambda\delta}\epsilon_{abcd} e^c_\lambda e^d_\delta = 2e\, e^{[\mu}_a e^{\nu]}_b$ where e is the determinant of the vierbein, $e = \frac{1}{4!}\epsilon^{\mu\nu\lambda\delta}\epsilon_{abcd} e^a_\mu e^b_\nu e^c_\lambda e^d_\delta$, $g = e^2$ and the scalar curvature is $R = \delta^\sigma_\mu g^{\nu\rho} R^\mu_{\nu\rho\sigma}(\Gamma)$.

The first term in (20) is just the topological density for the Gauss-Bonnet integral; the second term is the Hilbert action if we identify the gravitational constant κ as

$$\kappa^{-2} = 8M^5\alpha_H \tag{21}$$

and the third term is the cosmological term, $\Lambda\sqrt{g}$ with

$$\Lambda = 24 M^9 \alpha_C . \tag{22}$$

The original action has been completely reexpressed in terms of the metric and connection and in this way the relation to general relativity in the first order formalism is established. Notice, however, that unlike conventional relativity, our assumptions on the original 0(5) invariant action do not permit dynamical R^2 type terms in the action.

Sometimes one encounters statements to the effect that if instead one considers for the gauge group the Wigner-Inonü contraction of the 0(5) group to the Poincaré group then the cosmological term is absent. Such statements are misleading. They generally ignore the full 0(5) gauge invariance which implies a possible third term in the action (2), a term which upon performing the Wigner-Inonü contraction survives and can be identified with the cosmological term. Furthermore, for topologically non-trivial field configurations the action is infinite in the contraction limit. The Wigner-Inonü contraction is considered in detail in Appendix A and we do not consider it further here.

4. Equations of Motion

The equations of motion may be obtained by variation of the action (10) with respect to the ten independent 0(5) gauge fields ϵ^a_μ, ω^{ij}_μ and the four independent components of the vector field ϕ^a. After this variation is carried out we express our results in the physical gauge. The result may be expressed in terms of the currents $T^{\mu i}, T^{\mu ij}$ and S^i defined by

$$\delta S = \int \Big(T^{\mu i}\delta\epsilon^i_\mu + T^{\mu ij}\delta\omega^{ij}_\mu + S^i\delta\phi_i\Big)d^4x \,. \tag{23}$$

We find

$$T^{\delta d} = M\epsilon^{\mu\nu\lambda\delta}\epsilon^{abcd}\Big(2\alpha_H\,\hat{R}^{ab}_{\mu\nu}\,\epsilon^c_\lambda + 4\alpha_C\epsilon^a_\mu\epsilon^b_\nu\epsilon^c_\lambda\Big)$$

$$T^{\nu ab} = -2\alpha_H M\epsilon^{\mu\nu\lambda\delta}\epsilon^{abcd}S^c_{\mu\lambda}\epsilon^d_\delta \tag{24}$$

$$S^d = \epsilon^{\mu\nu\lambda\delta}\epsilon^{abcd}\Big(+\frac{4\alpha_H}{M}S^a_{\mu\nu}\epsilon^b_\lambda\epsilon^c_\delta - \alpha_H M\,\hat{R}^{ab}_{\mu\nu}\,S^c_{\delta\lambda} + 6\alpha_C M S^a_{\mu\nu}\epsilon^b_\lambda\epsilon^c_\delta\Big)$$

with

$$S^a_{\mu\nu} = \dot{D}_{[\mu}\,\epsilon^c_{\nu]} \tag{25}$$

where $\dot{D}_\mu$ is the 0(4) gauge field derivative and $\hat{R}^{ab}_{\mu\nu}(\omega)$ is the 0(4) field strength. Equation (25) just defines the torsion; from the tetrad postulate (12) we obtain the relation

$$2M^4 S_{\alpha\mu\nu} = \epsilon^c_\alpha S^c_{\mu\nu}\ . \tag{26}$$

The equations of motion, (24), are not all independent. We see directly from (24) that the sources are related by

$$2\epsilon^a_\nu T^{\nu ad} + M^2(\dot{D}_\delta\,T^{\delta d} + S^d) = 0\ . \tag{27}$$

Use is made of the 0(4) Bianchi identity $\epsilon^{\mu\nu\lambda\delta}\,\dot{D}_\nu\,\hat{R}^{ab}_{\lambda\delta} = 0$ in proving this result. This result implies that if the first two equations of motion are satisfied then so is the equation for S^i. But this comes as no surprise since we used the 0(5) gauge freedom to fix the field $\phi^a = 0$. Thus its associated equation of motion is redundant.

In the absence of matter the sources all vanish. Then one finds that the torsion vanishes, $S^a_{\mu\nu} = 0$. This condition enables us to solve for the connection purely in terms of the metric and the standard source free Einstein equations with a cosmological constant are obtained. In the presence of matter the torsion does not in general vanish and the equations for ϵ^{α}_{μ} and ω^{ij}_{μ} are completely independent.

Next we consider the coupling of matter. Our fundamental criterion is that the matter contribution to the action must not only be 0(5) gauge invariant but also be independent of the metric and connection. Remarkably, all the renormalizable matter field theories can satisfy these requirements.

5. Matter Couplings

We consider separately scalars, spin one-half fermions and Yang-Mills fields and show that their 0(5) invariant matter actions, which are independent of the geometry, reduce to the conventional field theories in flat space.

5.1 Scalar Field

For simplicity we consider a single scalar field σ; the generalization to include internal symmetries is trivial. The single scalar field is introduced as an 0(5) vector σ^A so that $\sigma = \phi^A\sigma^A/M$. In the physical gauge $\sigma = \sigma^5$. The other four components σ^a, we will see, are identified with the derivatives $4Me^a_\mu\sigma^a = \partial_\mu\sigma$. The action we obtain for the kinetic form of the scalar field will be in the first order formalism. We do not consider the most general possible action, it being sufficient to show the conventional theory is obtained from at least one action.

Consider

$$\begin{aligned} S_{s_1} &= \int d^4x\epsilon^{\mu\nu\lambda\delta}\epsilon^{ABCDE}\epsilon^A_\mu\epsilon^B_\nu\epsilon^C_\lambda\nabla_\delta\sigma^D\sigma^E \\ S_{s_2} &= \int d^4x\epsilon^{\mu\nu\lambda\delta}\epsilon^{ABCDE}\epsilon^A_\mu\epsilon^B_\nu D_\lambda(\phi^C\sigma^D)D_\delta\sigma^E\ . \end{aligned} \tag{28}$$

In the physical gauge, introducing the vierbein as in (16) these actions become

$$S_{s_1} = \int d^4x\, 6eM^6(\sigma e^{\sigma d}\, \dot{D}_\sigma \sigma^d - e^{\sigma d}\partial_\sigma \sigma \sigma^d)$$

$$S_{s_2} = \int d^4x\, [6eM^6(\sigma e^{\sigma d}\, \dot{D}_\sigma \sigma^d + e^{\sigma d}\partial_\sigma \sigma \sigma^d \tag{29}$$

$$- M\sigma^d\sigma^d) + 2eM^5 e^{\rho[c} e^{\sigma d]}\, \dot{D}_\rho \sigma^c \dot{D}_\sigma \sigma^d] \, .$$

In the flat space limit $e^{\rho c} = \delta^{\rho c}, e^c_\rho = \delta^c_\rho, \dot{D}_\sigma = \partial_\sigma$, etc. and one finds

$$S_{s_1} - S_{s_2} = \int d^4x 6M^6(-\partial_\rho \sigma \sigma^\rho + 2M\sigma^\rho\sigma^\rho) \tag{30}$$

where $\sigma^\rho = e^{\rho a}\sigma^a$. This is just the first order form for the scalar field kinetic energy. Variations with respect to $\delta\sigma_\rho$ implies $\sigma_\rho = \partial_\rho \sigma/4M$ and we see that the conventional kinetic energy term of the scalar field is

$$S_S^{KE} = \frac{2}{3M^5}(S_{S_2} - S_{S_1}) = \int d^4x\, \frac{1}{2}\,(\partial_\rho\sigma)^2\, . \tag{31}$$

The potential term for scalar fields may be written

$$S_S^P = \int d^4x\, \epsilon^{\mu\nu\lambda\delta}\epsilon^{ABCDE} V(\phi^A \cdot \sigma^A)\epsilon^A_\mu \epsilon^B_\nu \epsilon^C_\lambda \epsilon^D_\delta \phi^E \tag{32}$$

where $V(\phi^A \cdot \sigma^A)$ is a fourth order polynomial. So the conventional, renormalized scalar theory can be cast into the required form.

5.2 Fermi Field

To include fermions we must consider $Sp(4)$ instead of $0(5)$ and introduce the $Sp(4)$ hermitian 4×4 matrices

$$\gamma^\alpha = \begin{pmatrix} 0 & \sigma^a \\ \sigma^a & 0 \end{pmatrix} \quad \gamma^4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \gamma^5 = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{33}$$

where $a = 1, 2, 3$ and σ^a are the Pauli matrices. These five matrices satisfy

$$[\gamma^A, \gamma^B]_+ = 2\delta^{AB} \tag{34}$$

and

$$\sigma^{AB} = \frac{1}{4i} [\gamma^A, \gamma^B]_- \tag{35}$$

transforms like the adjoint representation of 0(5). Spinors transform in the fundamental 4 dimensional representation of $Sp(4)$ and their $Sp(4)$ gauge derivative is

$$D_\mu \psi = \left(\partial_\mu + \frac{1}{2}\, \omega_\mu^{AB} \sigma^{AB}\right)\psi \tag{36}$$

which satisfies

$$i[D_\mu, D_\nu]\psi = \frac{1}{2}\, \sigma^{AB} R_{\mu\nu}^{AB} \psi\ . \tag{37}$$

It is useful to introduce the hermitian matrix γ_6 which is defined by

$$M\gamma_6 = \frac{i}{4!}\, \epsilon^{ABCDE} \phi^A \gamma^B \gamma^C \gamma^D \gamma^E \tag{38}$$

satisfying

$$\begin{aligned} M[\gamma_6, \gamma^A]_+ &= 2\phi^A \\ iM[\gamma_6, \sigma^{AB}]_- &= \phi^{[A} \gamma^{B]} \\ \nabla_\mu \gamma_6 &= 0\ . \end{aligned} \tag{39}$$

This matrix, in the physical gauge, is $\gamma_6 = \gamma^5$. It generates chiral transformations according to

$$\delta\psi = i\gamma_6 \theta \psi\ . \tag{40}$$

The chiral invariant action for the fermi field is now easily written down,

$$S_F^{KE} = \frac{-i}{6M^7} \int d^4x \, \epsilon^{\mu\nu\lambda\delta} \epsilon^{ABCDE} \psi^+ \gamma^A \nabla_\mu \psi \phi^B \epsilon_\nu^C \epsilon_\lambda^D \epsilon_\delta^E \ . \tag{41}$$

In the physical gauge and flat space limit

$$S_F^{KE} = i \int d^4x \, \psi^+ \not{\partial} \psi \tag{42}$$

the conventional result. Yukawa couplings to the scalar field or mass terms may be accommodated by writing

$$S_F^Y = \frac{1}{24M^9} \int d^4x \, \epsilon^{\mu\nu\lambda\delta} \epsilon^{ABCDE} \Gamma \, \epsilon_\mu^A \epsilon_\nu^B \epsilon_\lambda^C \epsilon_\delta^D \phi^E \psi^+ \psi \tag{43}$$

where Γ can stand for 0(5) invariants such as either $\sigma^A \cdot \phi^A$ or m_F, the fermion mass. The generalization to include internal indices for the fermi field is obvious.

5.3 Yang-Mills Fields

Denoting the gauge field transforming as the adjoint representation of G, the Yang-Mills group, by $A_\mu^{\bar{A}}$, (we put a bar over the gauge group index to indicate that it is not an 0(5) index) the field strength is

$$F_{\mu\nu}^{\bar{A}} = \partial_{[\mu} A_{\nu]}^{\bar{A}} + C^{\bar{A}\bar{B}\bar{C}} A_{[\mu}^{\bar{B}} A_{\nu]}^{\bar{C}} \tag{44}$$

where $C^{\bar{A}\bar{B}\bar{C}}$ are the Lie structure constants of G.

In order to include Yang-Mills fields in our formalism we will introduce a tensor $G^{AB\bar{C}}$ transforming as the adjoint under G and the adjoint under 0(5)

$$G^{AB\bar{C}} = -G^{BA\bar{C}} \tag{45}$$

Using ϕ^A we can reduce the number of 0(5) components of $G^{AB\bar{C}}$ from 10 to 6 by the construction

$$T^{AB\bar{C}} = G^{AB\bar{C}} + \frac{1}{M^2} \phi^{[A} G^{B]D\bar{C}} \phi^D \tag{46}$$

so that

$$\phi^A T^{AB\bar{C}} = 0 \ . \tag{47}$$

Then $T^{AB\bar{C}}$ has the same number of components as the field strength and we can apply the first order formalism.

We find

$$\begin{aligned} S_{YM} = \int d^4x\, \epsilon^{\mu\nu\lambda\delta} \epsilon^{ABCDE} \Big(\frac{1}{4M^5} F^{\bar{A}}_{\mu\nu}(A) T^{AB\bar{A}} \epsilon^C_\lambda \epsilon^D_\delta \phi^E \\ - \frac{1}{2M^9} T^{AR\bar{H}} T^{BS\bar{H}} \epsilon^R_\mu \epsilon^S_\nu \epsilon^C_\lambda \epsilon^D_\delta \phi^E \Big) \end{aligned} \tag{48}$$

upon reduction becomes

$$S_{YM} = \int d^4x \Big(F^{\bar{A}}_{\mu\nu}(A) T^{\bar{A}}_{\mu\nu} - T^{\bar{A}}_{\mu\nu} T^{\bar{A}}_{\mu\nu} \Big) \tag{49}$$

where $T^{\bar{A}}_{\mu\nu} = \epsilon^A_\mu \epsilon^B_\nu T^{AB\bar{A}}$. Variation of (49) with respect to $\delta T^{\bar{A}}_{\mu\nu}$ and eliminating $T^{\bar{A}}_{\mu\nu}$ implies the equivalent action is

$$S_{YM} = \frac{1}{4} \int d^4x\, F^{\bar{A}}_{\mu\nu}(A) F^{\bar{A}}_{\mu\nu}(A) \tag{50}$$

the usual Yang-Mills action.

If the fermions or scalar fields transform as an irreducible representation of G they can couple to the Yang-Mills gauge field. This coupling is accomplished by changing the covariant derivatives D_μ and ∇_μ to include the gauge coupling of the Yang-Mills fields. Abelian gauge fields can have a mass term and still be renormalizable and such terms can also be easily written in this formalism.

We conclude that all renormalizable field theories can be included into this formalism. The representations we found satisfy the criteria of metric and connection independence and 0(5) invariance. Upon using the physical gauge and the geometrical interpretation of gravitational gauge fields, we showed that in the flat space limit these representations reduced to the familiar ones. These expressions are not unique. One can certainly construct actions with matter-fields including $\hat{R}_{\mu\nu}^{AB}$ that vanish in the flat space limit. The action for the matter fields is more complicated than that with just the gravitational gauge fields for which our constructive criteria led to a unique action of only a few terms. It remains an unsolved problem if there exist constructive criteria that might limit the form of the action for matter fields as was the case for the gravitational gauge fields.

6. Cosmological Symmetry

We return to the case of pure gravitational gauge fields without matter. Are there any symmetries of the action, given by (10), other than 0(5) invariance?

In order to answer this question we examined field transformations that depend on an infinitesimal 0(5) vector $\alpha^A(x)$. We separate α^A into longitudinal and transverse parts according to

$$\begin{aligned} &\alpha^A = \alpha_T^A + \alpha_L^A \\ \phi^A \alpha_T^A = 0 \quad &\alpha_T^A = \alpha^A - \frac{\alpha^B \cdot \phi^B}{M^2}\, \phi^A \\ &\alpha_L^A = \frac{\alpha^B \cdot \phi^B}{M^2}\, \phi^A \end{aligned} \tag{51}$$

Next we consider all possible variations in the field variables $\delta\omega_\mu^{AB}$ and $\delta\phi^A$ which are proportional to α^A, but which do not require the introduction of a metric. We may independently consider the longitudinal and transverse variations.

— Since $\phi^A \delta\phi^A = 0$, $\delta\phi^A$ cannot depend on the longitudinal parameter α_L^A and for $\delta\omega_\mu^{AB}$ we can have

$$\delta_L\omega_\mu^{AB} = \alpha_L^{[A} D_\mu \phi^{B]} \ . \tag{52}$$

The change in the action is

$$\begin{aligned}\delta_L S = \int d^4x\, \epsilon^{\mu\nu\lambda\delta} \epsilon^{ABCDE}\Big(&-2\alpha_H(\phi\cdot\alpha_L) R_{\mu\nu}^{AB}\epsilon_\lambda^C\epsilon_\delta^D\phi^E \\ &- 4\alpha_C(\phi\cdot\alpha_L)\epsilon_\mu^A\epsilon_\nu^B\epsilon_\lambda^C\epsilon_\delta^D\phi^E\Big) \ .\end{aligned} \tag{53}$$

We find no invariance for the longitudinal variations unless $\alpha_C = \alpha_H = 0$.

For the transverse variations there are several possibilities

$$\begin{aligned}\delta_1\phi^A &= M\alpha_T^A \\ \delta_2\omega_\mu^{AB} &= \phi^{[A} D_\mu \alpha_T^{B]} \\ \delta_3\omega_\mu^{AB} &= \alpha_T^{[A} D_\mu \phi^{B]} = \alpha_T^{[A}\epsilon_\mu^{B]} \\ \epsilon_{[\lambda}^{[C}\delta_4\omega_{\mu]}^{AB]} &= M^2\alpha_T^{[C}\hat{R}_{\lambda\mu}^{AB]}\end{aligned} \tag{54}$$

where $[ABC]$ means anti-symmetrization in all three indices. We note that the combination $\delta_1 + \delta_2 - \delta_3$ is just an infinitesimal gauge transform with gauge transformation parameters $\theta^{AB} = \phi^{[A}\alpha_T^{B]}$. It is straightforward to calculate the change in the action S due to these variations. The result is

$$\begin{aligned}\delta_1 S &= -\frac{1}{4}\alpha_H L_1 - (2\alpha_H + 3M^2\alpha_C)L_2 \\ \delta_2 S &= \frac{1}{4}\alpha_H L_1 + 3M^2\alpha_C L_2 \\ \delta_3 S &= -2\alpha_H L_2 \\ \delta_4 S &= -\alpha_C M^2 L_2 + \frac{1}{2}\alpha_H L_1\end{aligned} \tag{55}$$

where

$$L_1 = M^4 \int d^4x \, \epsilon^{\mu\nu\lambda\delta} \epsilon^{ABCDE} \alpha_T^A \, \hat{R}_{\mu\nu}^{BC} \, \hat{R}_{\lambda\delta}^{DE}$$

$$L_2 = M^2 \int d^4x \, \epsilon^{\mu\nu\lambda\delta} \epsilon^{ABCDE} \alpha_T^A \, \hat{R}_{\mu\nu}^{BC} \, \epsilon_\lambda^D \epsilon_\delta^E$$

and α_H and α_C are parameters appearing in S as coefficients of the Hilbert action and cosmological term respectively. It is evident from these relations that additional symmetry beyond simple gauge invariance may be present.

For example we note that (55) implies

$$\delta' S = \Big(\frac{5}{4}\alpha_C M^2 \delta_3 + \alpha_H \delta_2 - \frac{1}{2}\alpha_H \delta_4\Big) S = 0 \; . \tag{56}$$

This symmetry depends explicitly on the parameters α_H and α_C.Symmetries that depend explicitly on Lagrangian parameters are not expected to survive renormalization; yet as a classical symmetry of the action it might be interesting to investigate (56).

There are symmetries that imply that either the Hilbert action is absent, $\alpha_H = 0$

$$\delta_3 S = 0 \tag{57}$$

or the cosmological term is absent, $\alpha_C = 0$

$$\delta S = \Big(\delta_2 - \frac{1}{2}\delta_4\Big) S = 0 \tag{58}$$

or both (leaving only the topological invariant). These are the only interesting consequences besides (56) that follow from (55). Since we want to retain the Hilbert action but remove the cosmological term, in conformity with its experimental absence, we will require $\delta S = 0$. We refer to this symmetry as "cosmological symmetry", since it kills the cosmological term.

If we go to the physical gauge in which the action density has the form (17),

$$\mathcal{L} = M\,\epsilon^{\mu\nu\lambda\delta}\epsilon^{abcd}\Big(\alpha_T\,\hat{R}^{ab}_{\mu\nu}\,\hat{R}^{cd}_{\lambda\delta} + M^4\alpha_H\,\hat{R}^{ab}_{\mu\nu}\,e^c_\lambda e^d_\delta + M^8\alpha_C e^a_\mu e^b_\nu e^c_\lambda e^d_\delta\Big) \tag{17}$$

The the effect of the cosmological symmetry in the action is equivalent to

$$\begin{aligned} \delta e^a_\mu &= \dot{D}_\mu\,\alpha^a \\ \delta\omega^{[ij}_{[\mu}\,e^{e]}_{\delta]} &= -\,\hat{R}^{[ij}_{\mu\delta}\,\alpha^{e]}\,. \end{aligned} \tag{59}$$

It is easily checked that $\delta S = 0$ implies the cosmological constant $\alpha_C = 0$. The equation (59) for the 24 variations $\delta\omega^{ij}_\mu$ is not explicit; but (59) is 24 independent equations that can be explicitly solved for $\delta\omega^{ij}_\mu$. This is done in Appendix B.

Using the Noether procedure we can find the conservation law on the sources implied by cosmological symmetry. Defining

$$\begin{aligned} R^{ed}_{ij} &= e^\mu_i e^\nu_j\,\hat{R}^{ed}_{\mu\nu} \\ \overset{**}{R}{}^{ed}_{ij} &= \frac{1}{4}\epsilon^{ed\ell m}\epsilon_{ijrs}R^{\ell m}_{rs} \\ T^{kji} &= e^k_\mu T^{\mu ij} \end{aligned}$$

we obtain from Appendix B, (23) and (59)

$$\dot{D}_\mu\,T^{\mu e} = T^{kij}\Big(\overset{**}{R}{}^{ek}_{ij} + \frac{1}{2}\delta^k_j\overset{**}{R}{}^{er}_{ri} - \frac{1}{2}\delta^k_i\overset{**}{R}{}^{er}_{rj}\Big)\,. \tag{60}$$

If the sources satisfy this equation then the cosmological term vanishes.

Fundamentally, the Hilbert action and the cosmological term are distinguished by the two tensors $\hat{R}^{ab}_{\mu\nu}$ and $T^{ab}_{\mu\nu} = e^{[a}_\mu e^{b]}_\nu$ which, although they have the same index symmetries, $\hat{R}^{ab}_{\mu\nu}$ satisfies the Bianchi identity $\epsilon^{\mu\nu\lambda\delta}\,\dot{D}_\nu\,\hat{R}^{ab}_{\lambda\delta} = 0$ while $T^{ab}_{\mu\nu}$ does not, $\epsilon^{\mu\nu\lambda\delta}\,\dot{D}_\nu\,T^{ab}_{\lambda\delta} = \epsilon^{\mu\nu\lambda\delta}S^{[a}_{\nu\lambda}e^{b]}_\delta$ because of the presence of torsion. In Riemannian geometry there is no torsion and no way to distinguish the cosmological term from the Hilbert action. But in general, if torsion is present, cosmological symmetry is not empty. We note that in the absence of matter the equations of motion imply the torsion vanishs. For this reason it is important to extend the cosmological symmetry to the matter action as well (something we have not done).

The astute reader might note that it is possible to generalize the transformation (59) as follows

$$\begin{aligned} \delta' e^a_\mu &= \dot{D}_\mu \alpha^a \\ \delta' \omega^{[ij}_{[\mu} e^{e]}_{\delta]} &= -R^{[ij}_{\mu\delta} \alpha^{e]} + \Gamma e^{[i}_\mu e^j_\delta \alpha^{e]} \end{aligned} \tag{61}$$

with Γ to be determined by the requirement that $\delta' S = 0$. One finds $\Gamma = -6M^2(\alpha_C/\alpha_H)$ and (61) is equivalent to the previous parameter dependent transformation (56). However, as we remarked before, parameter dependent transformations do not survive renormalization. Furthermore, and more to the point of arguing for the naturalness of cosmological symmetry, we see that only with $\Gamma = 0$ does the transformation (61) satisfy

$$\epsilon^{\lambda\mu\nu\delta} \epsilon^{ijem} \dot{D}_\lambda \left(\frac{\delta\omega^{ij}_\mu e^e_\nu}{\delta\alpha^k} \right) = 0 \tag{62}$$

a consequence of the fact that $\hat{R}^{ab}_{\mu\nu}$ but not $T^{ab}_{\mu\nu}$ satisfies the Bianchi identity. We conclude, on the basis of those observations, that cosmological symmetry is a natural symmetry although only detailed calculations in the quantum theory can ultimately decide the question of the absence of anomalies.

7. Quantum Theory

Here we will make a few speculative observations about the quantum theory. The action for the pure gravitational gauge field is the most general that is consistent with the stated symmetries and metric independence. In order to quantize the theory it is first necessary to remove the gauge degrees of freedom (since these are not to be quantized). The gauge fixing can also be done in a metric independent fashion. We need ten independent conditions since there are ten gauge degrees of freedom. four of these ten are fixed by the physical gauge condition $\phi^A = 0$. The remaining six conditions $A^{ij} = -A^{ji} = 0$ can be of the form

$$A^{ij} = \epsilon^{\mu\nu\lambda\delta}\epsilon^{rst\ell}\omega_\mu^{ij}\epsilon_\nu^r\epsilon_\lambda^s\epsilon_\delta^t\eta^\ell$$

where the four vector η^ℓ defines this axial gauge or

$$A^{ij} = \epsilon^{\mu\nu\lambda\delta}\epsilon_\mu^{[i}\partial_\nu\omega_\lambda^{j]\ell}\epsilon_\delta^\ell$$

in the Lorentz-type gauge.

In view of the fact that no terms can be added to the action without violating metric independence we would conclude that *if a metric independent quantization procedure exists then the effective quantum action and the classical action are identical*, the theory is trivially renormalizable. However such a strong conclusion only follows if the quantization procedure is also metric independent. In the canonical formalism, since it is based on the existence of a Hamiltonian, a component of the stress-energy tensor, the procedure is explicitly metric dependent. In the path integral formulation the metric does not explicitly enter. Yet the path integral is only rigorously defined on a space-time lattice or Euclidean simplex and such a simplex depends on the metric.

If the path integral for the gravitional gauge fields given by

$$Z(J) = \int d[\phi^A]\, d\,[\omega_\mu^{AB}]\delta(\phi^i)\delta(A^{ij})e^{-(S+\int d^4x\epsilon^{\mu\nu\lambda\delta}\omega_\mu^{AB}J_{\nu\lambda\delta}^{AB})}$$

can be rigorously defined without invoking metric concepts then it can only depend upon the topology characterized by some indices. Conceivably it could be done exactly.

Our action S, however, has the characteristic problem of almost all gravity theories — there exist field configurations for which the Euclidean action S is unbounded from below. Then the path integral is undefined because the exponential diverges. In our theory this problem is aggravated by the fact that one cannot even add R^2 type terms to the action to bound it from below.

One point of view that might be adopted is that the effective gravitational gauge field quantum action *is* the classical action. That effective action is also metric independent and is consistent with our previous remarks. However, once one includes matter this can no longer be the case. The constant α_H, in the effective quantum action now becomes a field dependent function of 0(5) invariants like $\phi^A \cdot \sigma^A$, $T^{AB\bar{C}} T^{AB\bar{C}}$ arising from the matter sector. Such functions, however, are metric independent; hence if they are computed in one metric they are known for all metrics. Then one could in principle compute the effective quantum action in perturbation theory. One assumes a suitable background metric and uses this to determine to some order in perturbation theory the unknown field functions appearing in the metric independent effective quantum action. This is analogous to calculating a Lorentz invariant quantity in a special frame of reference — one has, in fact, done a frame independent calculation. This procedure warrants further investigation; it has the promise of becoming the basis of a renormalizable theory of gravity.

8. Conclusions

Our fundamental assumption that the action is independent of the metric and connection, and 0(5) gauge invariant implies a very restrictive form for the gravitational gauge field action. The 0(5) gauge fields admit of a geometrical interpretation consistent with conventional general relativity. All the renormalizable matter theories can be included in this formalism. A simple "cosmological symmetry" has been found that requires the cosmological constant to vanish.

All of the equations we have written down are far more simple in the elegant notation of Cartan's calculus of exterior forms.[7] We have avoided this notational convenience because most physicists are unfamiliar with it.

Several important problems which this work raises remain unsolved. The cosmological symmetry we found is only a partial solution to the "cosmological term problem"[8] because we have not yet extended this symmetry to matter fields. The matter fields can produce a vacuum energy which adds to the conventional cosmological term a constant. Yet the fact that the symmetry exists at all in the gravitational gauge fields is encouraging. Simple models including matter ought to be examined.

The cosmological symmetry is local. Does this imply that for a larger gauge symmetry than 0(5) and its associated gauge field theory the cosmological symmetry is automatic — no cosmological term will appear if the larger gauge theory is interpreted geometrically? The cosmological symmetry might have a gauge theory interpretation in the context of a larger gauge group.

A major problem, and an intriguing one, is whether or not a metric independent quantization procedure exists or if this even makes sense. If so, then the usual problem of quantum gravity might be circumvented. Trying to quantize gravity is like trying to quantize a spin 2 hadron—it is the wrong problem. We made a few speculations about solutions to this problem in the last section.

A strong virtue of gauge theories of gravity is that they imply that gravity, like the strong and electro-weak interactions, is also basically due to gauge fields. Gravity is no longer the "odd man out." This opens the door to total unification of all interactions on an equal footing. Metric theories of gravity do not have this property. The simplest group that has $0(5) \times SU(5)$ as a subgroup is $SU(9)$ and this provides a possible toy model.[9] One may extend the idea of this paper to local supersymmetry by gauging $0Sp(1,5)$ and that may lead to yet further unification.

While such ideas are clearly speculative, in light of the current status of quantum gravity theory, any new outlook seems worthy of investigation.

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Appendix A

Here we consider the Wigner-Inonü contraction of the 0(5) gauge group as applied to our action (2). If the generators of 0(5) are $L^{AB} = -L^{BA}$ and the parameters $\theta^{AB} = -\theta^{BA}$ then the contraction is accomplished by setting

$$P^a = \lambda L^{a5} \quad \theta^{a5} = \lambda \alpha^a$$

$$L^{ij} = L^{ij} \quad \theta^{ij} = \theta^{ij}$$

and letting the contraction parameter, $\lambda \to 0$ with P^a, L^{ij}, α^a and θ^{ij} held fixed. This implies we should also redefine our fields according to

$$\omega_\mu^{5a} = \lambda e_\mu^a \quad \phi^i = \lambda \hat{\phi}^i$$

$$\omega_\mu^{ij} = \omega_\mu^{ij} \quad \phi^5 = \phi^5$$

and as $\lambda \to 0$ $e_\mu^a, \omega^{ij}, \hat{\phi}^i$ and ϕ^5 are held fixed. These fields are then representations of the Poincare group.

In what follows we will not fix the gauge so that the full ten parameter gauge group invariance is maintained. This will be instructive. We may substitute the fields with the λ contraction parameter into the action (2). We also define the constants α_i appearing in (2) to be

$$\alpha_1 = a_1/\lambda^2 \, , \alpha_2 = a_2/\lambda^2 \, , \alpha_3 = a_3/\lambda^4$$

with a_i held fixed as $\lambda \to 0$.

The first term of the action (2) requires some care because it is singular in the contraction limit and one can lose full gauge invariance if it is not treated correctly. In order to treat it correctly one can either write the action in the form (10) and carry out the contraction (in which case the gauge invariance is retained directly) or expand the field ϕ^5 according to

$$\phi^5 = M\sqrt{1 - \frac{\lambda^2}{M^2}\hat{\phi}^2} \simeq M - \frac{\lambda^2}{2M}\hat{\phi}^2 + \cdots$$

substitute into the action (2) and then take the $\lambda \to 0$ limit. If this is done the action density becomes as $\lambda \to 0$

$$\mathcal{L} = \epsilon^{\mu\nu\lambda\delta}\epsilon^{abcd}\Big(\frac{b_1}{\lambda^2}\epsilon^{\mu\nu\lambda\delta}\epsilon^{abcd}\,\hat{R}^{ab}_{\mu\nu}\,\hat{R}^{cd}_{\lambda\delta} + b_2\,\hat{R}^{ab}_{\mu\nu}\,\gamma^c_\lambda\gamma^d_\delta + b_3\gamma^a_\mu\gamma^b_\nu\gamma^c_\lambda\gamma^d_\delta\Big)$$

where the b_i are linear combinations of the a_i and

$$\gamma^a_\mu = e^a_\mu - \frac{1}{M}\dot{D}_\mu(\omega)\,\hat{\phi}^a$$

and $\hat{R}^{ab}_{\mu\nu}$ is the 0(4) field strength. The first term in the contracted action is the Gauss-Bonnet topological density. It is singular as $\lambda \to 0$ but does not contribute to the equations of motion. However, field configurations with index $n \neq 0$ contribute an infinite action as $\lambda \to 0$.

The second and third terms can be identified with the Hilbert action and cosmological term. While it is true that a contribution to the cosmological term arising from the second term of (2) vanishes as $\lambda \to 0$ the third term of (2) contributes a non-vanishing cosmological term as $\lambda \to 0$.

the action we obtain is fully Poincaré invariant. It is obviously 0(4) invariant. Under translations we have

$$\delta\, e^a_\mu = \dot{D}_\mu\,\alpha^a$$

$$\delta\,\hat{\phi}^a = M\alpha^a$$

so that $\delta\gamma^a_\mu = 0$ and the action is trivially translationally invariant. We see that $\hat{\phi}^a$ is a complete gauge artifact under translations and so we can fix the gauge with $\hat{\phi}^a = 0$. Hence $\gamma^a_\mu = e^a_\mu$. But now the translational freedom has been fixed. Either the translational freedom is absent (if we fix the gauge) or invisible (if we do not).

We conclude, in summary, that the Wigner-Inonü contraction (i) produces a singular action proportional to the topological index; (ii) does not eliminate the cosmological term; (iii) the translational gauge invariance is either absence or invisible.

Appendix B

The cosmological symmetry transformation is

$$\begin{aligned} \delta\, e^i_\mu &= \dot{D}_\mu\, \alpha^i \\ \epsilon^{ijed} \delta\, \omega^{ij}_{[\mu} e^e_{\delta]} &= -\epsilon^{ijed}\, \hat{R}^{ij}_{\mu\delta}\, \alpha^e \end{aligned} \qquad (B.1)$$

and here we will solve the second equation for $\delta\omega^{ij}_\mu$. We make use of the inverse vierbein, $e^{i\mu} e^j_\mu = \delta^{ij}$ and define $e^{k\mu}\delta\omega^{ij}_\mu = \delta\omega^{ij}_k$, $e^{\ell\mu} e^{k\delta}\, \hat{R}^{ed}_{\mu\delta} = R^{ed}_{\ell k}$ and also the dual

$$\overset{*}{\xi}{}^{\ell m} = \frac{1}{2}\epsilon^{\ell m r s}\xi^{rs} \ .$$

Then (B.1) reads

$$\delta\overset{*}{\omega}_k{}^{\ell d} - \delta\overset{*}{\omega}{}^{kd}_\ell = -\overset{*}{R}{}^{ed}_{k\ell}\alpha^e$$

so that

$$\epsilon^{stk\ell}\overset{*}{\omega}{}^{\ell d}_k = -\overset{**}{R}{}^{ed}_{st}\alpha^e$$

This is equivalent to

$$\delta\omega^{st}_d + \delta^s_d\omega^{tk}_k - \delta^t_d\delta\omega^{sk}_k = \overset{**}{R}{}^{ed}_{st}\alpha^e$$

from which follows upon contraction and elimination

$$\delta\omega^{st}_d = \left(\overset{**}{R}{}^{ed}_{st} + \frac{1}{2}\delta^t_d\overset{**}{R}{}^{er}_{rs} - \frac{1}{2}\delta^s_d\overset{**}{R}{}^{er}_{rt}\right)\alpha^e \qquad (B.2)$$

which is the explicit solution. We remark that a standard identity relates $\overset{**}{R}$ to R.

References and Notes

1. Without the restriction to polynomial actions we could allow actions of the form $S = \int d^4x(\epsilon^{\mu\nu\lambda\delta}\epsilon^{\alpha\beta\sigma\tau}L_{\mu\nu\lambda\delta\alpha\beta\sigma\tau})^{1/2}$ with L independent of the metric and connection.
2. A related development is found in S. W. MacDowell and F. Mansouri, Phys. Rev. Lett. 38, 739 (1977). These authors consider only an 0(4) subgroup of 0(5).
3. This term, upon reductions described in Section 3, is a torsion-torsion interaction of the form $S^a_{\mu\nu}S^a_{\lambda\delta}\epsilon^{\mu\nu\lambda\delta}$ with $S^a_{\mu\nu}$, the torsion tensor.
4. Theories of this kind have been described by A. Zee, Phys. Rev. Lett 42, 417 (1979). See also L. Smolin, Nucl. Phys. B160, 253 (1979).
5. Such a gauge choice can produce singularities in the 0(5) gauge fields if they have a non-trivial topology.
6. These are reviewed in F. W. Hehl, Paul van der Heyde and G. D. Kerlick, Rev. Mod. Phys. 48, 393 (1976).
7. A review which describes this is T. Eguchi, P. B. Gilkey, A. J. Hanson, Phys. Rep. 66, 213 (1980).
8. Reviewed by A. Zee, Report to 1983 Coral Gables Conference.
9. See the remarks in H. Pagels, Report to 1983 Coral Gables Conference.